AFFTC-TN-60-28
July 1960

USAF FLIGHT TEST PILOT SCHOOL

VOLUME I

USAF
EXPERIMENTAL
FLIGHT TEST PILOT SCHOOL

AERODYNAMICS HANDBOOK FOR PERFORMANCE FLIGHT TESTING

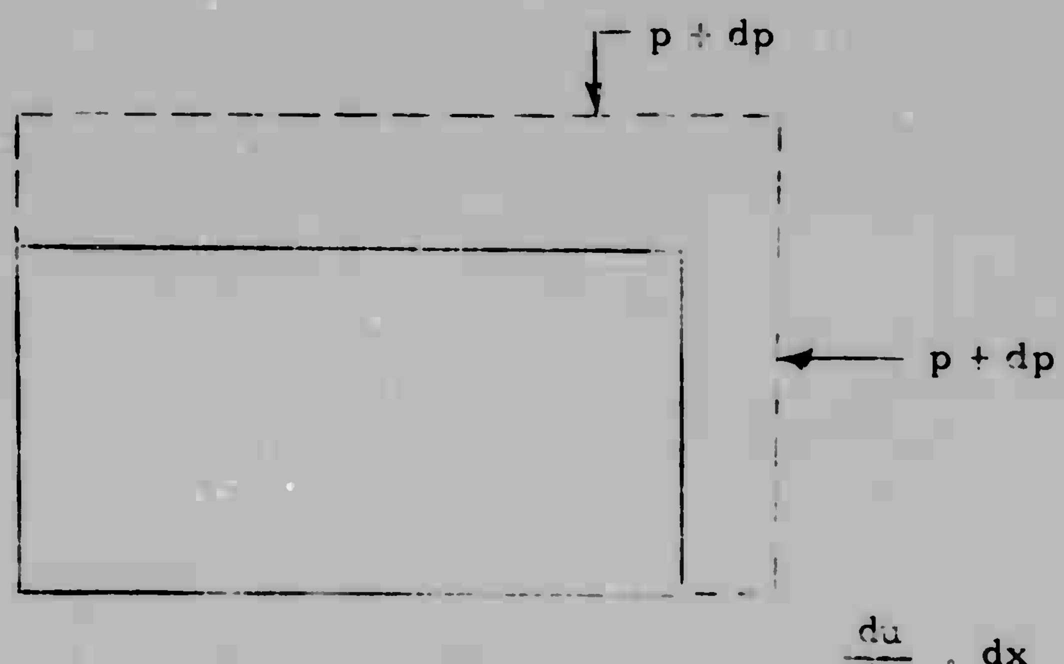
UNITED STATES AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
AIR FORCE FLIGHT TEST CENTER
EDWARDS AIR FORCE BASE, CALIFORNIA

No DDC limit

AERODYNAMICS HANDBOOK

ERRATA SHEET - AFFTC-TN-60-28

PAGE	CHANGE
i	a = Slope of lift curve
	C_{D_p} = Parasite or profile drag coefficient
	C_{D_π} = Proper drag coefficient
vii	add τ = Shear stress # lb/ft^2
	ν (nu) Kinematic viscosity ft^2/sec
1-18	line 4 change "it's" to "it is"
1-24	line 12 to read "number of nondimensional parameters"
	line 19 spelling "independent"
1-26	line 4 "with out" to one word "without"
1-31	line 2 spelling "moleculor" to "molecular" Figure 1.3.1 change axis g to y
1-33	Figure 1.3.3



PAGE	CHANGE
1-34	line 12 forces should be "faces"
1-37	line 16 should read "in the absence of the conversion of mass to energy through nuclear means, the mass....etc"

1-55 lines 19 and 20 $F_{inertia} = q\ dA = 1/2\ \rho V^2\ \blacksquare \ell^2$
where dA = cross sectional area of the stream = ℓ^2

1-58 line 4 Figure 1.3.9 should read Figure 1.3.14

line 6 $\delta_L = 4.94\sqrt{\frac{\mu x}{\rho u}} = \frac{4.94\ x}{\sqrt{R_{e_x}}}$

1-62 Figure 1.3.16 NOTE: Arrows signify magnitude of negative pressure

1-68 lines 2 and 7 to read:

$pV = R\ T = \frac{P}{\rho g}$

where p is the pressure lb/ft^2

$\rho = \frac{1}{gV^1}$ is the density $\sim$ $slugs/ft^3$

V is the specific volume $\sim$ ft^3/lb

T is the temperature (degrees absolute)

R is the universal gas constant $\frac{ft}{^oR}$ or $\frac{ft}{^oK}$

1-74 line 16 should read "considered only in an open...etc."

1-86 line 19 perse is "per se."

1-87 last line spelling porcess should be "process"

2-1 line 13 spelling nitrogen should be "nitrogen"
line 14 correct monoxide to read "dioxide"
line 17 spelling apporoximately should be "approximately"

2-5 line 23 "slug, ($lb\ sec^2/ft$)"

2-6 line 8 $V = 1/W = \frac{1}{\rho g}$

2-14 line 22 $P_2\ dA = \left[P_1 + (dp/dL)\ (dL)\right]\ dA$

2-15 line 3 spelling "boundary"

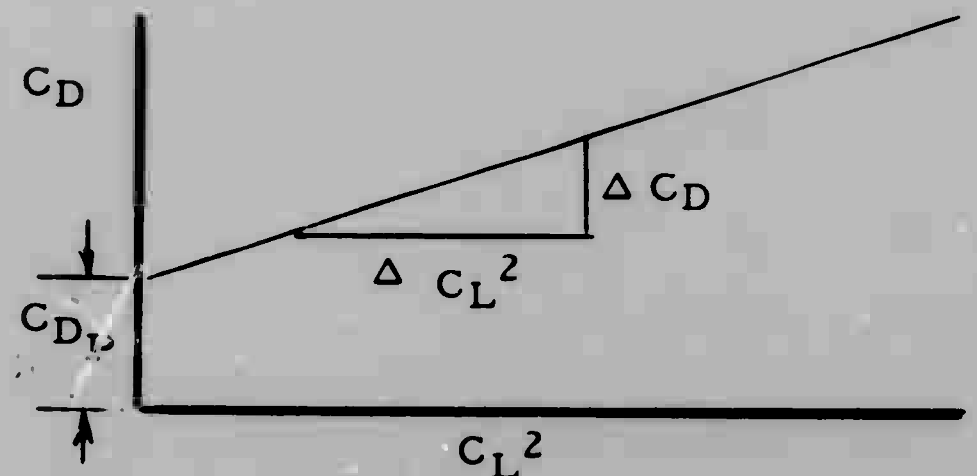
line 5 $F = P_1\ dA - \left[P_1 + dP/dL\ (dL)\right]\ dA$

2-31 below Eq 2.3.2

$q = 1/2\ \rho_\delta \cdot V_t^2$

2-39 line 9 The term $\frac{\rho V \ell}{\mu}$ defined as the Reynolds number and V/a is the Mach number. The term ρV^2 is twice the dynamic pressure etc........

2-63 line 5 should read "The minimum drag coefficient is seen....etc." Figure 2.3.17 replot



2-64 line 2 y = m x + b

line 7 $m = \frac{dC_L^{\,2}}{dC_D} = \frac{1}{\pi ARe}$

$b = C_{D_f}$

line 8 and 9 ELIMINATE

line 13 change to read FIG. 2.3.18a and b

2-71 renumber bottom figure FIG. 2.3.22A

label ordinate C_p not C_L

label abscissa C_L not n

2-72 line 16 the theoretical <u>chord</u> <u>for</u> <u>a</u> wing which has....etc.

2-76 line 16 Simply stated <u>then</u>...... etc..

line 18 designing smooth airfoils.......etc..

2-79 Fig 2.4.1.2 spelling on label <u>boundary</u>

2-80 line 5 and 7 C_L's

2-81 line 4 spelling porous

2-82 line 5 change profile to "parasite"

2-83 After Fig 2.4.2.1 eliminate rest of page

2-84 eliminate page

2-85 eliminate first twelve lines

The following replaces pages 2-83, 2-84, 2-85, 2.4.2(A)

PARASITE DRAG COMPARISON METHODS

Parasite drag is composed of skin friction drag and pressure drag. Comparison of the parasite drag of aerodynamic shapes ranging from a flat plate (if this can be called an aerodynamic shape) to a complete airplane is often desirable. Several methods exist for making this comparison.

2.4.2(A) 1 EQUIVALENT FLAT PLATE AREA

Wind tunnel experiments have shown that a drag coefficient of approximately 1.28 is a good average figure for a flat plate for Reynolds numbers in the flight range. The total parasite drag of a flat plate, which is almost entirely pressure drag, is computed by the equation:

$$D_p = 1.28\, q\, S_p \qquad \text{Equation 2.4.2.1}$$

where S_p = plate area

The parasite drag of an airplane can be expressed in terms of an equivalent flat plate area. That is, a flat plate of such an area that its drag will be equivalent to the drag of the airplane. Since the aerodynamic coefficient C_{D_p} is arbitrary and based on any convenient area (generally wing area), then:

$$\text{Parasite Drag} = D_p = C_{D_p}\, q\, S_w = 1.28\, Q\, S_p \qquad \text{Equation 2.4.2.2}$$

$$S_p = \frac{C_{D_p}}{1.28}\, S_w = A_e$$

where S_p = Flat Plate Area

A_e = Equivalent Flat Plate Area

Equivalent Flat Plate Area is obtained by substituting the C_{D_p} and S_w (Wing Area) of each particular aircraft into equation 2.4.2.2. The resultant number, equivalent flat plate area, A_e, gives a direct parasite drag comparison between two different types of airplanes.

2.4.2(A) 2 EQUIVALENT PARASITE AREA

Equivalent Parasite Area, f, is defined as that area which results assuming that the drag coefficient is 1.0 or:

$D_p = 1.0\ q\ f$ Equation 2.4.2.3

f = equivalent parasite area

2.4.2(A) 3 C_{D_π}, PROPER DRAG COEFFICIENT

Another Parasite drag coefficient, useful to aircraft designers in making drag estimates is C_{D_π}, which is defined as the drag coefficient based on frontal area. Therefore;

$D_p = C_{D_\pi} q\ S_\pi$ Equation 2.4.2.4

S_π = Frontal Area

2.4.2.(A) 4 RELATION BETWEEN THE VARIOUS COMPARATIVE DRAG TERMS

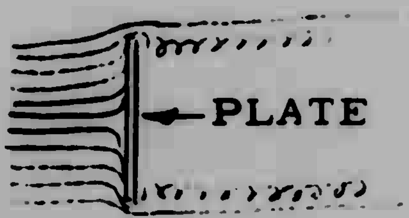
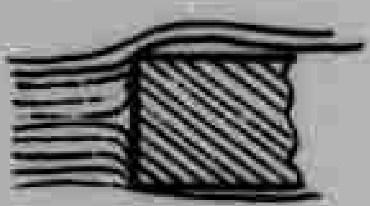
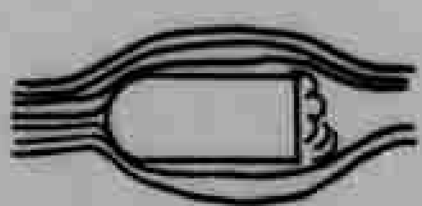
$D_p = C_{D_p}\ q\ S_w = 1.28\ q\ A_e = 1.0\ q\ f = C_{D_\pi}\ q\ S_\pi$

or

$C_{D_p}\ S_w = 1.28\ A_e = f = C_{D_\pi}\ S_\pi$ Equation 2.4.2.5

Drag comparison for different shapes may be made by comparing equivalent flat plate area, equivalent parasite area, proper drag coefficient or drag coefficient.

In figure 2.4.2.2, comparisons are made for bodies of unit (1 ft^2) cross section.

OBJECT	Equivalent Flat Plate Area A_e	Equivalent Parasite Area f	Proper Drag Coefficient C_D	Parasite Drag Coefficient $C_{D_p}(S_w=100ft^2)$
	$1.0\ ft^2$	$1.28\ ft^2$	1.28	.0128
	$.80-.94\ ft^2$	$1.02-1.20\ ft^2$	1.02-1.28	.0102-.0128
	$0.23\ ft^2$	$0.295\ ft^2$	0.295	.0295
	$0.035\ ft^2$	$0.045\ ft^2$	0.045	.0045

2-85 line 18 eliminate "and conversely"

3-6 1st line below figures spelling parallel

3-14 line 1 straight and level flight unaccelerated, the thrust etc...

Equation 3.2.1

$D_p = C_{D_p} = 1/2 \rho\ V_t^2\ S$

line 11

ρ = density of air

line 18

$D_i = C_{D_i}\ 1/2\ \rho\ V_t^2\ S$

3-15	line 1 $C_L = \frac{2L}{\rho_0 V_t^2 S}$	
	line 14 $C_L = \frac{2W}{\rho_0 V_t^2 S}$	
3-19	line 4 If we consider the thrust curve with drag curves etc. .	
3-22	line 4 provided the engine and duct efficiency etc.	
3-33	title on sec. 3.2.2 relabel to "ENGINE-AIR FRAME PERFORMANCE"	
3-35	line 8 spelling different	
3-37	line 7 change left to "right"	
3-60	line 10 $W_1 = W_0 - F.U.$ = final gross weight at end of endurance or cruise W_0 = Initial gross weight at start of endurance or cruise	
3-61	ρ in all equations is ρ_0	
3-63	line 19 delete rest of paragraph and add - In actual practice, because of in service variations in engine/airframe performance and because of operational conditions which reduce range such as turbulence and turning or maneuvering requirements, the operational values are usually not taken as greater than 80-85 percent of the theoretical values.	
3-64	line 1 Investigate first endurance. Divide the coefficient of drag equation by $C_L^{3/2}$ and invert. line 11 and 12 Divide the coefficient of drag equation by C_L and invert.	
3-70	Paragraph 4 relabel paragraph 3.	
3-82	line 7 and 9 ΔF should be ΔF_t, also line 11 and 21.	
3-85	line 5 Charts A-72 to A-73.	

3-87	line 5 $T_s = \frac{W_t}{V_{t_s}} \quad R/C_s + D_{test}$
	line 7 $\frac{d\ T_s}{dW} = \frac{W_t}{V} \cdot \frac{d\ (R/C_s)}{dW} + \frac{R/C_s}{V_{t_s}} + \frac{dD_t}{dW}$
	line 10 $\frac{d\ (R/C_s)}{dW} = - \left[\frac{R/C_s}{W_t} + \frac{d\ D_t}{dW} \cdot \frac{V_{t_s}}{W_t}\right]$
	equation 3.3.18
	$\Delta\ R/C_{weight} = - \left[R/C_s \quad \frac{dW_t}{W_t} + \frac{d\ D_t}{W_t} \quad V_{t_s}\right]$
3-88	line 12
	$dD = \frac{.006429}{Pa_s\ b^2\ M^2\ e} \left(\frac{N_t^2\ W_t^2}{1} \quad \frac{\delta s}{\delta t} - \frac{N_s^2\ W_s^2}{1}\right)$
	line 16 delete subtracting add <u>substituting</u>.
	line 18 equation 3.3.20
	$\Delta\ R/C_3$ = etc...
3-90	line 16 spelling <u>algebraically</u>
3-97	line 16 In order for an aircraft to descent <u>without</u> <u>accelerating</u>, the thrust, T, etc.....
3-107	equation 3.5.4
	$\Delta C_D = \frac{.000675}{M^2\ S} \times \frac{\Delta\ F_n}{\delta}$
	line 14
	where $\frac{\Delta F_n}{\delta}$ is determined etc.....
3-114	equation 3.6.7
	$\left(\frac{dh}{dt}\right)_{app} = \left(\frac{(T-D)\ V}{W}\right) \left(\frac{1}{1 + (V/g)\ (dV/dh)}\right)_{acc\ cor}$
3-115	line 1 spelling <u>envelope</u>
3-121	line 1 below Fig. 3.6.5 spelling <u>dimensional</u>
3-122	top left corner spelling <u>dimensional</u>
3-127	line 5
	$dW/dt = TC_i$

3-134 line 3

$$R = \left(C_{D_p} + \frac{C_L^2}{\pi A R e}\right) q S + \quad (W - C_L q S)$$

3-143 equation 3.7.11

$$S_a = \int_{E_{TO}}^{E_{50}} \frac{V\,dE}{(T-R)\,V}$$

change all integral sign to

$$\int_{E_{TO}}^{E_{50}} \quad \text{etc.}\ldots\ldots$$

3-145 line 12 spelling upward

3-152 line 11 spelling exponential

3-154 near bottom of page - Amount proportional to the sine of

3-158 line 7 1. Wind

3-162 For Fig. 3.7.5a and 3.7.5b use following code:

Total Resistance - High C_L

Total Resistance - Low C_L

Aerodynamic Resistance - High C_L

Aerodynamic Resistance - Low C_L

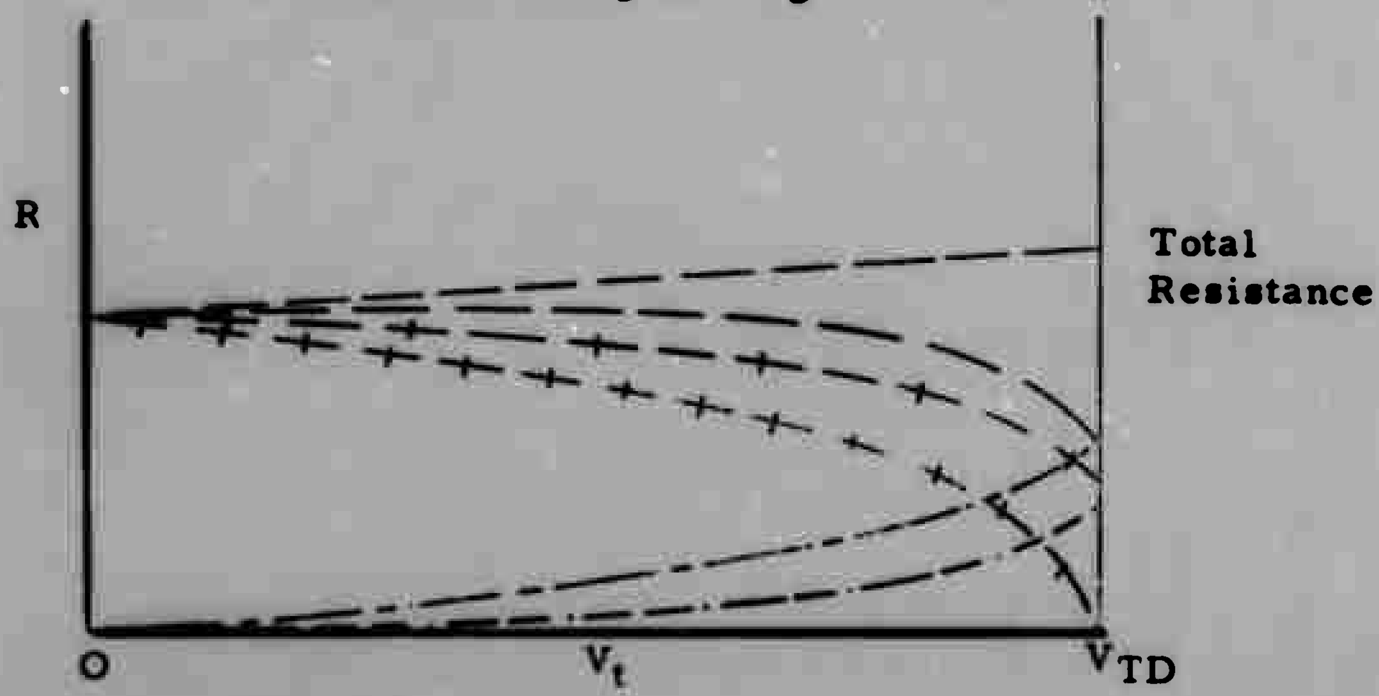
Braking Resistance - High C_L

Braking Resistance - Low C_L

orr

redraw and relabel Fig. 3.7.5B

Low Aerodynamic Drag and High Friction



4-35

equation 4.4.7

$$\text{Thrust (T)} = A\Delta P = A \rho \Delta V_2 \left(V + \frac{\Delta V_2}{2}\right)$$

AFFTC-TN-60-28
JULY 1960

AERODYNAMICS HANDBOOK
FOR
PERFORMANCE FLIGHT TESTING
VOLUME I

JAMES J. KING
Captain, USAF
Chief, Performance Section

PHILIP E. NEALE, JR.
Captain, USAF
Instructor, Performance Section

WILLIAM G. SCHWEIKHARD
Aero Research Engineer

THOMAS P. STAFFORD
Captain, USAF
Instructor, Performance Section

This handbook has been reviewed and approved.

Richard C. Lathrop
RICHARD C. LATHROP
Major, USAF
Commandant

This is a preliminary copy of the Aerodynamics handbook which will be published in the near future. Your comments and criticism will help make the finished article a more useful volume. Suggested changes should be submitted to William Schweikhard, FTTO or phone 27081 or 41761.

TABLE OF CONTENTS

FORWARD

This Aerodynamics handbook was written by the staff of the Performance Section, USAF Experimental Flight Test Pilot School, Edwards Air Force Base, California. The purpose is to consolidate all the material into one handbook so that all the material covered during the course is readily available to the student. The contents of this handbook are not as technical as that required by an aerodynamist or aeronautical engineer, but does present the theoretical basis of the material at the level that the average pilot can understand.

The reader should bear in mind that this handbook was written to satisfy school conditions and that in some cases the material maybe somewhat simplified; however, the material covered should be applicable anywhere.

It is hoped that this handbook will not only be an aid to the student attending the school but will be a ready reference for him and others engaged in flight testing.

LIST OF SYMBOLS AND ABBREVIATIONS

SYMBOL	NOMENCLATURE	UNITS
a	Speed of sound = $38.967 \sqrt{T_a {}^{\circ}K}$	kts
a	Acceleration	ft/sec/sec
A	Area	ft^2
A_e	Equivalent Flat Plate Area	ft^2
AR	Aspect ratio, b^2/S, b/c	
b	Wing span	ft
BHP	Brake horsepower	550 ft-lb/sec
BHP_a	Brake horsepower (available std)	550 ft-lb/sec
BHP_c	Brake horsepower chart	550 ft-lb/sec
BHP_t	Brake horsepower test	550 ft-lb/sec
BHP_s	Brake horsepower standard	550 ft-lb/sec
BMEP	Brake mean effective pressure	lb/in^2
BSFC	Brake specific fuel consumption	lb/bhp-hr
BTU	British Thermal Unit	778 ft-lbs
c	Wing chord	ft
C	Constant	
$^{\circ}C$	Degrees centigrade	
C_D	Drag coefficient	
C_{D_i}	Induced drag coefficient	
C_{D_m}	Mach drag coefficient	
C_{D_p}	Parasite drag coefficient	
C_L	Lift coefficient	
C_p	Pressure coefficient = $\Delta P_p/q_{c_{ic}}$	

C_Q	Torque coefficient	
C_T	Thrust coefficient	
D	Propeller diameter	ft
D	Aerodynamic drag	lb
D	Distance	ft
dh/dt	Rate of climb (tapeline)	ft/min
dhe/dt	Rate of change of specific energy	ft/sec
e	Oswald efficiency factor	
e	Internal energy	ft-lbs
E_t	Total energy	ft-lb
E_k	Kinetic energy	ft-lb
E_p	Potential energy	ft-lb
ESHP	Equivalent shaft horsepower	550 ft-lb/sec
ETHP	Equivalent thrust horsepower	550 ft-lb/sec
f	Equivalent parasite area	ft^2
F	Thrust	lb
F	Force	lb
^oF	Degrees fahrenheit	
F_e	Excess thrust	lb
F_g	Gross thrust	lb
F_n	Net thrust	lb
FAT	Free air temperature	degrees
F.U.	Fuel used	lbs
g	Acceleration due to gravity	32.174 ft/sec^2
h	tapeline height	ft

h_e	Energy height, specific energy	ft
H	Height	ft
H_c	Calibrated height	ft
H_{ic}	Instrument corrected height	ft
H_i	Indicated height	ft
ΔH_{pc}	Altitude position error correction	ft
ΔH_{ic}	Altimeter instrument correction	ft
"Hg	Inches of mercury	
hp or HP	Horsepower	550 ft-lb/sec
in	Inches	
IAS	Indicated airspeed	kts
J	Advance ratio, V/ND	
K	Constant	
k	Constant	
°K	Degrees kelvin	
L	Length	ft
L	Lift	lbs
lb	Pounds	
m	Mass	
$\dot{m}$	Mass flow rate	lbs/sec
M	Mach number, free stream	
min	minutes	
MP	Manifold pressure	inches Hg
MAC	Mean aerodynamic chord	ft
n	Load factor	

N	Engine speed	rpm
P	Pressure	inches Hg
P	Power	ft-lb/sec
P_a	Ambient pressure	inches Hg
P_s	Static pressure	inches Hg
P_t	Total pressure	inches Hg
P_{t_2}	Compressor inlet total pressure	inches Hg
P_{t_7}	Tailpipe total pressure	inches Hg
PIW	Brake horsepower corrected to sea level, standard weight	
PSI	Pounds per Square Inch	
Q	Torque	ft-lbs
q	Dynamic pressure	lb/ft^2
q_c	$P_t - P_a$	in Hg
$q_{c_{ic}}$	$P_t - P_s$	in Hg
R	Gas constant	$ft^2/^oK\ sec^2$
^oR	Degrees rankine	
R	Radius of turn	Nautical miles
RN, R_e	Reynolds number	
R/C	Rate of climb	ft/min
R/D	Rate of descent	ft/min
S	Wing area	ft^2
S_p	Flat plate frontal area	ft^2
S_a	Air distance from takeoff to 50 feet or air distance from 50 feet to touchdown	ft
S_g	Ground roll	ft

std, s	Standard	
SFC	Specific fuel consumtion	$\frac{lb/hr}{SHP}$
SHP	Shaft horsepower	
T	Temperature, absolute	^{o}K or ^{o}R
t	temperature	^{o}C or ^{o}F
T	Thrust	lb
t	Time seconds, minutes, hours	
T_a	Ambient temperature	^{o}R or ^{o}K
T_i	Indicated temperature	^{o}R or ^{o}K
T_{ic}	Instrument corrected temperature	^{o}R or ^{o}K
T_o	Std sea level temperature	^{o}R or ^{o}K
T_s	Std day temperature	^{o}R or ^{o}K
T_t	Test day temperature	^{o}R or ^{o}K
T_{a_s}	Std ambient temperature	^{o}R or ^{o}K
T_{a_t}	Test ambient temperature	^{o}R or ^{o}K
$T_{t_{2_t}}$	Test Total Temperature at Compressor Inlet	^{o}R or ^{o}K
$T_{t_{2_s}}$	Std Total Temperature at Compressor Inlet	^{o}R or ^{o}K
T. D.	Touchdown	
T.O.	Takeoff	
u	velocity	ft/sec
V, v	Velocity, airspeed	ft/sec or kts
V	Volume	ft^3 or in^3
V_{avg}	Average velocity	ft/sec

	Indicated airspeed	kts
c	Instrument corrected airspeed	kts
	Calibrated airspeed	kts
	Equivalent airspeed	kts
	True airspeed	kts
	Ground speed	kts
V_c	Airspeed compressibility correction	kts
V_{ic}	Instrument correction for airspeed	kts
V_{pc}	Airspeed position error correction	kts
ax	Maximum airspeed	kts
in	Minimum airspeed	kts
	Stall speed	kts
	Wind speed	kts
	Specific wt	lb/ft^3
	Air flow	lb/sec
	Fuel flow	lb/sec
	Std weight	lb
	Test weight	lb
	Angle of attack	degrees
	Propeller blade angle	
	Angle between propeller plane of rotation and the chord line	degrees
	Chord wise circulation	
	Climb or descent angle	degrees
	Ratio of specific heat	
	An Incremental change	

δ	Pressure ratio, $P_a/29.92$	
ϵ	Downwash angle	degrees
η	Efficiency	
η_o	Overall efficiency	
η_p	Propulsive efficiency	
η_t	Thermal efficiency	
η_{me}	Mechanical efficiency	
η_R	Ram efficiency	
θ_a	$T_a/288$	
θ_{t_2}	$T_{t_2}/288$	
ϕ	Helix angle	degrees
μ	Coefficient of viscosity	$lb\text{-}sec/ft^2$
π	3.14159	
ρ	Density	$Slugs/ft^3$
σ	Density ratio	
ω	Rate of turn	degrees/sec

SUBSCRIPTS

a	Ambient or free air
Crit	Critical
e	Exit conditions
ic	Instrument corrected
Min	Minimum
Max	Maximum
o of SL	Sea Level std. conditions
s	Std day conditions
t	Test day conditions
t	Total or stagnation conditions

SECTION I

FUNDAMENTALS REVIEW

1.1 PHYSICS

1.1.1 INTRODUCTION:

This section is designed to review elementary physics for the new student at the USAF Experimental Flight Test Pilot School.

Physics is primarily concerned with phenomena that can be described in terms of energy and matter. It is therefore important that an accurate measurement be made of this energy and matter. Three fundamental quantities time, length and mass are the units of measurement.

1.1.2 UNITS:

1.1.2 a TIME: The unit of time is called the second. Sixty seconds make up a minute and sixty minutes are equal to one hour.

1.1.2 b LENGTH: The English system unit of length is the foot. Twelve inches comprise a foot and three feet are equal to one yard. 5280 feet are equal to one statute mile.

1.1.2 c MASS: The English system unit of mass is the pound. Sixteen ounces comprise the pound and 2,000 pounds are equal to one ton. Mass is in a sense the body itself or the amount of matter that it contains. Hence the mass is independent of the position of the body with relation to the earth or space.

1.1.2 d VOLUME: The English system unit of volume is the cubic foot and 27 cubic feet are equal to one cubic yard.

1.1.2 e DENSITY: The density of a body is defined as mass per unit volume and is expressed as pounds per cubic foot.

1.1.3 QUANTITIES: Two types of quantities are used in the measurement of energy and matter: the scalar quantity and the vector quantity.

1.1.3 a SCALAR QUANTITY: The scalar quantities have only magnitude. No direction is required with this type of quantity. Time, length, speed, mass and volume are typical scalar quantities. Scalar quantities can be added and subtracted numerically.

1.1.3 b VECTOR QUANTITY: The vector quantity must have both magnitude and direction to be completly known. Force, velocity, acceleration and weight are examples of vector quantities. A vector quantity can be represented by a straight line, the direction of which represents the direction of the vector and the length represents the magnitude. It is common practice to let an arrowhead represent the direction of the vector. Vector quantities must be added and subtracted vectorially. Consider three forces acting on the center of gravity of an object. (Figure 1.1.1a)

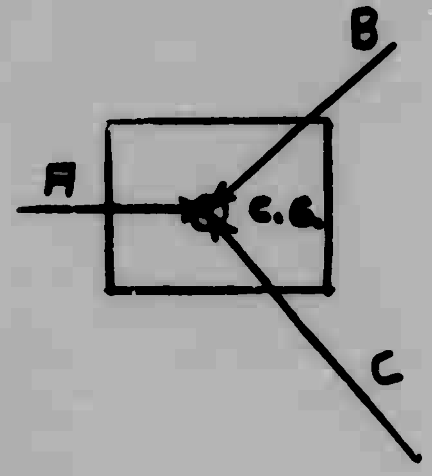


Figure 1.1.1a

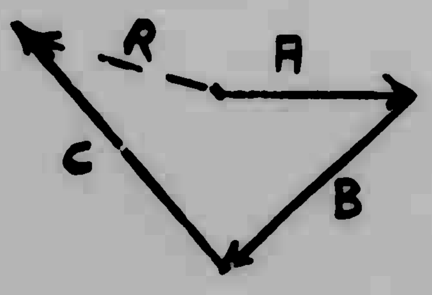
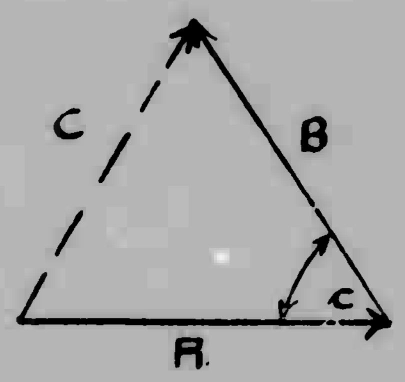


Figure 1.1.1b

These vectors can be added together vectorially. Figure 1.1.1b illustrates the method where the three vectors are combined to give the resultant R. The general rule is that the tail of one vector is placed at the head of the preceeding vector. The resultant of the vectors is then drawn from the tail of the first vector to the head of the last vector. Now all three of the original vectors can be replaced by the resultant R with no change on the system. Many problems are concerned with only two vectors. The resultant can be solved using the law of cosines or the law of parallelogram as illustrated in Figure 1.1.2a and 1.1.2b.

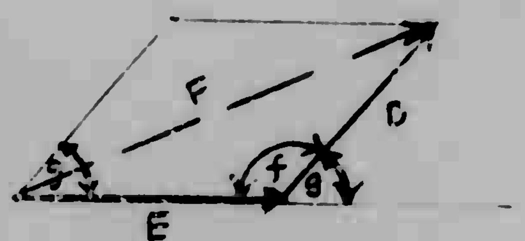
Law of Cosines



$C^2 = A^2 + B^2 - 2AB \cos C$

Figure 1.1.2a

Law of Parallelograms



$F^2 = D^2 + E^2 - 2DE \cos f$

$F^2 = D^2 + E^2 + 2DE \cos g$

Figure 1.1.2b

If more than two vectors are to be added it is recommended that each vector be broken down into horizontal and vertical components. All horizontal and vertical components are then summed up and the resultant is found. This brings out the fact that a vector may be broken down into a number of components. The resultant aerodynamic force on an airfoil can be broken down into components that are parellel and perpendicular to the relative wind. Figure 1.1.3a. These forces are known as drag and lift. The resultant can also be broken down into components that are parallel to and perpendicular to the chord of the airfoil. These force are refered to as the chordwise drag force and the normal lift force. Figure 1.1.3b.

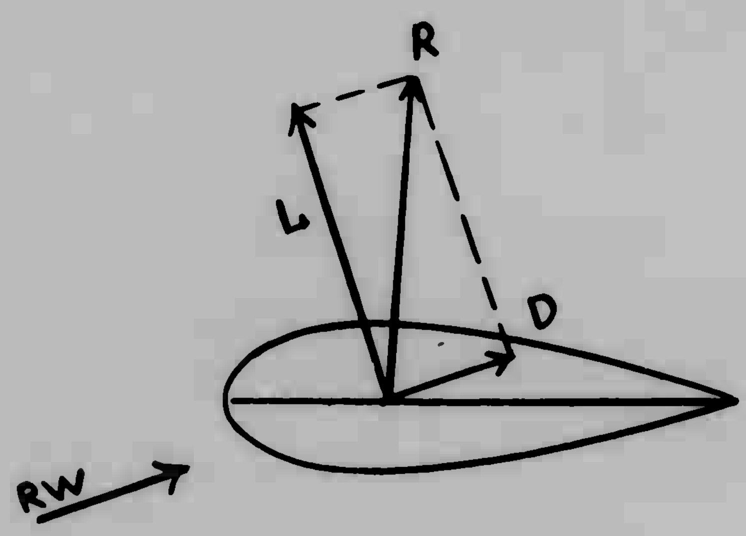


Figure 1.1.3a

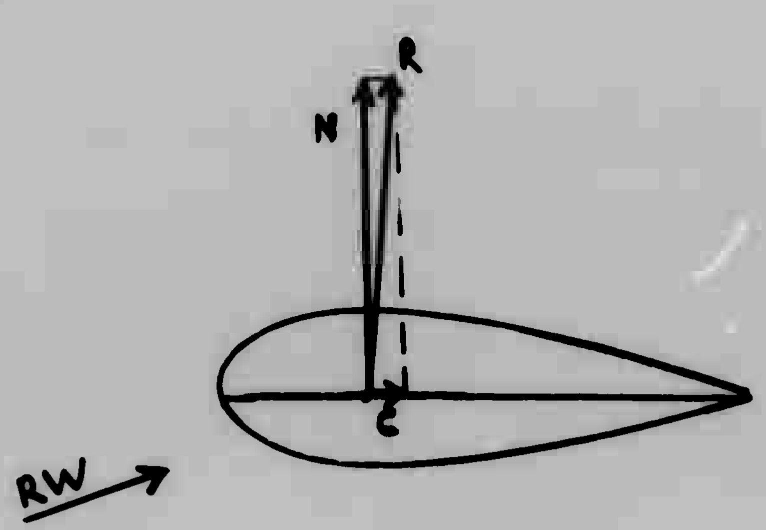


Figure 1.1.3b

1.1.4 COMPOSITION OF FORCES: A force is an action exerted by one body on another that tends to change the state of motion of the body acted upon. The pull of the earth on bodies which are at or near its surface is easily observed and is used as a system of units in which to measure other forces. These units of force are known as gravitational units of force. A force of one pound is a force equal to the force with which the earth attracts a mass of 1 pound. The weight of a body is the measure of the attraction of the earth of the amount of mass that the body contains. When any doubt exists as to the specification of a pound of mass or a pound of force the term pound-weight will be used to describe the unit of force.

1.1.5a MOMENT OF FORCE AND TORQUE: The moment of a force (or what is simply called a moment) is equal to the product of the force times the perpendicular distance from the force to the reference axis or center of rotation. The moment is often called torque. Consider Figure 1.1.4 in which a force F has been resolved into components F_y and F_x. The moment (written M) of the force

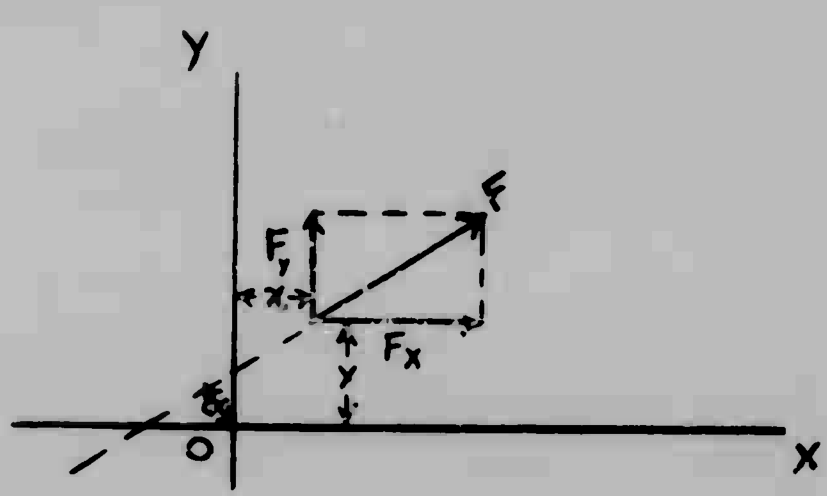


Figure 1.1.4

F about an axis through O is

$$M_o = F \cdot d$$

In order to facilitate bookkeeping, so to speak, a sign convention must be adapted. Clockwise moments about an axis are positive in this discussion. Accordingly, since force F is equivalent to the force system F_y plus F_x in Figure 1.1.4, then also

$$M_o = F \cdot d = (F_x)y - (F_y)x$$

Examples of moments are known to all. For instance, a water wheel simply utilizes gravitational pull (force) on water contained in buckets on the wheel circumference to turn the wheel and do useful work. Consider the familiar balance scale consisting of a beam free to rotate about some mid point, one end of which is loaded by an object of unknown weight and the other with a collection of known weights. The amount of the known weights is varied until the system is in balance or equilibrium. As a last example, consider the torque wrench of Figure 1.1.5.

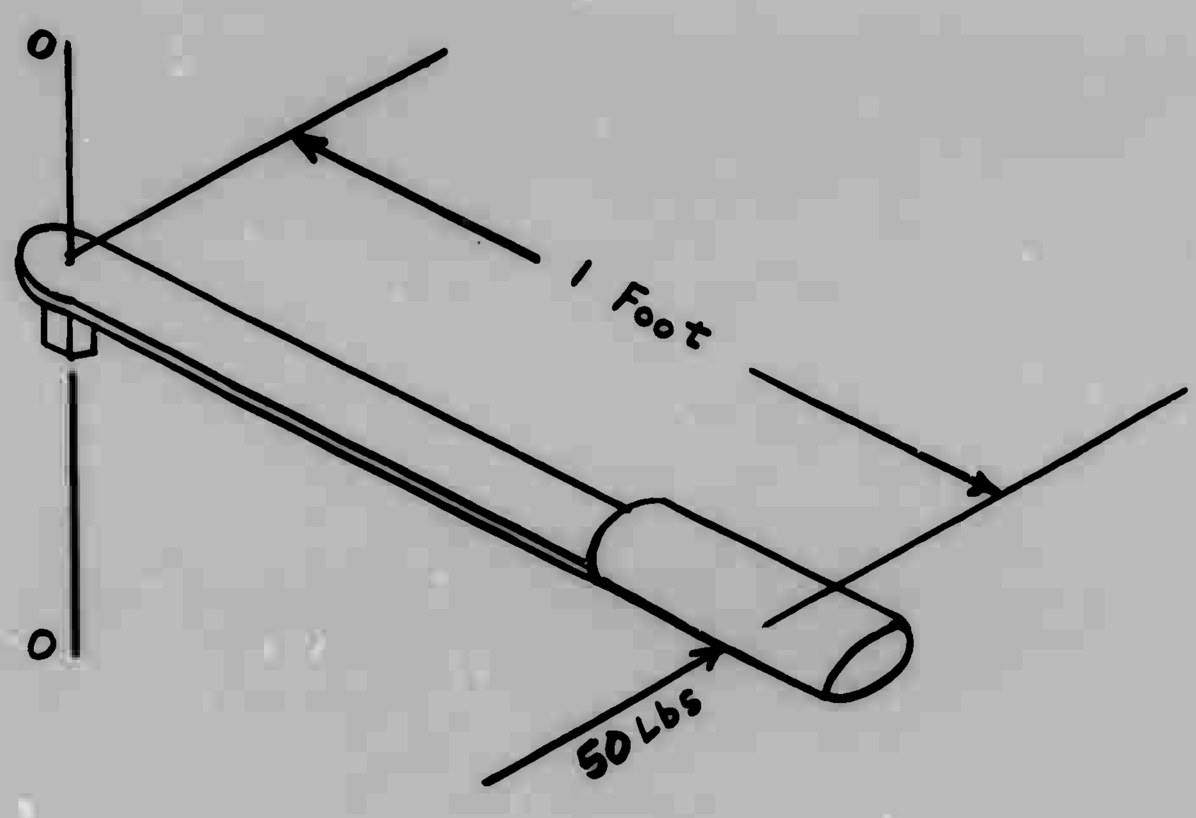


Figure 1.1.5

In this case a force of fifty pounds is being applied at a moment arm of one foot about axis O-O. In other words the applied torque to axis O-O is 50 ft. lbs. (50 lbs x 1 foot).

1.1.5 b RESOLUTION OF A FORCE INTO A FORCE AND A COUPLE:

Occasionally the analysis of a problem may be simplified if the line of action of a force can be shifted. Figure 1.1.6a portrays an object under the influence of force P as shown. For analytic purposes, it is desired to shift the line of action of force P from its original location to pass through point O'. To accomplish this, forces parallel to P and of the same magnitude and of

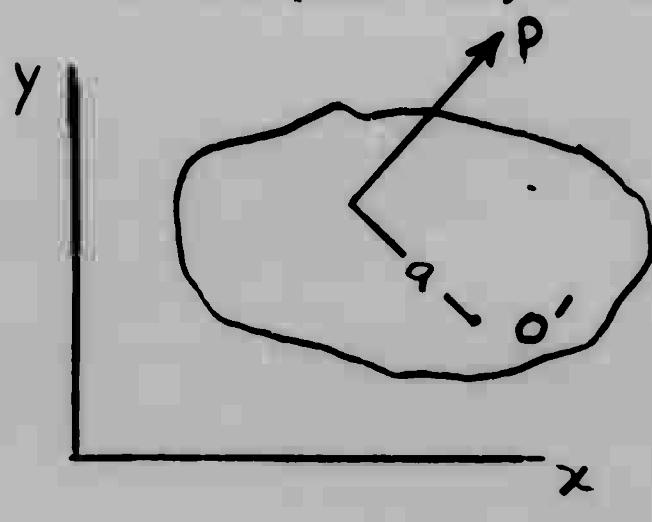


Figure 1.1.6a

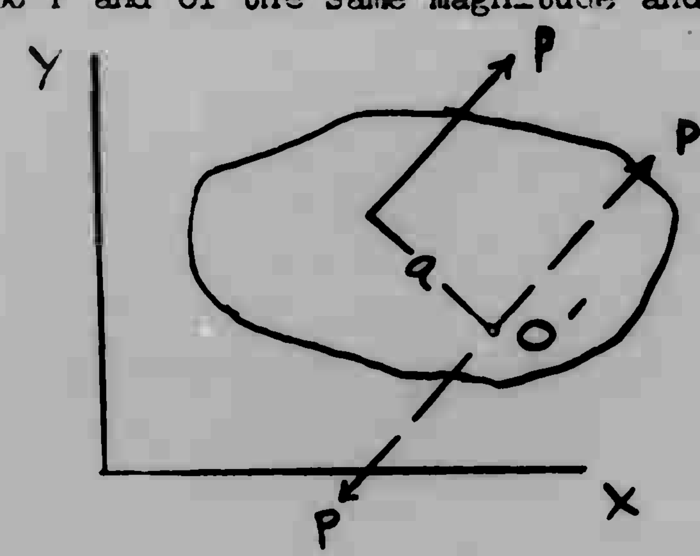


Figure 1.1.6b

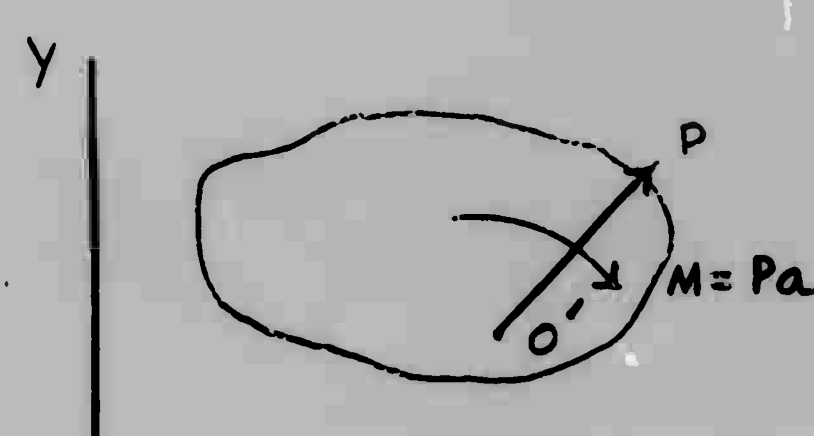


Figure 1.1.6c

the same and opposite sense can be drawn through O' without changing the force system in any way, since the resultant of the added system is zero. This is illustrated in Figure 1.1.6b. If a is the perpendicular distance between the line of action of force P and point O', it is seen that force P is equivalent to the same force P acting through O' plus a clockwise moment equal to Pa. This is shown in Figure 1.1.6c.

As a practical illustration of this process, consider the airfoil section of Figure 1.1.7. The line of action of the resultant aerodynamic force on the airfoil passes through the center of pressure. The chordwise location of the center of pressure on a non-symetrical airfoil is subject to change with changes in angle of attack. This tends to complicate the analysis of the lift effects on the overall flying qualities of the aircraft for reasons which will be explained later in the course.

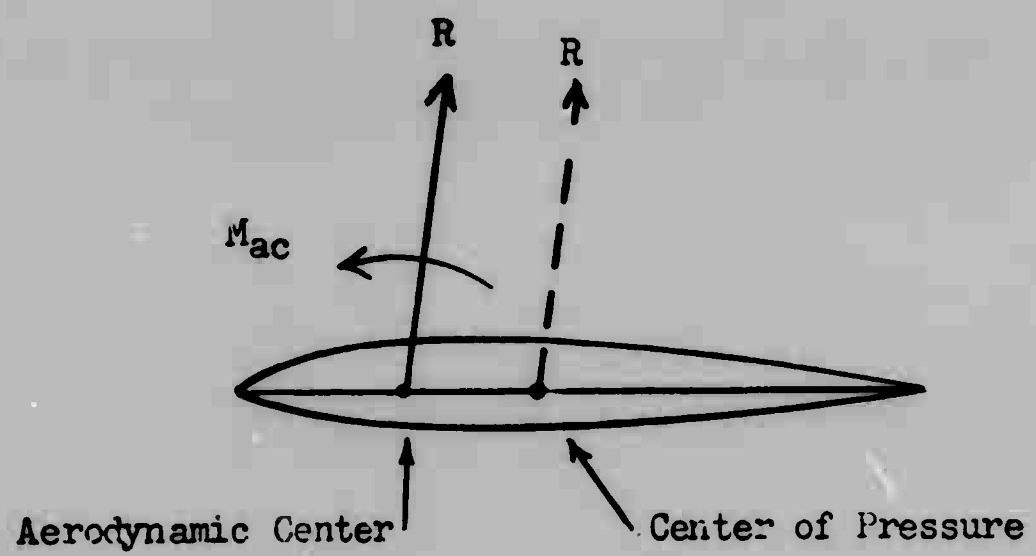


Figure 1.1.7

Current practice is to change the point of application of the resultant aerodynamic force from the center of pressure to the aerodynamic center. This can be done by moving the force R from the c.p. to the a.c. and supplying a moment M_{ac} equal in magnitude to R times the perpendicular distance from c.p. to a.c. For slow speed flight the aerodynamic center remains relatively fixed at different angles of attack and this is found to be a definite advantage in stability work.

1.1.6 a EQUILIBRIUM:

If the forces acting on a body neither tend to change the body's translation or rotation, the body is said to be in equilibrium. In a three dimensional order (length, width, and depth) it is convenient to use a three axis system to catalogue all the forces acting on a body. Consider Figure 1.1.8 in which an airplane is drawn about its accepted axis system. The origin of the axis system is the aircraft's center of gravity while the positive direction of each axis is denoted by the arrowheads. Obviously then, the necessary conditions for

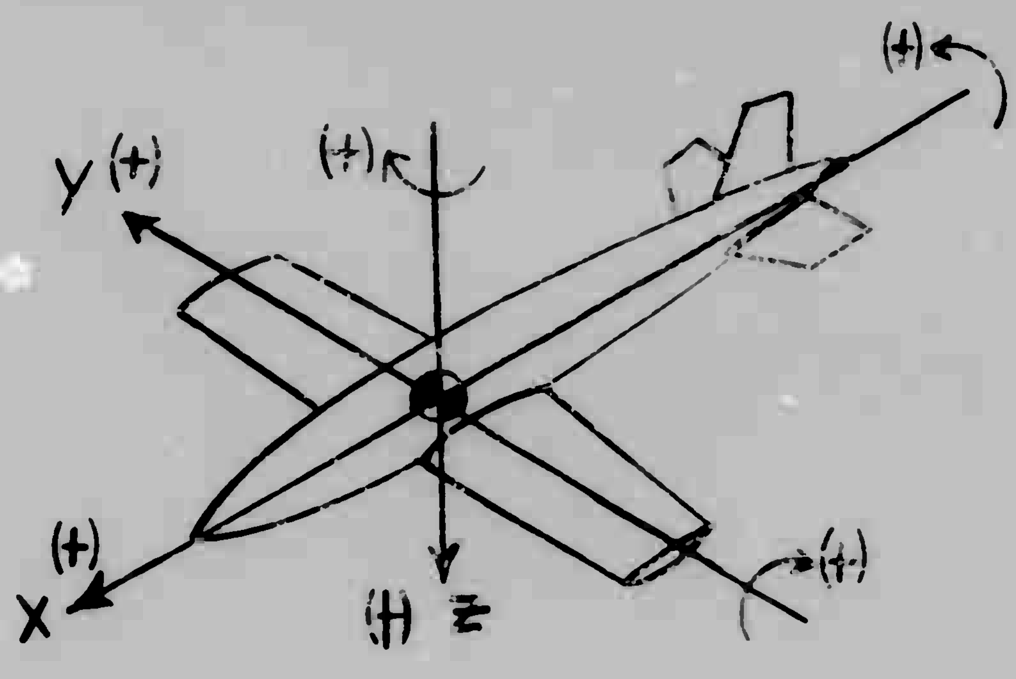


Figure 1.1.8

static equilibrium of the aircraft or any other three dimensional body are:

$$\sum F_x = 0 \qquad \sum M_x = 0$$
$$\sum F_y = 0 \qquad \sum M_y = 0$$
$$\sum F_z = 0 \qquad \sum M_z = 0$$

The symbols $F_x = 0$ are to be read as "the summation of all the forces in the x direction equals zero" while the symbols $M_x = 0$ are to be read as "the summation of the moments of all the forces about the x axis equals zero." These are the conditions under which the aircraft must operate if the aircraft is stationary or moving along a straight and level flight path at a constant velocity.

1.1.6 b EQUILIBRIUM OF COPLANAR FORCE SYSTEMS:

Although this is a three dimensional world, it is often possible to simplify an analysis of a problem by working with two dimensional models. Consider the weightless beam of Figure 1.1.9 in which all the forces are acting in the plane of the paper. The problem is to determine the magnitude, point of application and inclination of force R required to bring the system into equilibrium. For the two dimensional case

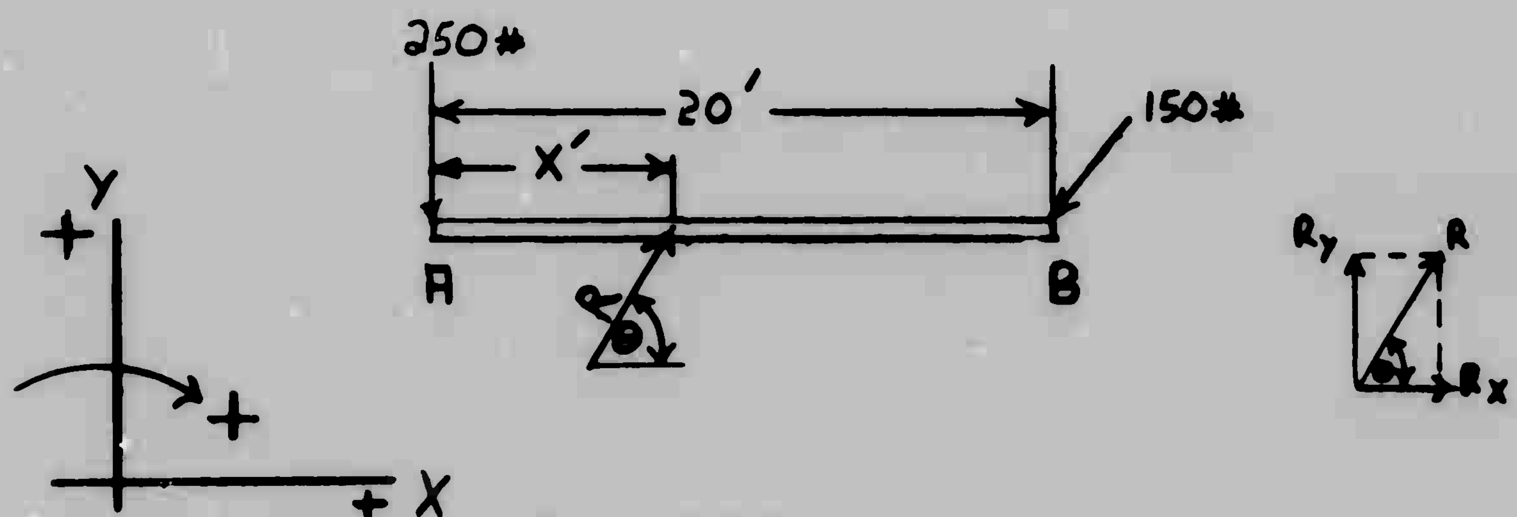


Figure 1.1.9

presented, the conditions for equilibrium are:

$$\sum F_x = 0 \qquad \sum M_A = 0$$

$$\sum F_y = 0 \qquad \sum M_B = 0$$

$$\sum M_{\text{any point}} = 0$$

By the numbers, a general method of solving these type problems would be as follows:

1. Choose an axis system and sign convention (clockwise moments are plus, etc).

2. Sum the forces in each applicable direction (should equal zero if the system is in equilibrium).

3. Select a point about which to take moments. It is usually most convenient to select this point under a known force.

4. Sum the moments about the selected point (should equal zero if the system is in equilibrium). Repeat steps as necessary until all the unknown quantities are clear.

Using the above method, the problem of Figure 1.1.7 would be worked in the following manner:

(1) The selected axis system and sign convention is as indicated in the Figure.

(2) $\sum F_y = 0$

$$-250 - 150 \cos 45^\circ + R_y = 0$$

$$-250 - 106 + R_y = 0$$

$$R_y = 356 \text{ lbs}$$

$\sum F_x = 0$

$$-150 \sin 45^\circ + R_x = 0$$

$$R_x = 106 \text{ lbs}$$

$$R = \sqrt{R_x^2 + R_y^2} = 10^2\sqrt{1.12 + 12.67} = 372 \text{ lbs}$$

$$\theta = \tan^{-1} R_y/R_x = \tan^{-1} \frac{356}{106} = 73.4^\circ$$

(3, 4) $\sum M_B = 0$

$$-250\ (20) + R_y\ (20 - x) = 0$$

$$x = \frac{-5000 + 7120}{356} = 5.97 \text{ feet}$$

check

$\sum M_A = 0$

$$5.97(356) - 150\ (\cos 45^\circ)\ (20) = 0$$
$$2120 - 2120 = 0$$
$$0 = 0$$

If the student is unfamiliar with these methods, it would be worthwhile to rework the above.

1.1.7 CENTER OF GRAVITY:

The center of gravity of an object has been defined as that point where the entire mass of an object may be considered to be concentrated without having gravity affect the system. The weight of an object is simply the force of gravitational pull on a body by the earth. A body is made up of a number of small parts, each of which is attracted to the earth. The weight of the body is therefore the resultant force of all the gravitational forces acting on each of the small parts. The line of action of this resultant gravitational force passes through a point in the body called the center of gravity. Consider a weightless beam with two weights at either end as in Figure 1.1.10

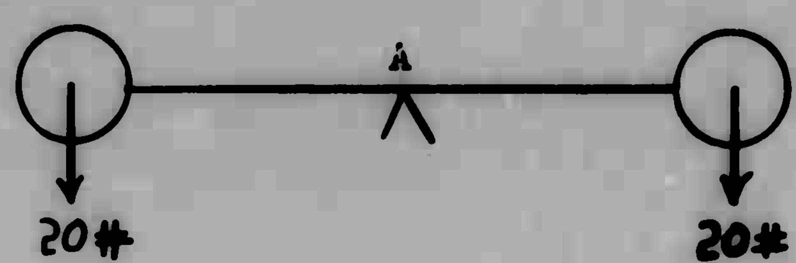


Figure 1.1.10

The bar is pivoted at point A. If point A coincides with the center of gravity of the mass system consisting of the bar and two weights, then the bar and two weights could be replaced with a resultant force of 40 pounds, with no apparent change in the equilibrium of the system. Obviously, it would be impossible to balance the bar and weights upon knife edge A at some point not coincident with the center of gravity of the system, since the line of action of the weight (force) of the system would have an unbalanced moment about the new pivot point.

1.1.8 TYPES OF MOTION:

When a body moves continuously in the same direction it is said to have a motion of TRANSLATION. An airplane moving down a runway or a piston sliding in a cylinder are illustrations of motions of translation. If a body turns about a fixed axis it has a motion of ROTATION. The flywheel of a gasoline engine or the rotation of a ball about its centroid as it rolls down an incline are examples of motions of rotation. In the latter case, the ball is exhibiting both rotation and translation simultaneously. If a body reverses its motion from time to time and returns to its original position at regular or irregular intervals, it is said to have a motion of VIBRATION or OSCILLATION. A weight bouncing on the end of a spring or the bob weight on the pendulum of a clock exhibit motions of vibration. Occasionally a body will exhibit all three types of motion simultaneously. A spinning aircraft for example may translate toward the ground as it rotates about some fixed vertical axis while the bank and pitch angles vary cyclically between fixed limits.

1.1.9 NEWTONS THREE LAWS OF MOTION:

Newton's three laws of motion are basic tools of the physicist or anyone concerned with the motion of any type of matter - gaseous, liquid or solid. Briefly stated, the laws read as follows:

1. Every body continues in a state of rest, or uniform motion in a straight line, or uniform motion of rotation about a fixed axis, unless compelled to change the state of motion by the action of some external force.

Inherent in the First Law is the concept of inertia which is that property of matter by virtue of which it tends to remain at rest or in uniform motion unless acted upon by external forces. Bodies show opposition to being translated or rotated. The magnitude of this opposition to translation is proportional to the mass of the body. The magnitude of the inertial opposition to rotation is dependent on the mass distribution as well as total mass.

2. The time rate change of momentum of a body is proportional to the impressed force, or

$$F = \frac{d\,(m\,v)}{dt} \qquad \text{Equation 1.1.1}$$

The linear momentum of a body is equal to the product of its mass times velocity. Thus a one pound bullet moving with a velocity of one hundred (100) feet per second would have a linear momentum of 100 ft. pounds per second (1 x 100 = 100). If a constant drag force of ten (10) pounds were suddenly to be applied to the one pound bullet, the velocity would decrease at the rate of 322 ft/sec^2. As shown, assuming the mass m remained constant:

$$F = \frac{d(m\,v)}{dt} = m\left(\frac{d\,v}{d\,t}\right) = m\,a$$

or

$$F = \frac{W}{g}\,a \qquad \text{or} \quad a = \frac{F\,g}{W} = \frac{(-10)(32.2)}{1}$$

The minus sign in front of the ten signifies a negative or decelerating force. Perhaps a word is due on units. The standard units of mass in the work here is the slug. A mass of one slug will have a weight of 32.2 pounds at the earth's surface and the weight-mass relationship is given as:

$$m = W/g \qquad \text{Equation 1.1.2}$$

where g is the acceleration due to gravity. The normal acceleration due to gravity is 32.2 ft/sec^2 at the earth's surface, although this value will decrease with altitude. The units of linear acceleration should be apparent from its definition. Acceleration is the time rate of change of velocity. If a graph is made of the time history of the velocity of a body as in Figure 1.1.11

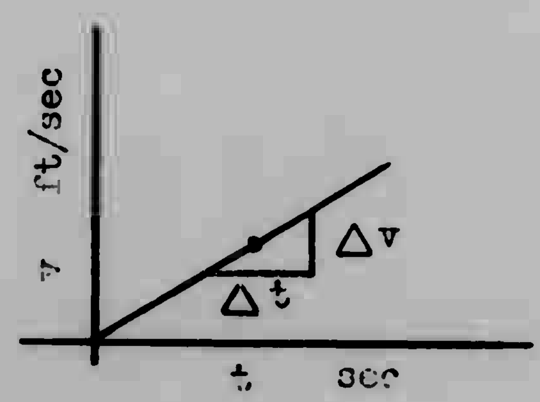


Figure 1.1.11

the acceleration at any time t is equal to

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{\text{ft/sec}}{\text{sec}} = \text{ft/sec}^2$$

The conservation of momentum can be stated that in an enclosed system the total momentum will remain unchanged if no external forces are applied.

3. To every action, there is an equal and opposite reaction.

Newton's Third Law is familiar to every thinking man. If a man presses on a wall with a force of ten pounds, the wall must press back with the same force. As the jet pilot increases his engine rpm preparatory to taxiing, the airplane remains stationary. The engine thrust force acting forward is resisted by the friction on the wheels and the inertia of the airplane. Finally the forward thrust attains a value sufficient to overcome the resisting forces, and the aircraft starts to move in accordance with Newton's Second Law.

1.1.10 EQUATIONS OF LINEAR MOTION:

In flight test work we are concerned with equations of linear motion. The basic equation is that a body traveling at a constant velocity for given period of time will traverse a specified distance.

$$S = Vt \qquad \text{Equation 1.1.3}$$

If the body started with a constant velocity and accelerated at a constant rate of acceleration for a period of time a constant final velocity would be obtained.

$$V_f = V_o + at$$

$$a = \frac{V_f - V_o}{t}$$

Now if $V_o = 0$ it is seen that $V_f = at$. The average velocity =

$$\frac{V_o + V_f}{2} = \frac{0 + (0 + at)}{2} = \frac{at}{2}$$

Distance traveled at the average velocity for the period of time, $S = V_{avg} \cdot t$

$= \frac{at}{2}\ (t) = \frac{at^2}{2}$. Now if the body was moving at a constant velocity at the start of the acceleration.

the total distance covered would be

$$S = V_0 t + \frac{at^2}{2}$$

Going back to the original equations

$$S = \frac{1}{2} at^2 \text{ and } t = \frac{V_f}{a} \text{ then } t^2 = \frac{V_f^2}{a^2}$$

substuting $S = \frac{1}{2} a \frac{V_f^2}{a^2} = \frac{V_f^2}{2a}$

$$\text{or } 2aS = V_f^2$$

if the body had an original velocity

$$V_0^2 + 2aS = V_f^2$$

The above equations are summarized as follows:

$S = V_{avg}t = \frac{at^2}{2}$ (if $V_0 = 0$) Equation 1.1.4

$V_f = V_0 + at$ Equation 1.1.5

$S = V_0 t + \frac{at^2}{2}$ Equation 1.1.6

$V_0^2 + 2aS = V_f^2$ Equation 1.1.7

1.1.11 ENERGY:

Energy is defined as the ability to do work. The work required to stretch a spring is stored up as potential energy in the spring. The important principle of the conservation of energy merely states that in any body or system of bodies, the total amount of energy will remain unchanged if the system is neither giving up or receiving energy. The energy may be transformed from one form to another such as heat and light, but the total amount of energy in the system will remain unchanged. POTENTIAL energy is the energy a body has because of its position. Lifting a mass above the surface of the earth stores potential

energy in the body, since the pull of gravity drawing the body back to the earths surface is capable of being used to do useful work. KINETIC energy is the energy a body has because of its motion. Any body in motion is able to move other bodies by colliding with them and it's possible to refine this collision so that useful work is done.

1.1.12 WORK AND POWER:

Work is equal to the product of the force doing the work times the distance through which the force moves the object of the work. As such, the units of work in our convention are foot pounds. Power is simply the time rate of doing work. Power is therefore measured as foot pounds per unit time. The English unit of power is called horsepower which is equal to 550 ft-pounds of work per second.

1.1.13 THE MEASUREMENT OF ENERGY:

The measure of the potential energy which a body has by virtue of its position is equal to the work spent in lifting the body. The increase in potential energy of a 500 pound weight lifted ten feet in the air for example is equal to 5000 ft-lbs as shown.

Potential Energy = Work spent lifting the weight

Potential Energy = F x d = W h Equation 1.1.8

Potential Energy = 500 x 10 = 5000 ft. lbs.

The measure of the kinetic energy which a body has by virtue of its motion is equal to the work expended in order to move the body at a certain speed. In stopping, the body will give up an amount of energy equal to the work done in starting the motion if losses due to friction, drag and so on are neglected.

Kinetic Energy = Work spent in accelerating body to velocity v

Kinetic Energy = F x S = (ma) $(\frac{1}{2} at^2)$ assuming uniform acceleration

or Kinetic Energy = $(m \frac{v}{t})(\frac{1}{2} \frac{v}{t} t^2) = \frac{1}{2} m v^2$ Equation 1.1.9

Thus, the Kinetic Energy a 500 lb weight would possess by virtue of its velocity of 10 ft/sec is 776 ft-lbs. as shown:

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} \frac{W}{g} v^2 = \frac{1}{2} \frac{500}{32.2} (10)^2 = 776 \text{ ft. lbs.}$$

1.1.14 TOTAL ENERGY:

The total energy of a body may be composed of potential energy, kinetic energy and rotational energy. An artillery projectile in flight has a total energy which consists of all three types. For flight test applications we are primarily concerned with potential and kinetic, and will assume that the rotational energy is zero. An expression for the total energy is now

E_t = potential energy + kinetic energy

$$E_t = Wh + \frac{W V^2}{2g}$$ Equation 1.1.10

Units

E = ft - lbs

W = lbs

h = ft

V = ft/sec

g = ft/sec^2

The introduction of the turbojet and rocket engine has expanded the flight envelop of an aircraft to values where an energy analysis is often convenient in determining climb schedule, zoom and dive potentials, acceleration time intervals, turning performance and range. Section 3.6 describes in detail the energy concept of flight test work.

1.2 DIMENSIONAL ANALYSIS

1.2.1 INTRODUCTION

Dimensional analysis is a tool which in its simplest form is used every day in checking the validity of algebraic expressions. In its more sophisticated forms it is a powerful tool for reducing a large number of variables in a problem to a <u>minimum</u> number of nondimensional parameters. Under certain conditions when there are not too many variables it is possible to derive an equation directly from the functional relation $f\ (l,m,n) = 0$.

It is the purpose of this section to present some of the uses of dimensional analysis along with some principles and techniques required to apply it to specific problems.

1.2.2 DIMENSIONAL CHECKING

We all should be familiar with the process of dimensionally checking algebraic equations. It simply involves the principle that the dimensions of the quantities that appear on the left side of an equation must equal those on the right. For instance, the distance traveled is equal to the velocity times the time enroute

$$d = v\ t$$

To dimensionally check this expression we substitute the appropriate dimensions where L is the units of length and t, is the units of time. Substituting the appropriate units in the above expression we obtain

$$L = \frac{L}{t} \times t$$

$$\text{or} \qquad L = L$$

Thus the expression is dimensionally correct.

1.2.3 DERIVATION OF EQUATIONS FROM FUNCTIONAL RELATIONSHIPS

We now want to become familiar with the more advanced techniques of dimensional analysis. Two things are required to attack a particular problem. First, we must know all of the variables in the problem, f (l,m,n,....). It does not matter if we have too many variables in the functional relationship but we must not fail to include the primary ones. It is well to keep unnecessary variables to a minimum since it complicates the mathematics in the derivation. Second, we must set up a consistant dimensional notation. Two sets of basic dimensions are available. One is the mass, length, time, (M,L,t) system and the other is the force, length, time (F,L,t) system. Regardless of which system is used the final result will be the same. Since the beginning student has a better feel for units of force than for units of mass the (F,L,t) system will be used. To facilitate the use of dimensional analysis we will include a table of commonly occurring variables in terms of their basic dimensions.

TABLE I

DIMENSION	UNITS
Length	L
Area	L^2
Volume	L^3
Velocity	L/t
Acceleration	L/t^2
Mass $= \frac{w}{g}$	$\frac{Ft^2}{L}$
Pressure	F/L^2

Rotational Speed (RPM)	$1/t$
Temperature $\propto$ Energy	L^2/t^2
Density	$\frac{Ft^2}{L^4}$
Viscosity	Ft/L^2

To illustrate the principle of dimensional analysis, let us derive the equation of state and include an unnecessary variable. Let us suppose that we have no idea of the form of the equation. Further more let us say that we observe that pressure varies with density and temperature but we are not sure whether it varies with viscosity or not, so let's include it as a part of the analysis.

$$P = f\ (\rho, T, \mu)$$

From our table we find the basic dimensional forms

$$P = F/L^2 \qquad \rho = \frac{Ft^2}{L^4}$$

$$T = L^2/t^2 \qquad \mu = \frac{Ft}{L^2}$$

We can express the above functional relationship as an exponential equation with unknown exponents, (a,b,c,d,.......).

$$P = C\left[\rho^a\ T^b \mu^c\right]$$

where C = a dimensionless constant

We can find the value of these exponents by simultaneously adjusting the exponents of each basic dimension in such a way that it is the same on both sides of the equation. This is done by solving a set of simultaneous linear equations.

Substituting in the basic dimensions for ρ , T and μ , the equation becomes,

$$\frac{F}{L^2} = \left[\left(\frac{Ft^2}{L^4}\right)^a \left(\frac{L^2}{t^2}\right)^b \left(\frac{Ft}{L^2}\right)^c\right].$$

Equating exponents of like dimensions we get

$$F: \quad 1 = a + c$$

$$L: \quad -2 = -4a + 2b - 2c$$

$$t: \quad 0 = 2a - 2b + c$$

simultaneous solution of these three equations gives the values of the exponents:

Multiplying the "t" equation by 2 and adding to the "L" equation gives

$$-2b = -2$$

$$b = 1$$

subtracting the "F" equation from the "t" equation and substituting the value for b gives

$$a - 2 = -1$$

$$a = 1$$

Then from the F equation $c = 0$. Thus, we see that the equation of state is not a function of the viscosity and that even though it was included in the dimensional analysis it did not affect the final result.

Substituting the values of "a" and "b" into the exponential equation gives the equation of state, where the constant c is really the gas constant R.

$$P = C \rho T$$

$$P = \rho R T$$

This result can be dimensionally checked as before where R is a dimensionless constant. NOTE: In general, the exact equation can not be explicitly obtained if more than three variables are involved. Also note that each of the variables are not dimensionless.

1.2.4 BUCKINGHAM Pi THEOREM

A variation of the previous method is commonly known as the Buckingham π Theorem. It differs from the previous method in that it yields a group of dimensionless parameters called π_1, π_2, π_3,etc.,rather than the variables themselves to some exponential power. It also allows the prediction of the number of these parameters that will be obtained. In short, it is simply a more systematic means of reducing a large number of variables to a minimum number nondimensional parameters.

It should be remembered that in general dimensional analysis will not give the complete equation desired but will only give the proper functional relationship. In the Buckingham π notation the relationship will be of the form

$$\pi = f(\pi_1, \pi_2, \pi_3, \ldots.) = 0$$

where $\pi_1, \pi_2, \pi_3, \ldots$ are the dimensionless parameters and π denotes the functional relationship as a whole.

The number of independant dimensionless parameters that will occur for any specific problem is given by n , (the number of variables, velocity, pressure, density, etc) minus m (the number of basic dimensions (F,L,t) occurring in the variables.) Thus, if $n - m = 3$ then three dimensionless parameters π_1, π_2, π_3 can be obtained. As an example, let us consider the variation of velocity as a function of distance and time. There are three variables

(v, d, t) and two basic units (L, t); therefore, n - m = 1 indicating that one dimensionless parameter will be obtained. That is the function

$$\pi = f\,(v, d, t) = 0$$

can be reduced to

$$\pi = f\,(\pi_1) = 0$$

by use of the Buckingham π Theorem.

It should be noted that the above functional relationship can be written in any number of forms all of which are correct.

$$f\,(v, d, t) = 0$$

$$v = f\,(d, t)$$

$$d = f\,(v, t)$$

$$t = f\,(d, v)$$

In general, the functional relationship derived from the Buckingham Theorem will be of the form

$$\pi = f\,(\pi_1, \pi_2, \pi_3, \ldots \pi_{(n - m)})$$

where the number of parameters is given by the factor n - m.

The techniques of the π Theorem are best explained by a simple example. Let us consider the previous equation of state problem. First of all the independent variables are pressure, density, temperature and viscosity (i.e. n = 4)

All three basic dimensions are involved so that m is 3. Thus, the number of nondimensional parameters that can be expected is n - m = 1.

Writing in functional notation

$$\pi = f(\pi_1) = f(P, T, \rho, \mu)$$

or in exponential form

$$\pi = P^a \, T^b \rho^c \mu^d = \text{constant}$$

any one of the exponents can be set equal to <u>one</u> with out changing the result.

letting $a = 1.0$

$$\pi_1 = (P \; T^b \; \rho^c \; \mu^d)$$

or in dimensional form

$$\pi_1 = \left[\frac{F}{L^2}\right] \left[\frac{L^2}{t^2}\right]^b \left[\frac{Ft^2}{L^4}\right]^c \left[\frac{Ft}{L^2}\right]^d = \text{constant}$$

Equating exponents of F, L and t

$$F: \quad 1 + C + d = 0$$

$$L: \quad -2 + 2b - 4C - 2d = 0$$

$$t: \quad -2b + 2C + d = 0$$

Simultaneous solution of these equations give

$a = 1 \qquad C = -1$

$b = -1 \qquad d = 0$

Thus

$$\pi_1 = \frac{P}{\rho T} = \text{constant}$$

or $$\pi = f(\pi_1) = f(P/\rho T) = 0$$

This is the same result obtained previously.

More advanced examples of dimensional analysis are found in section 2.3.1 and 3.2.3a

In summary, dimensional analysis may be used to check algebraic expressions, to derive simple equations from the functional relations such as f (l, m, n, ...) and to at least reduce functional relationships with a large number of variables to a minimum number of nondimensional parameters by the Buckingham π approach.

Dimensional analysis is especially useful in flight testing since there are a large number of aerodynamic and engine variables which must be considered with no known mathematical relations connecting them. Dimensional analysis provides a means of combining these variables into a smaller number of aerodynamic and engine parameters which can then be plotted and analyzed in a conventional manner.

1.3 FLUID MECHANICS

In mechanics the energy relationship and Newtons three laws were applied to rigid bodies. Now, we wish to apply these principles of mechanics to fluids in order to get a better understanding of fluid motion which later will be applied to the study of aerodynamics. Aerodynamics is merely a specialized branch of fluid mechanics which deals specifically with the flow of air as the working fluid. This section will consider some fundamental characteristics common to all fluids and where applicable an analogy will be drawn between the mechanical and fluid mechanical systems. This section is not meant to be a complete thesis of the science of fluid mechanics but it will provide the basic background information required for a better understanding of the material covered in the sections which follow.

1.3.1 THE STRUCTURE OF MATTER

It is well known that all matter is made up of atomic particles which themselves are made up of subatomic particles called protons, neutrons, electrons, etc. Those which are most significant for our consideration are the primary charged particles, namely the protons and electrons. These particles are responsible for the electrical bonds which hold matter together. They are the forces which determine whether matter will be in a solid, liquid or gaseous form and if in a gaseous form whether they will exist as molecules (i.e., combinations of atoms) or as free atoms or ions (charged particles).

Solids are characterised by a fixed crystaline structure which because of strong electrical bonds holds each atom in a given location relative to the other atoms. While each atom does have a certain degree of freedom such as vibration they can not migrate randomly through the crystaline structure to any extent as can molecules in a fluid or gas.

A liquid represents a state which is neither solid nor gaseous. It does not have a fixed crystaline state nor does it have a completely random motion. Inter molecular attraction is the dominating factor in liquid but these bonds are not so strong that they can not be easily broken or deformed. Therefore, fluids may be thought of as being crystaline at any instant but with the crystaline structure changing with time. Some evidence of this semicrystaline state has come to light in recent research.

Gases on the other hand are almost completely independent of intermolecular attraction. They are characterized by a completely random and chaotic motion so that their motion can be expressed only as a statistical probability. The basic element of a gas is the molecule which is generally made up of two or more tightly bonded atoms, so that, they have little capacity for bonding to other molecules. (Note: this in the absence of chemical reaction).

From this, it is seen that the difference between the solid, liquid or gaseous states is just a matter of degree and so, in the broadest sense all may be considered as fluids for all have some capability to deform or flow. Thus, we see that many of techniques used to evaluate the deformation properties of solids may be modified and generalized for application to liquids and gases. This again suggests a connection between mechanics and fluid mechanics. For the purposes of this writing the majority of the discussion will be directed toward fluids made up of diatomic gases as found in air (Oxygen, O_2, Nitrogen, N_2). However, an attempt will be made to keep the discussion as general as possible.

1.3.2 FLUID MOTION

Before launching into a detailed discussion of fluid motion it is well to get a worms eye view in order to define what is meant by fluid flow of a gas.

1.3.2.1 KINETIC THEORY OF GASES

When a gas is referred to as being at rest it does not mean that there is no motion within it. It merely means that there is no net transmission of mass past a given point. Gases are made up of molecules which are continuously in motion. These molecules may be visualized as being very tiny billiard balls which randomly travel about colliding with each other and with the wall of their container. The collisions with the walls and other molecules are perfectly elastic so that no energy is lost.

Since the direction of motion and the velocity of these molecules are completely random, a statistical sample would show that as many molecules are traveling in one direction as in the other and that the total energy * and momentum are equal. Remembering that pressure is proportional to momentum and temperature is proportional to the kinetic energy of the molecules, the temperature and pressure should be the same at any point in the gas. The latter two conditions are born cut by the fact that experimentally the temperatures and pressures are observed to be the same no matter where the measurements are taken in the container. Thus, it follows that the average velocity of the molecules are the same in all directions and that there are as many molecules going by a given point in one direction as are going in the opposite direction. This is the static or no flow condition.

Now, if there is flow, there will be a resultant average velocity of the molecules past this point in the direction of flow which is equal to the flow

* This concept is called "equipartician of energy".

velocity. In other words, the fluid motion is merely superimposed on the random moleculor motion.

1.3.2.2 THE FLUID ELEMENT

Ultimately it is our purpose to discuss specific types of fluid flow. But first it is necessary to gain a basic understanding of the nature of fluids, the means by which forces are transmitted, and the effects of these forces on the fluid. A fluid may be visualized as being made up of an infinite number of tiny differential elements. These fluid elements are the same as the free body in mechanics; that is, the sum of the external forces acting on the element result in velocity and acceleration which is proportional to the mass of the element. The fluid element differs from the free body in that it is not rigid and therefore can be deformed as well as translated. For our purposes the fluid element will be defined as a very small cubic differential element whose sides, dx, dy and dz, are mutually perpendicular to each other.

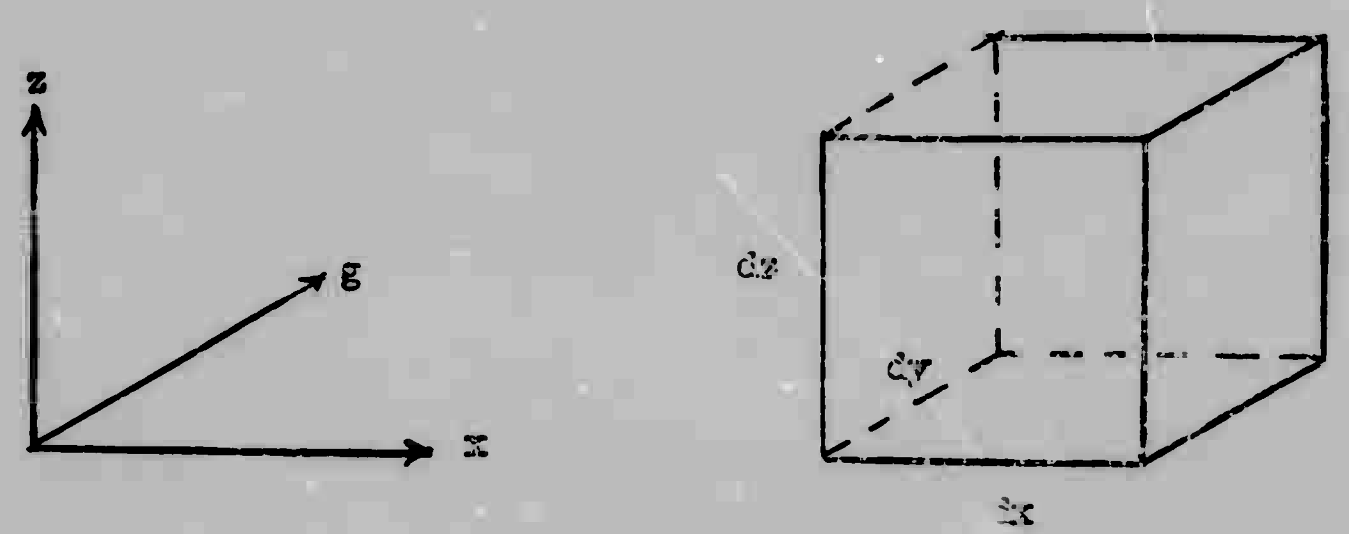


Figure 1.3.1 Fluid Element

The volume of this element is given by

$$dV = dx \cdot dy \cdot dz \qquad \text{Equation 1.3.1}$$

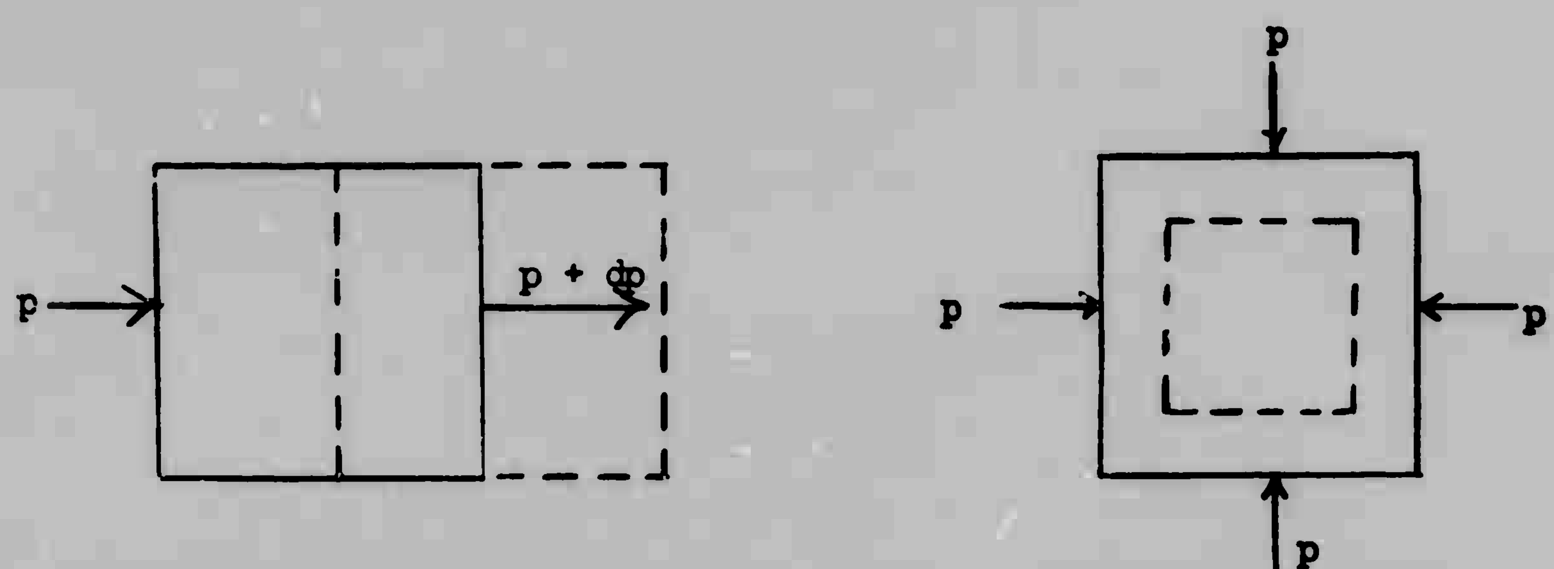
Using the fluid element concept we can apply forces to the element and draw conclusions concerning the fluid motion.

1.3.2.3 FLUID FORCES

Two types of forces can act on the fluid element:

1. Hydrostatic or pressure forces and
2. Viscous or shear forces

The pressure force always acts normal to the surface being considered. If this were not so, the pressure measured on the wall of anything but a spherical pressure vessel would depend on the position of the pressure gage in the tank. We know that this is not the case. So we conclude that the pressure only exerts a force which is perpendicular to the surface. In terms of the fluid element then, it is seen that pressure can cause only translational motion if applied differentially, and compression and expansion if applied uniformly, Figure 1.3.2.



Differential Application of Pressure

Uniform Application of Pressure

Figure 1.3.2 Effects of Pressure on the Fluid Element (dotted lines indicate the results of the dp increase)

Viscous forces can act either parallel or perpendicular to the surface being considered. Viscous forces are caused by both normal and horizontal stress in a substance. By analogy to a solid, the normal stress is like the tensile stress and horizontal stress is the same as the shear stress. Stress is defined in terms of the strain (percent deformation) in a substance.

For uniformly applied pressure, or the normal viscous stress, Figure 1.3.3, one corner of the fluid element has a different velocity relative to the other. Therefore, a strain, ϵ, is developed which is given by

$$\epsilon = \frac{\frac{du}{dx} \cdot dx}{dx} = \frac{du}{dx} \qquad \text{Equation 1.3.2}$$

The stress - strain relationship for the pressure is then

$$dp = E\epsilon = E\frac{du}{dx} \qquad \text{Equation 1.3.3}$$

where E is a constant of proportionality called the bulk modulus or for metals, the modulus of elasticity.

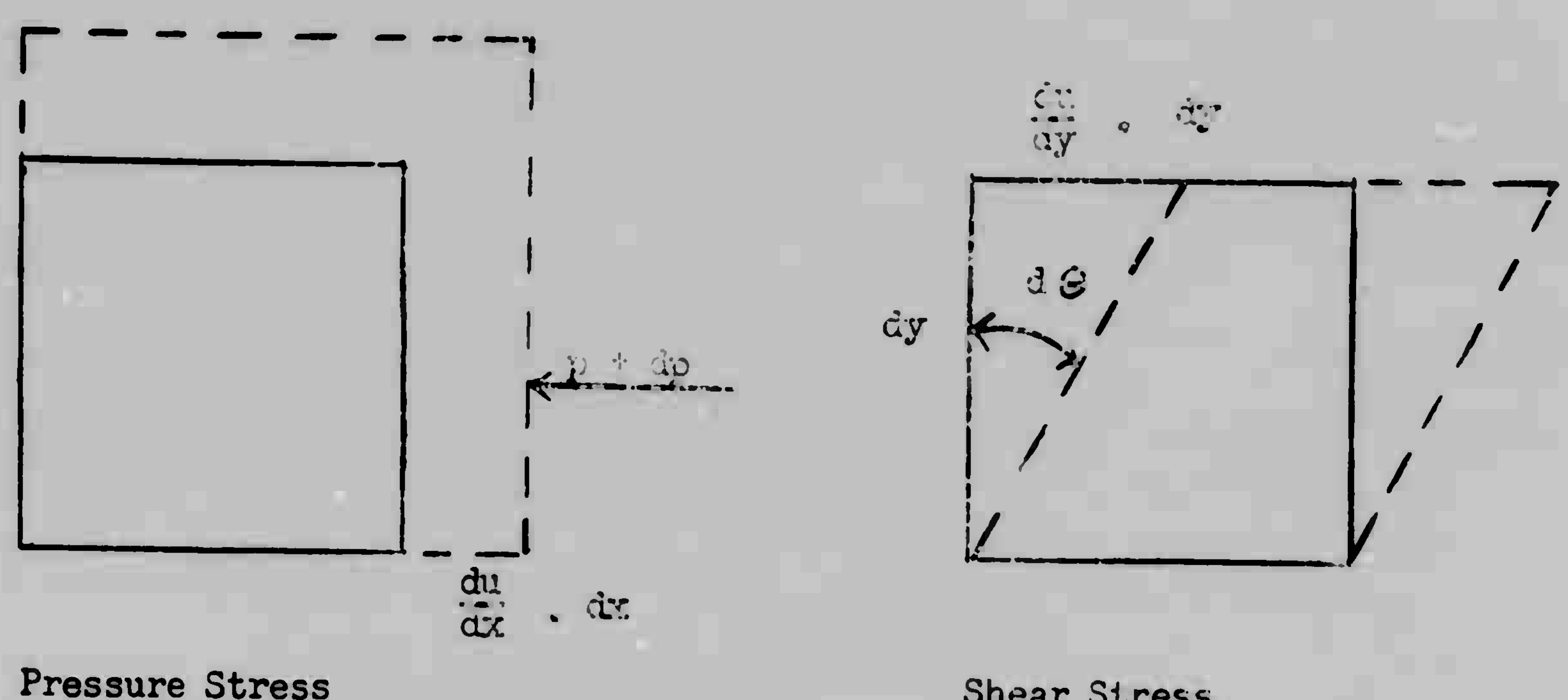


Figure 1.3.3

The shear stress is defined in terms of the angular strain, $d\theta$, Figure 1.3.3.

$$d\theta = \frac{\frac{du}{dy} \cdot dy}{dy} = \frac{du}{dy}$$

The stress - strain relationship is

$$\tau = k\,d\theta = k\,\frac{du}{dy} \qquad \text{Equation 1.3.4}$$

where k is a constant of proportionality which for fluids is called the coefficient of viscosity, μ, and for solids the shear modulus.

Thus, we see that shear stresses in fluids are proportional to the rate of strain $\frac{du}{dx}$, $\frac{du}{dy}$ or $\frac{du}{dz}$

From the previous discussion it is seen that there can be as many as three shear stresses acting on any face of a fluid element, one along each of the three axes, Figure 1.3.4. This makes a total of 18 stresses for the six forces of the element. On the otherhand, there are only six possible pressure forces acting on the element since pressure does not act parallel to the surface, Figure 1.3.4. The pressure and viscous stresses have been shown on separate diagrams in Figure 1.3.4, but of course they act at the same time in an actual viscous flow.

BLANK PAGE

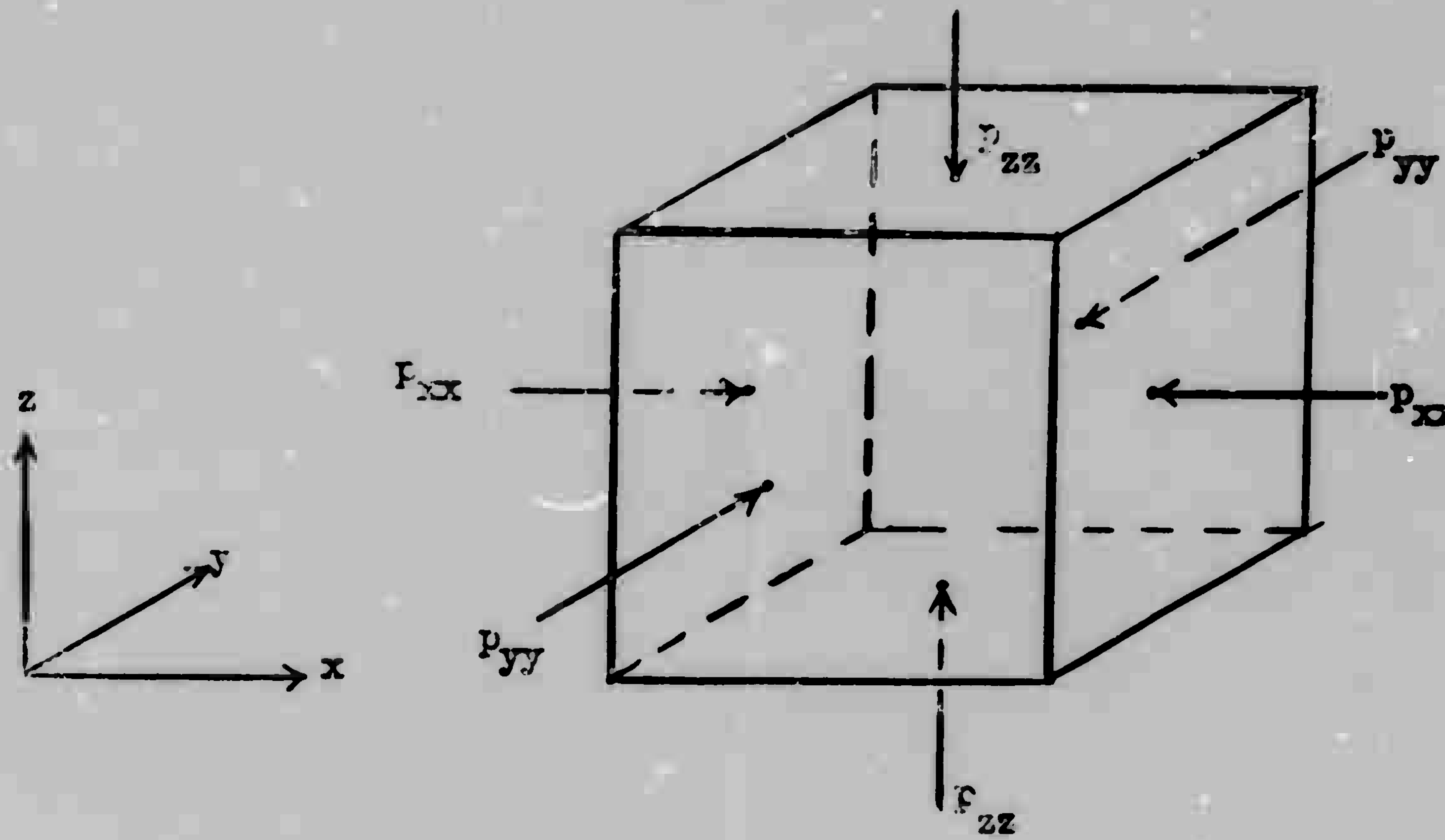


Figure 1.3.4 Pressure Stresses

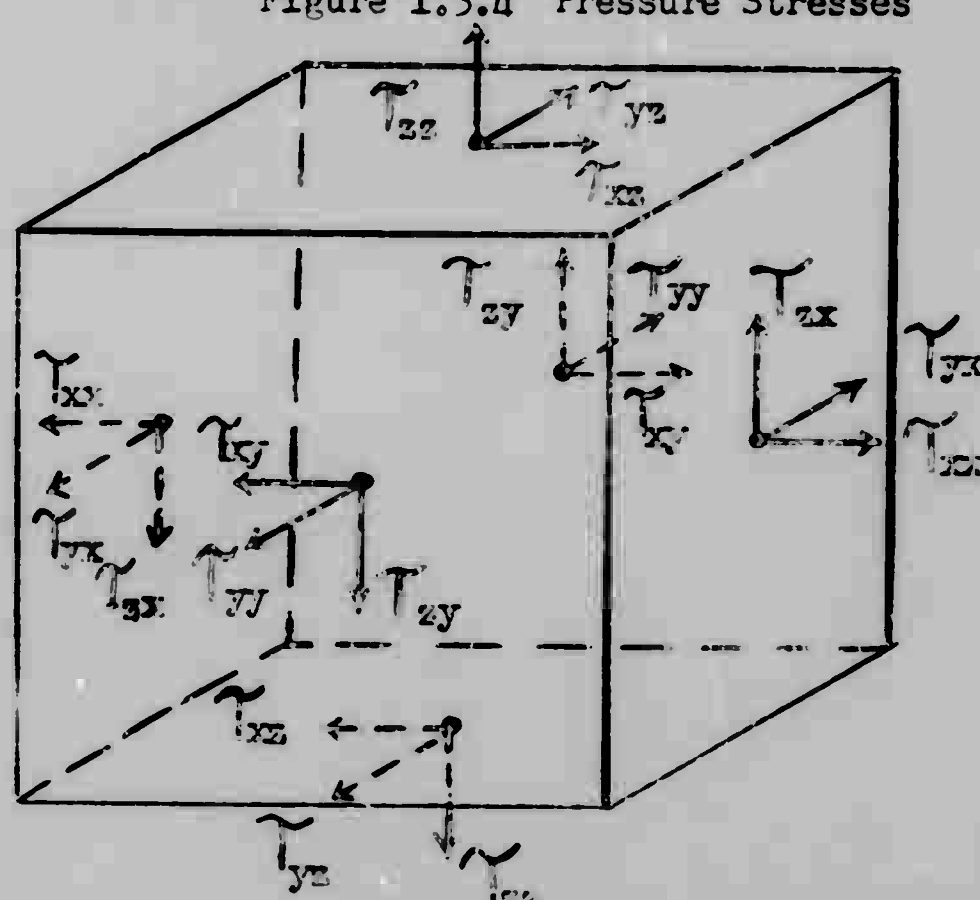


Figure 1.3.4 Viscous Stress

Note: A subscript notation has been used for Figure 1.3.4 to indicate the direction of the stress and the surface to which it is applied. For instance τ_{xy} indicates a stress in the x direction acting on a surface whose normal is along the y axis. Thus, τ_{xx}, τ_{yy} and τ_{zz} represent normal stresses which for a gas are very small compared to the shear stress. This is because the normal stress is primarily caused by inter molecular attraction which is almost negligable in gases.

In Figure 1.3.4 it is seen that the shear stresses act in pairs which result in the creation of a moment. Therefore, they are responsible for the rotation and deformation of the fluid element.

1.3.2.4 MOTION OF THE FLUID ELEMENT

In response to the pressure and shear stresses discussed above, the fluid element translates, rotates, dilates and deforms. Translation of the fluid is easily understood since an unbalanced pressure stress, dp, results in an unbalanced force which must from Newtons Second Law accelerate the fluid. Likewise, the rotational motion is conveniently attributed to the unbalanced moment caused by the shear stresses. Dilation is simply the expansion and contraction of the fluid element in response to the pressure stress and the normal viscous stress. Deformation is caused by the shear stress tending to skew the cubic fluid element into rombic configuration, Figure 1.3.5.

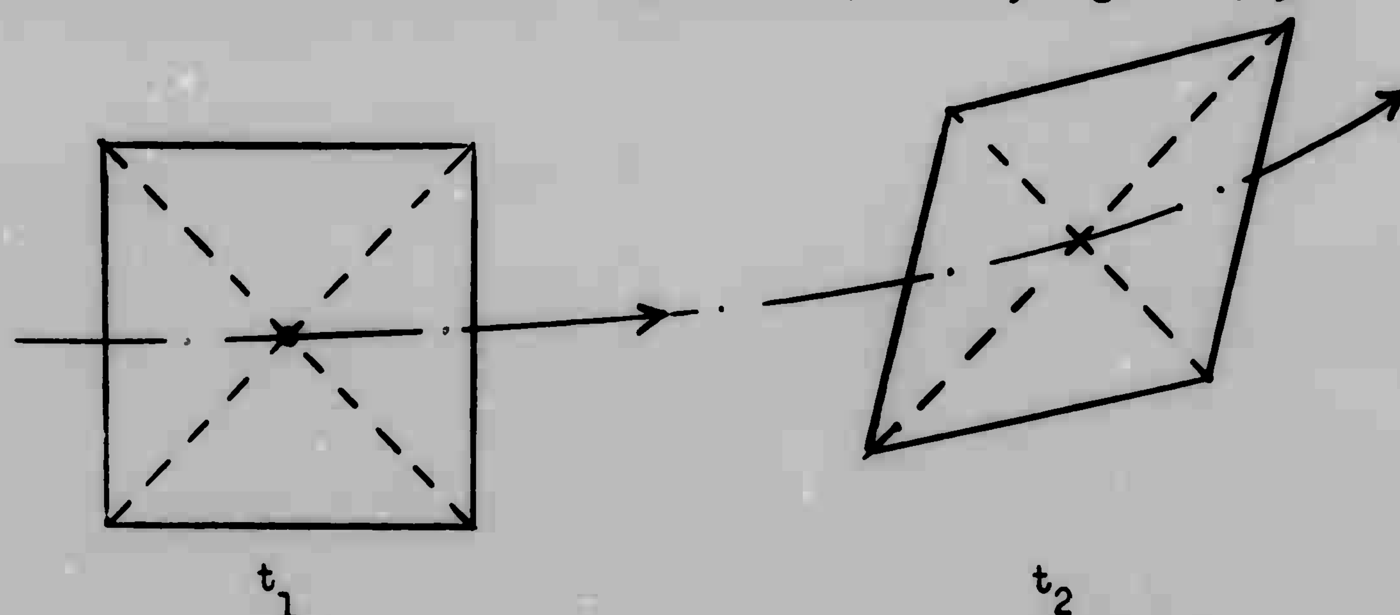


Figure 1.3.5

A careful distinction between deformation and rotation must be drawn for fluid flow for it is possible for the fluid to appear to rotate when actually it has deformed as in Figure 1.3.5. When a solid cube rotates its geometry

remains fixed so that the diagonals rotate the same amount as the sides. Since a fluid element is not rigid the sides do not maintain a fixed relationship to the diagonals, thus requiring a closer look at the concept of rotation. It can be shown that the angular change of the diagonals establishes a consistant criterion for the rotation of the fluid element. As shown in Figure 1.3.5, the angular relationship of the sides changed with respect to the flow direction but the diagonals did not; therefore, the fluid element has deformed from its original configuration at time, t_1, but it has not rotated.

1.3.3 CONSERVATION EQUATIONS

There are three basic fundamental concepts which are common to all of the various branches of science. These concepts are often referred to as the conservation relations.

1. Conservation of mass or continuity simply states that "mass is neither created nor destroyed". That is if mass disappears in one portion of a system it will appear elsewhere generally as mass but sometimes as energy. Otherwise stated, in the absence of the conversion mass to energy through nuclear means the mass that enters a system must eventually leave or be stored within the system. A basic example of this is shown through the following example: If ten marbles are thrown into an empty box which has a small hole in the bottom, some of the marbles will drop through the hole and some will remain in the box. The principle of conservation of mass requires that the total number of marbles entering the box must equal the number which passed through plus the number which are still in the box. A more formalized mathematical description of this concept will be applied to fluid flow.

2. Conservation of Momentum - This is known to most people as Newton's Second Law

$$F = ma$$

or more generally

$$F = \frac{d(m\,v)}{dt} \qquad \text{Equation 1.3.5}$$

where mv is the momentum.

While this principle is called the conservation momentum this is not strictly so, except in a frictionless system with no applied force. More generally it states the relationship between the change in momentum and the force required to produce this change. A general application of this principle to fluid flow will be presented in this section.

3. Conservation of energy is the third member of the fundamental equations. It will be discussed in more detail in section 1.4 but a short description is in order at this time. Like the conservation of mass, "conservation of energy" means that energy is neither created nor destroyed but is merely transformed. Energy under this concept is not only the kinetic and potential energy of mechanics but also the internal energy, heat or thermal energy, chemical energy and work or friction energy. It is well known that potential energy can be converted into kinetic energy, and chemical energy to heat and work; but it is sometimes not so well understood how kinetic energy or work is converted into heat. Actually all are interchangeable if a means can be devised to make the conversion.

Now it is desired to apply the principles of conservation of mass and momentum to fluid flow. To be completely general we should not consider the fluid flow as occurring along any one coordinate axis or to be constant with time, so where possible we shall consider three dimensional nonsteady flow. In order to help visualize and study three dimensional flow we shall resort to the concept of an elemental control volume which is fixed relative to the flow; that is, the fluid flows through it in some arbitrary direction as shown in Figure 1.3.6a. The control volume is different from the fluid element in that the fluid element encompassed a given mass of fluid and followed it through the flow, while the control volume is fixed in space and the fluid flow is observed as it passes through it. As shown in Figure 1.3.6a and b the velocity, u, of the fluid flowing through the volume can change with respect to x, y and z coordinates and with respect to time.

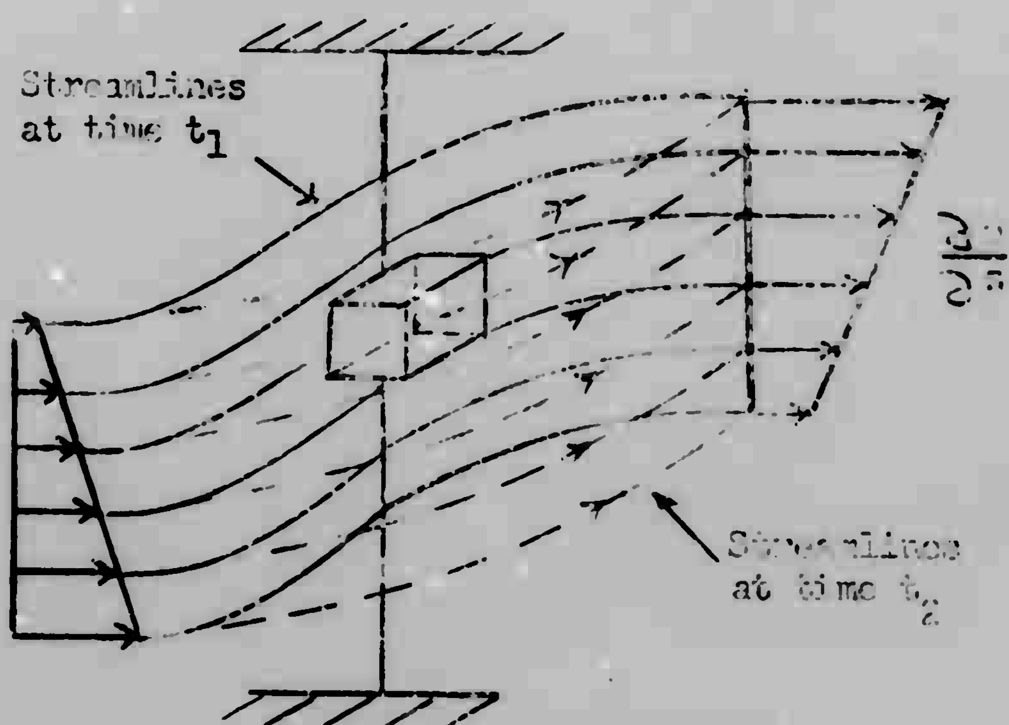


Figure 1.3.6a Streamlines Through the Control Volume

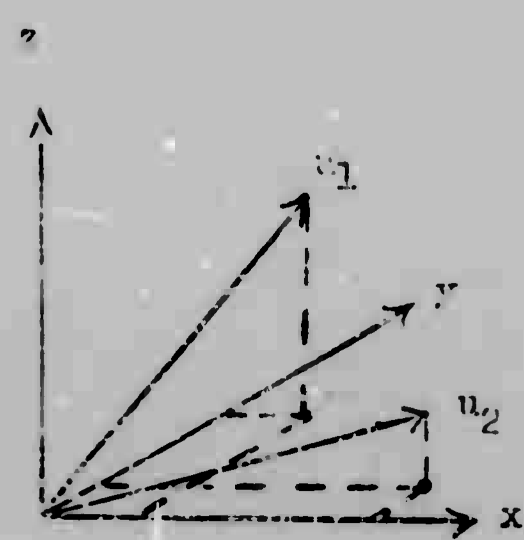


Figure 1.3.6b Coordinate System

This fact may be expressed

$$u = f(x, y, z, t) \qquad \text{Equation 1.3.6a}$$

or in differential form

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy + \frac{\partial u}{\partial z} dz + \frac{\partial u}{\partial t} dt \qquad \text{Equation 1.3.6b}$$

where: $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial z}$ and $\frac{\partial u}{\partial t}$ are partial derivatives of u, that is, all of the remaining variables not included in the derivative are held constant.

$\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial u}{\partial z}$ represent the velocity gradients within the fluid at the location of the control volume.

The partial differential form is useful here because its basic definition allows the change of the dependent variable, u, to be expressed as the sum of the changes in u caused by the separate change in each of the independent variables, x, y, z and t. To review: 1) Partial differentials require that the incremental changes du, dx, dy, dz, etc., be small. 2) The partial derivative, $\frac{\partial u}{\partial x}$, like the total derivative, $\frac{du}{dx}$, shows the slope or rate of change of u with respect to x, but in addition it implies that all of the variables, (y, z and t), other than those shown in the derivative are held constant. To emphasize this point the partial derivative is sometimes written as $\left.\frac{\partial u}{\partial x}\right|_{yzt}$ to indicate which variables are held constant. 3) Thus, it separates the individual effects of each independent variable. In the final analysis these separate effects can be added together as in Equation 1.3.6b to obtain the total resultant change, du.

A physical understanding of the partial derivative is best obtained by a graphical intrepretation of a function involving three variables.

$$z = f(x, y)$$

Such a function describes a surface in three dimensional space coordinates, Figure 1.3.7.

The partial derivative $\frac{\partial z}{\partial x}$ is the slope of the curve in the x-z plane (crosshatched in Figure 1.3.7a) which passes through the point n. Similarly, $\frac{\partial z}{\partial y}$ is the slope of the curve in the y-z plane, Figure 1.3.7a, which passes through n. The change in dz is given by the change in z due to moving in the x direction, dz_x, plus the change in z due to moving in the y direction, dz_y.

$$dz = dz_x + dz_y$$

or

$$dz = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy$$

The validity of this statement is graphically shown in Figure 1.3.7b.

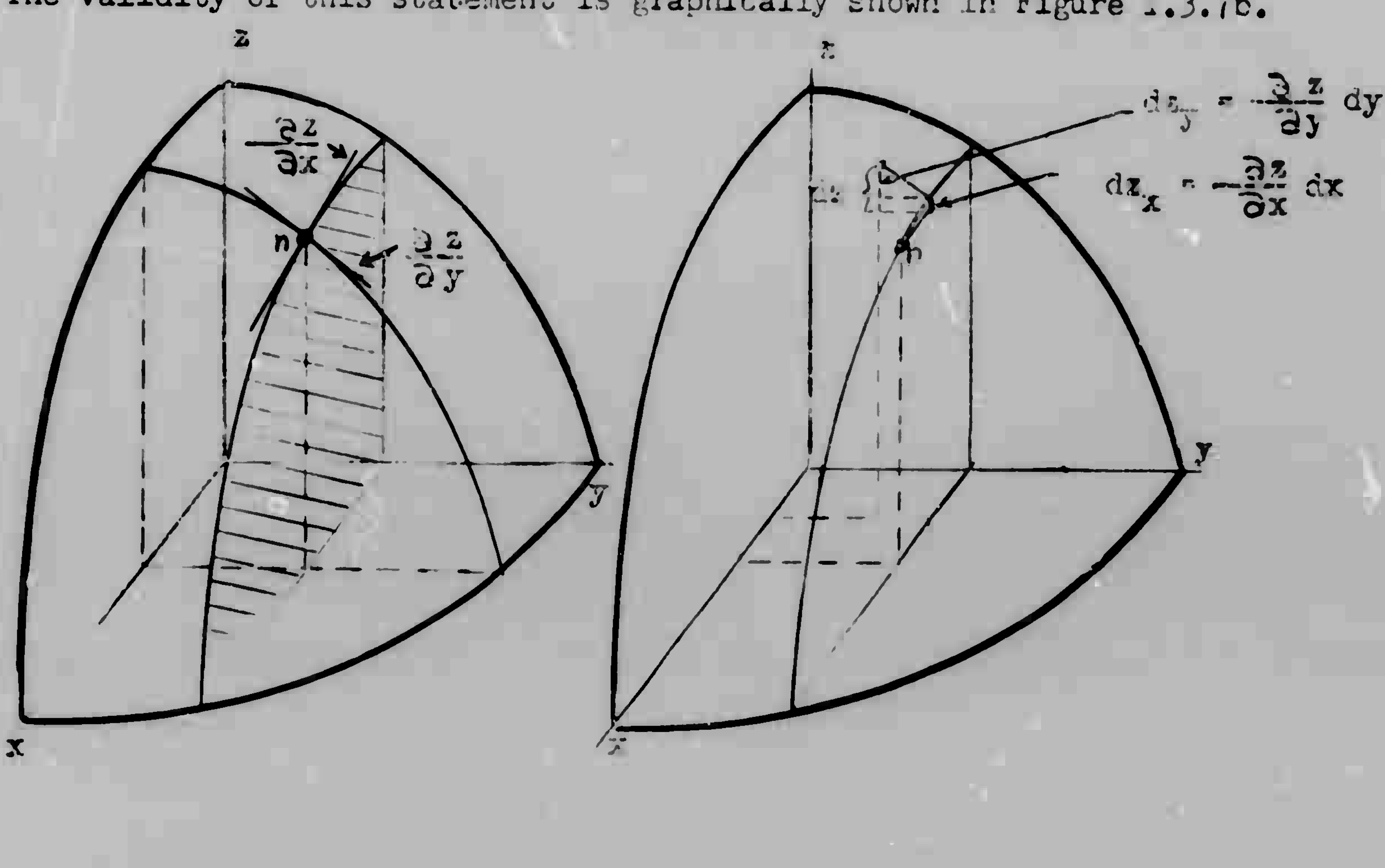


Figure 1.3.7 a and b Partial Derivative

While functions of four or more variables can not be easily interpreted in terms of a graph such as the one above, the concept of a partial derivative is none the less valid. It is just that four and five dimensional space is not readily visualized by our three dimensional minds. For a generalized fluid flow it will seldom be necessary to work with more than four independent variables, as in Equation 1.3.6a and b.

Returning now to Equation 1.3.6b and Figure 1.3.6 we see that the partial derivatives $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ and $\frac{\partial u}{\partial z}$ are the velocity gradients within the fluid at the location of the control volume and that $\frac{\partial u}{\partial x}dx$ represents the change in the velocity of the fluid while traversing the control volume in the x direction. Likewise $\frac{\partial u}{\partial y}dy$ and $\frac{\partial u}{\partial z}dz$ represent the change in the velocity while traversing the control volume in the y and z directions respectively. The variation of u with respect to time is given by $\frac{\partial u}{\partial t}dt$.

The total derivative of u with respect to time, (i.e., the acceleration) may be obtained by modifying Equation 1.3.6b

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial u}{\partial t}$$

or

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\cdot u_x + \frac{\partial u}{\partial y}\cdot u_y + \frac{\partial u}{\partial z}\cdot u_z + \frac{\partial u}{\partial t} \qquad \text{Equation 1.3.7}$$

where u_x, u_y and u_z are the velocity components of u at any given time along their respective axes indicated by the subscripts.
This equation is sometimes referred to as the "fluid derivative".

Using the concepts of the control volume and the partial differential notation we now want to obtain a general form of the conservation equations

1.3.3.1 CONSERVATION OF MASS

The "Conservation of Mass" or "continuity relationship" as it is called in fluid flow simply states that the amount of fluid entering a control volume in a given length of time is equal to the amount that passed through, plus the amount that was stored in the volume during the time interval. Since the fluid entering one face of the control volume does not necessarily leave by way of the opposite face a net increase or decrease in mass flow along any axis may result even if the flow does not vary with time. However, the sum of the inflow and outflow along the axis must equal the amount stored in the volume. Therefore, a valid criterion for the conservation of mass is obtained by subtracting the inflow from the outflow along each axis.

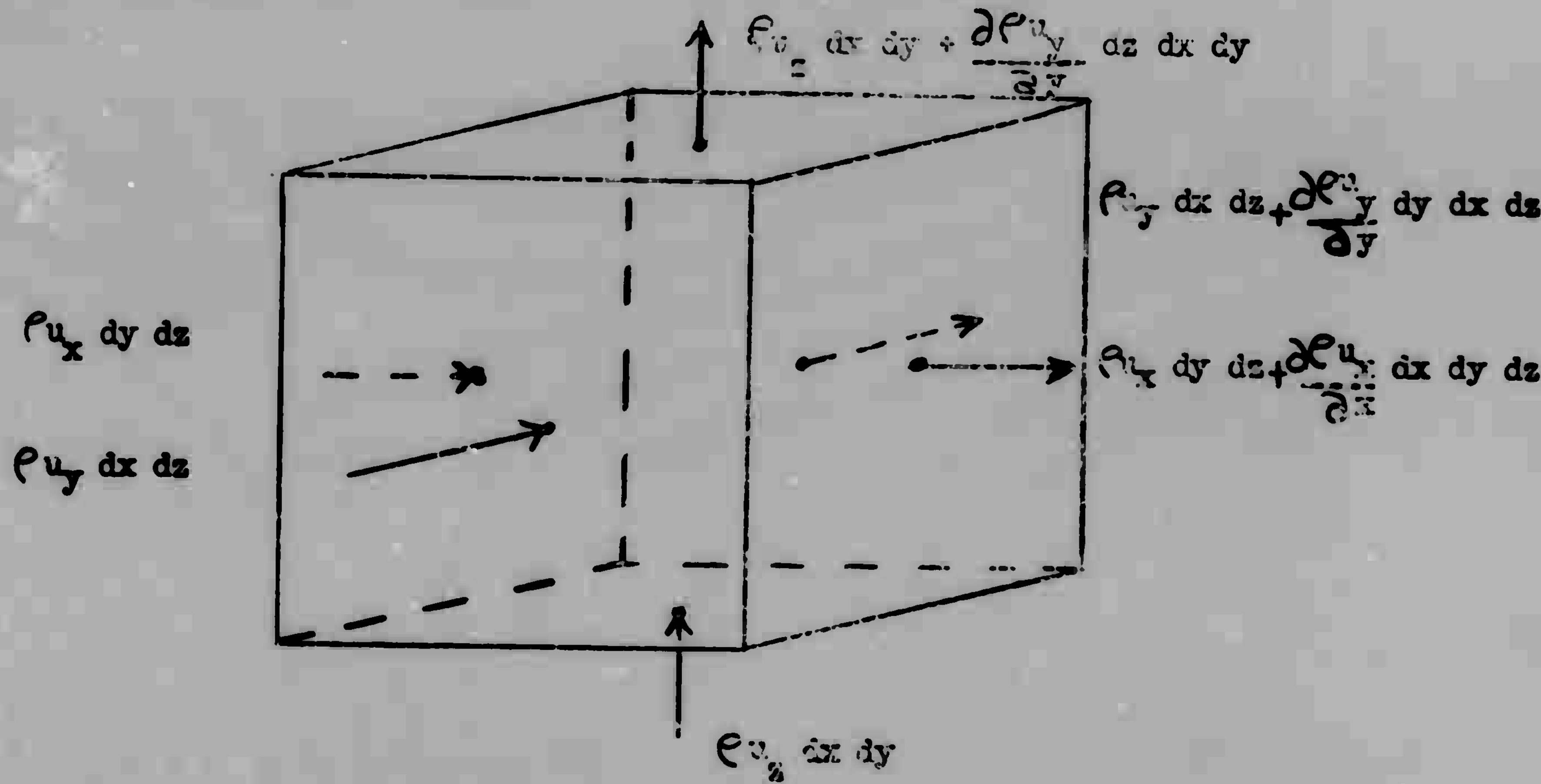


Figure 1.3.8

The inflow and outflow for each face is depicted in Figure 1.3.8 and their summation is as follows:

$$\sum X: -\rho u_x \, dy\, dz + \left[\rho u_x \, dy\, dz + \frac{\partial \rho u_x}{\partial x} dx\, dy\, dz\right] = \frac{\partial \rho u_x}{\partial x} dx\, dy\, dz$$

$$\sum Y: -\rho u_y \, dx\, dz + \left[\rho u_y \, dx\, dz + \frac{\partial \rho u_y}{\partial y} dy\, dx\, dz\right] = \frac{\partial \rho u_y}{\partial y} dy\, dx\, dz$$

$$\sum Z: -\rho u_z \, dx\, dy + \left[\rho u_z \, dx\, dy + \frac{\partial \rho u_z}{\partial z} dz\, dx\, dy\right] = \frac{\partial \rho u_z}{\partial z} dz\, dx\, dy$$

The conservation of mass is:

$$\sum X, Y, Z = \frac{\partial \rho}{\partial t} dx\, dy\, dz = \text{storage rate}$$

$$-\frac{\partial(\rho u_x)}{\partial x} dx\, dy\, dz - \frac{\partial(\rho u_y)}{\partial y} dx\, dy\, dz - \frac{\partial(\rho u_z)}{\partial z} dx\, dy\, dz = \frac{\partial \rho}{\partial t} dx\, dy\, dz$$

or per unit volume

$$-\frac{\partial(\rho u_x)}{\partial x} - \frac{\partial(\rho u_y)}{\partial y} - \frac{\partial(\rho u_z)}{\partial z} = \frac{\partial \rho}{\partial t}$$

The conservation of mass is

$$\frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0 \qquad \text{Equation 1.3.8}$$

A shorthand form of this equation is sometimes used as

$$\frac{\partial \rho u_i}{\partial x_i} + \frac{\partial \rho}{\partial t} = 0$$

where i is a generalized subscript which is alternately given the values x, y, z to expand it to the form of equation 1.3.8.

This then is the general continuity equation for three dimensional non-steady (i.e., varying with time) flow. For the purposes of this course one or two dimensional steady flow considerations are satisfactory. For more simple applications Equation 1.3.8 becomes

Two Dimensional Steady Flow:

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho u}{\partial y} = 0 \qquad \text{Equation 1.3.8a}$$

since $d\rho/dt$ and $\frac{\partial \rho u}{\partial z}$ are zero for the above conditions

One Dimensional Steady Flow:

$$\frac{\partial \rho u}{\partial x} = \frac{d \rho u}{\partial x} = 0$$

If a stream is passing through some cross sectional area other than a unit area, A must be included in the above relations, i.e.,

$$\frac{d(\rho u A)}{dx} = 0 \qquad \text{Equation 1.3.8b}$$

1.3.3.2 CONSERVATION OF MOMENTUM

The conservation of momentum relationship is obtained for fluid flow by applying Newtons Second Law to the control volume.

$$F = ma = m \frac{du}{dt} \qquad \text{Equation 1.3.9}$$

As previously stated, this does not necessarily imply that momentum is conserved but it states the relationship between the change in momentum and the force required to produce this change.

The mass of fluid within the control volume at any given time is

$$M = \rho\, dx\, dy\, dz$$

The acceleration of the fluid was previously shown in Equation 1.3.7. Thus for acceleration in the x direction, the right side of Equation 1.3.9 becomes

$$M \frac{d u_x}{dt} = (\rho\, dx\, dy\, dz)\left(\frac{\partial u_x}{\partial x} u_x + \frac{u_x}{\partial y} u_y + \frac{u_y}{\partial z} u_z + \frac{u_x}{\partial t}\right)$$

The left side of Equation 1.3.9 is obtained by considering the sum of the forces produced by the pressure and shear stresses. The resultant force acting on the control volume is a vector and therefore, can be resolved along the reference axis. The pressure and shearing forces acting in the x direction are shown in Figure 1.3.9.

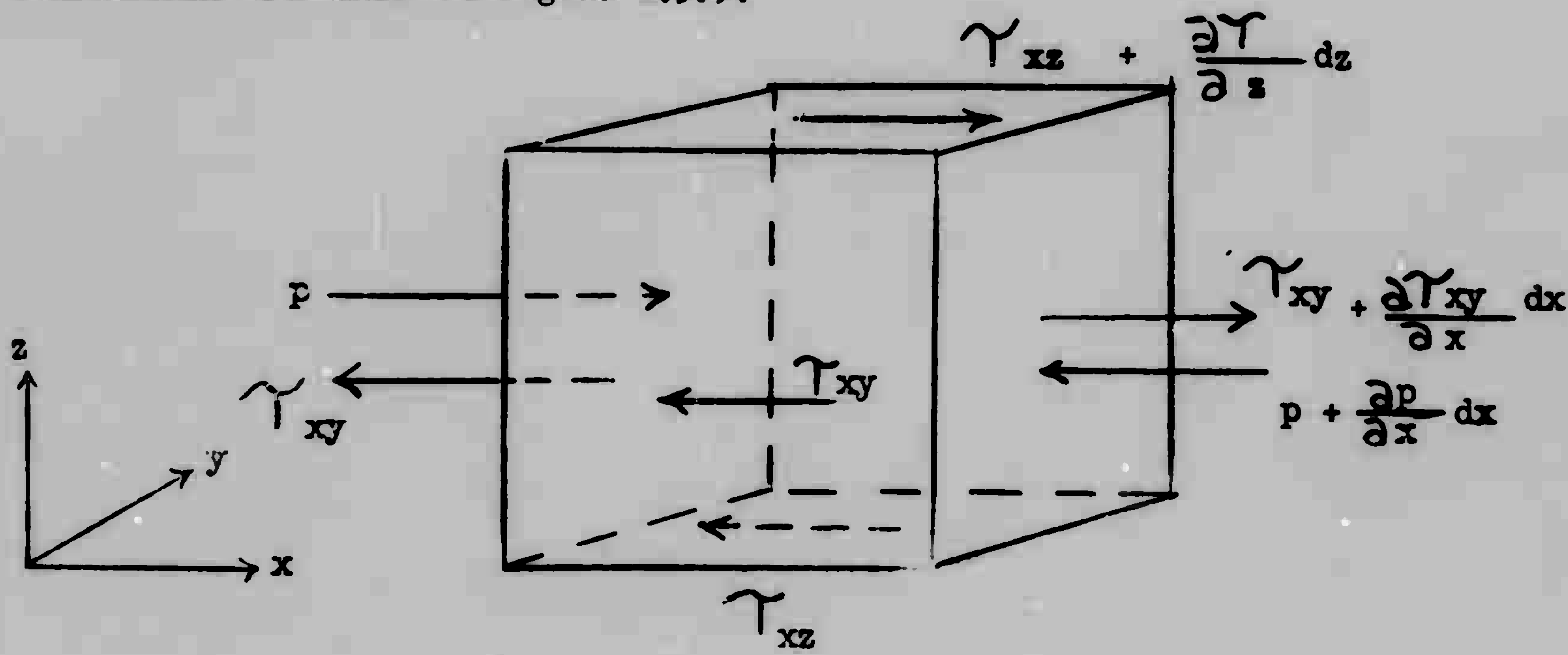


Figure 1.3.9

The sum of these forces are

$$F_x = -\frac{\partial p}{\partial x} dx\, dy\, dz + \frac{\partial \tau_{xx}}{\partial x} dx\, dy\, dz + \frac{\partial \tau_{xy}}{\partial y} dy\, dx\, dz + \frac{\partial \tau_{xz}}{\partial z} dz\, dx\, dy$$

Equation 1.3.9b

Setting the force (Equation 1.3.9b) equal to the momentum (Equation 1.3.9a) gives the momentum equation for the x direction

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho\left[u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} + \frac{\partial u_x}{\partial t}\right]$$

Equation 1.3.10

The other two momentum equations would be obtained by repeating this operation for the forces in the y and z directions.

Like the continuity equation a shorthand form of the momentum equation can be written in a general form where the equation for the cases is obtained by alternately substituting the appropriate subscripts, x, y and z

$$-\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} = \rho u_j \frac{\partial u_i}{\partial x_j} + \rho \frac{\partial u_i}{\partial t}$$

If i is x and j becomes x, y, z Equation 1.3.10 is obtained. If i is y and j becomes x, y, z the momentum equation for the y direction is obtained.

Again the momentum equation can be simplified considerably if steady, two dimensional flow is considered.

X Direction:

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = \rho\left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y}\right)$$

Equation 1.3.11a

Y Direction:

$$-\frac{\partial p}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = \rho\left(u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y}\right)$$

An even greater simplification is obtained if the flow can be considered as being steady, nonviscous and one or two dimensional

2 Dimensional, Nonviscous and Steady

$$\frac{\partial p}{\partial x} = \rho\left(u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y}\right)$$

$$\frac{\partial p}{\partial y} = \rho\left(u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y}\right)$$ Equation 1.3.11b

1 Dimensional, Nonviscous and Steady

$$-\frac{\partial p}{\partial x} = \rho u_x \frac{\partial u_x}{\partial x}$$ Equation 1.3.11c

or

$$dp + \rho u \, du = 0$$ Equation 1.3.11d

This equation is often referred to as Euler's equation.

1.3.4 VISCOUS FLUID FLOW

1.3.4.1 VISCOSITY

Viscosity, in simple terms, is nothing more than the stickiness cohesiveness of a fluid, such as cold molasses sticking to a spoon. It is analogous to friction in the mechanical system. In liquids, it is caused by the intermolecular attraction of the molecules while in gases it is due to the exchange of momentum of two adjacent parallel streams. For this discussion we will concern ourselves with gases.

We have seen in Section 1.3.2.1 that molecules in a gaseous fluid flow tend to migrate randomly through the fluid whether the fluid be in motion or at rest. It is this effect which is responsible for the viscosity of gases.

To understand the mechanism by which viscous forces are developed consider two parallel streams having different velocities, Figure 1.3.10a.

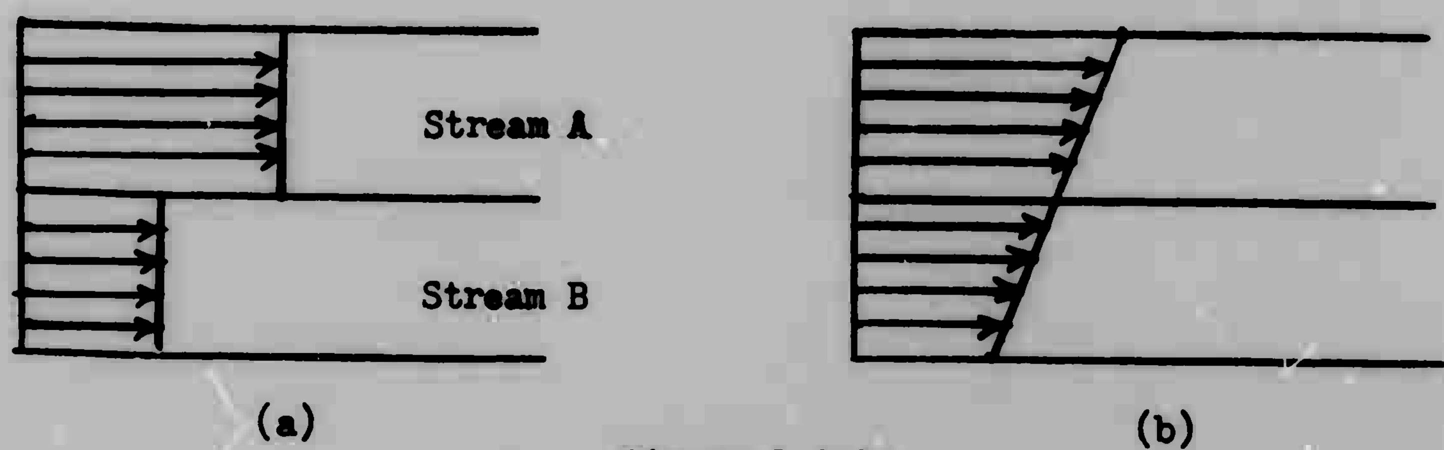


Figure 1.3.10

Due to the random molecular motion,molecules of stream A wander over into stream B and vice versa. The same number of molecules go from A to B as go from B to A so there is no net exchange of mass. However, the average velocity of those molecules going from A to B is greater than the average velocity of those going from B to A. Thus,the momentum (m v) in the direction of flow of the molecules of stream A is greater than the momentum of stream B. Hence the molecules coming from stream A tend to increase the velocity and momentum of stream B while the molecules of stream B tend to decrease the velocity and momentum of stream A. The end result of this process is that the stream velocities adjacent to the boundary adjust themselves to some average velocity as shown in Figure 1.3.10b. Thus, in the final analysis there is no discontinuity in velocity between the two streams. This process might be likened to two coal carrying freight trains traveling in the same direction at different speeds on parallel tracks with men on each train shoveling coal from one to the other. As a result of the difference in momentum of the coal exchanged, the faster train is slowed down and the slower trains speed is increased.

This then is the mechanism which causes shear or viscous forces in a gas. From this we see that the force is proportional to the difference in momentum.

$$F = k\, d(mv)$$

or since the net mass exchanged is zero

$$F = k\, m\, dv$$

Thus, it is seen the shear force is proportional to the number of molecules passing from one stream to another. The number of molecules exchanged can be increased by increasing the temperature of the gas. Therefore, as the temperature of a gas increases the viscosity likewise increases. This is contrary to what is experienced with liquids.

Regardless of the mechanism by which viscous forces arise, whether they be from momentum exchange as in a gas or from intermolecular attraction as in a liquid, they are manifested primarily as shear stresses which are transmitted between the various layers of the fluid. As previously mentioned, normal viscous stresses are generally negligable in gases. The shear stress in a fluid was given in Equation 1.3.4 as

$$\tau = k \frac{du}{dy} \qquad \text{Equation 1.3.4}$$

where in a gas k is equal to the coefficient of viscosity.

$$\tau = \mu \frac{du}{dy} \qquad \text{Equation 1.3.12}$$

Thus, the shear stress between any two layers of a fluid is proportional to the velocity profile $\frac{du}{dy}$, where μ is the constant of proportionality for a given fluid.

From this definition of viscosity, it is seen that the viscous force tends to resist fluid motion in any direction and therefore is the fluid counterpart of friction in mechanics. This friction effect of viscous flow is responsible for several fluid flow characteristics. First, it has a damping effect which tends to dissipate all flow disturbances, such as flow, pulsations, turbulence, etc. This may be witnessed by noting the reduction in the size of surface waves on water as they travel away from their source. This is one of the factors involved in Reynolds number. Second, it is primarily responsible for the formation of the boundary layer which is formed when a fluid flows over a surface.

1.3.4.2 BOUNDARY LAYER

The boundary layer is a thin sheet of retarded fluid immediately adjacent to the surface of a body. It is caused by the shear stress in the fluid which slows the flow next to the surface to zero velocity. As this shearing force is transmitted out into the stream it decreases in magnitude resulting in the familiar velocity profile of the boundary layer, Figure 1.3.11.

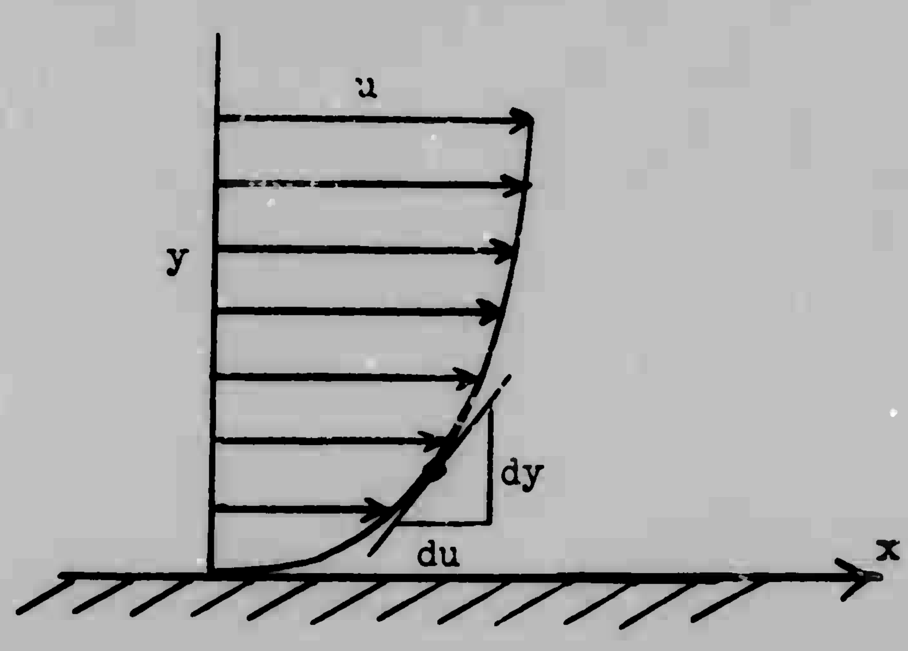


Figure 1.3.11 Typical Velocity Profile in the Boundary Layer

The characteristic features of the boundary layer are:

1. The somewhat parabolic shape indicating a loss of momentum in the boundary layer.

2. Zero velocity at the surface.

3. Free stream velocity at the top. (Actually the top of the boundary layer is defined as the point where 99% of the free stream velocity has been attained. This is because the velocity theoretically approaches but never quite attains free stream conditions).

The development of the boundary layer may be studied by periodically injecting drops of dye into a uniform stream as it approaches and passes over a flat plate as in Figure 1.3.12.

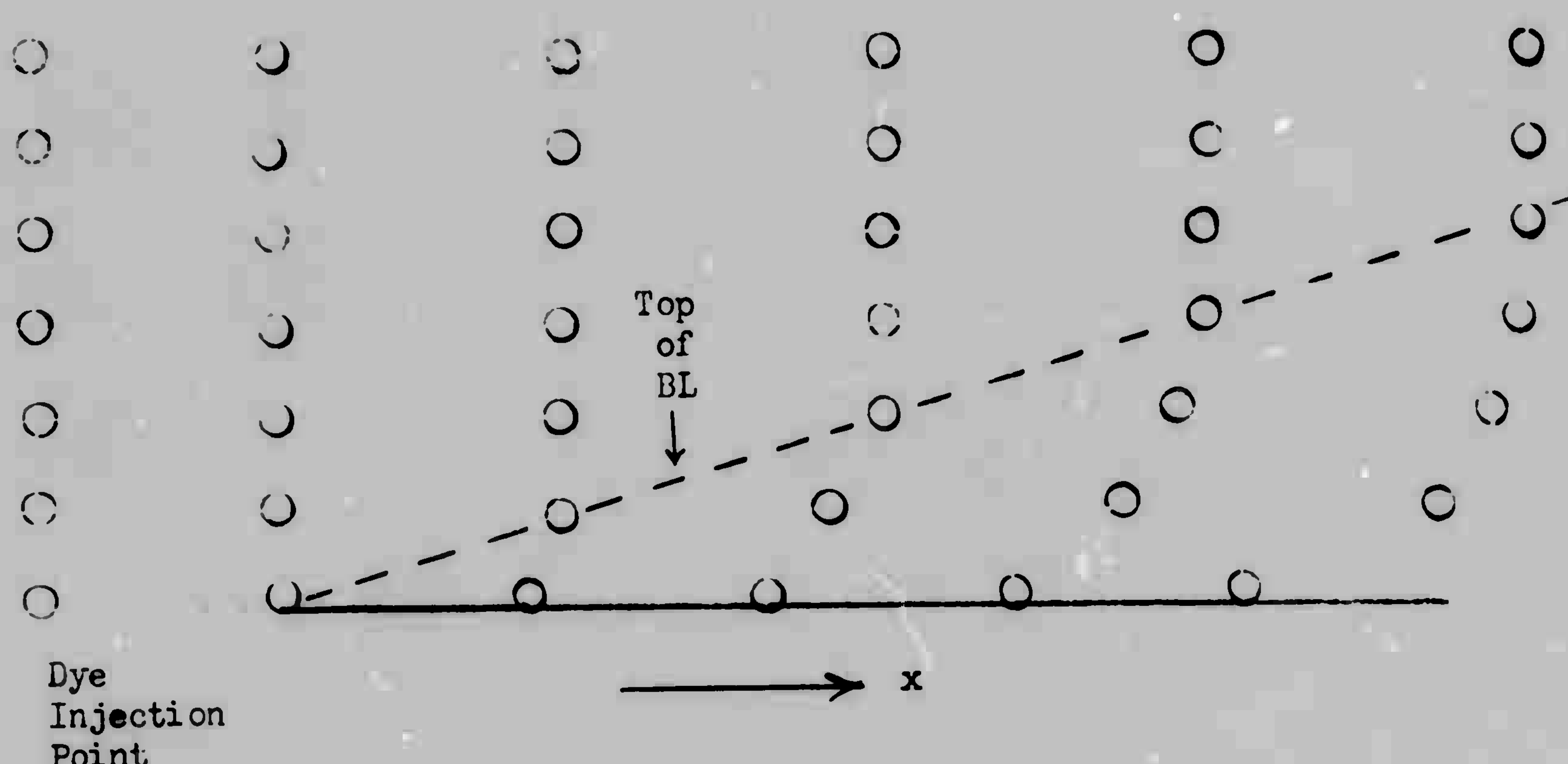


Figure 1.3.12

Observing the resulting patterns, several characteristics common to the growth of all boundary layers are noted

1. The boundary layer becomes thicker as the distance, x, from the leading edge increases.

2. The velocity profile changes with increasing x.

1.3.4.3 REYNOLDS NUMBER

Before discussing the more specific characteristics of the boundary layer and its contribution to aerodynamics, it is necessary to have a good physical understanding of the Reynolds number, its significance and its effect on fluid flow. The factors which go together to make up the Reynolds number consistantly appear in a specific arrangement in fluid flow analysis. The grouping of these terms and the physical intrepretation was first observed by Osborne Reynold for whom the quantity is named. Reynolds number is defined as

$$R_e = \frac{\rho V x}{\mu}^{*} \qquad \text{Equation 1.3.13}$$

where:

ρ = the density, slugs/ft^3

V = the velocity, ft/sec

μ = the coefficient of viscosity, $\frac{\text{lb - sec}}{\text{ft}^2}$

x = the distance, ft

Reynolds observed that when dye was injected into a fluid flow that a very straight fine line persisted for a ways down stream, but that after a certain distance the flow became unsteady and then turbulent causing the dye-line

* Note: In aerodynamics x is often replaced by some characteristic length, l, (where l = k)

first to oscillate and then to dissipate Figure 1.3.13. Furthermore, he observed that the point at which this transition occurred was directly related to the quantity $\rho Vx/\mu$.

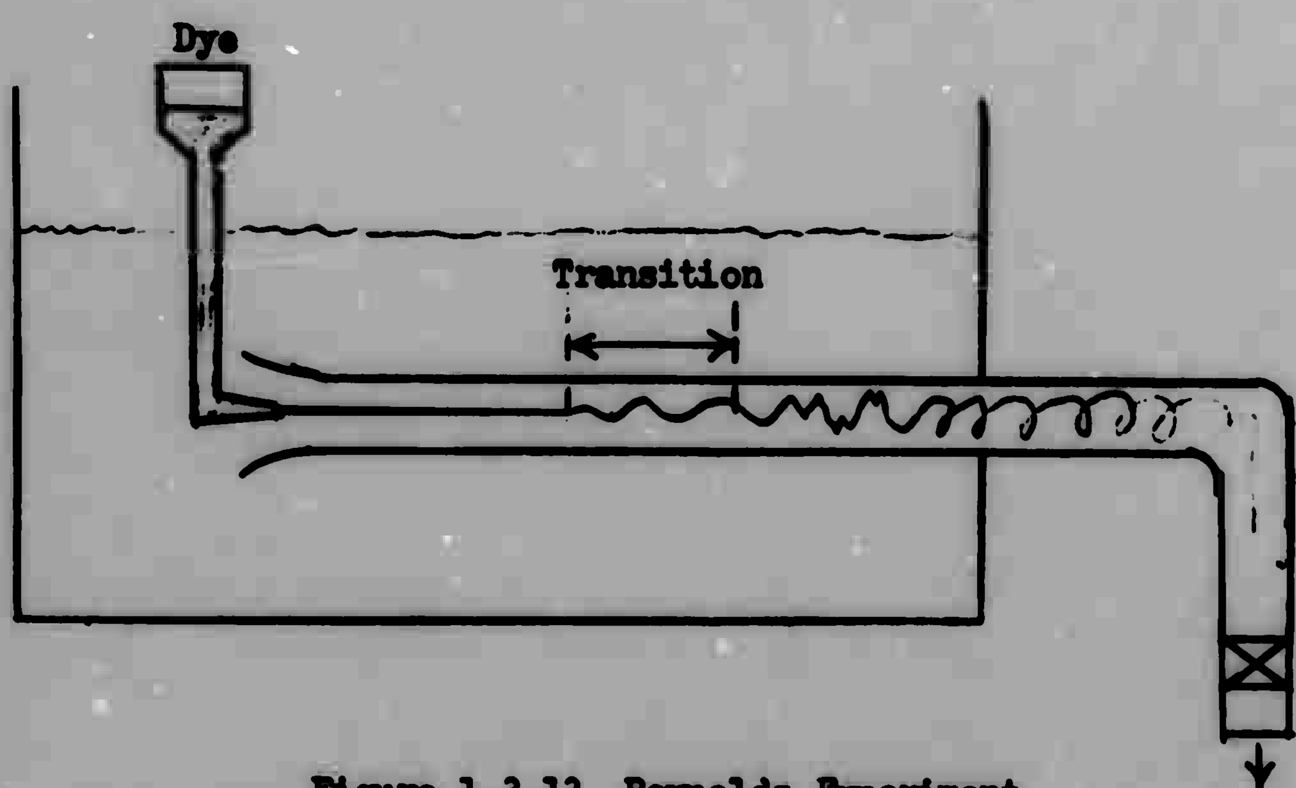


Figure 1.3.13 Reynolds Experiment

The flow prior to transition is said to be laminar; that is, consisting of specific lamina with no cross flow currents to disturb the dye. The flow down stream of the transition is said to be turbulent or consisting of random cross flow and rotational currents which disperse the dye. The transition region between these two conditions may be thought of as being part laminar and part turbulent. The entire significance of these two types of flow was probably not realized by Reynolds but it is of tremendous importance in determining the aerodynamics of an aircraft and in the extrapolation of wind tunnel model results to full scale aircraft. The latter is the reason that the Reynolds number is often referred to as a scaling factor.

A better physical grasp of the meaning of the Reynolds number is obtained if it is viewed in terms of the forces acting on the fluid. There are four forces which act on a fluid:

1. Inertia forces or dynamic pressure
2. Viscous forces
3. Static pressure
4. Gravitation forces

The latter two are generally small as compared with the first two; therefore, they can be neglected. From this it follows that the Reynolds number is proportional to the ratio of the inertia forces to the viscous forces.

$$R_e \approx \frac{\text{inertia force}}{\text{viscous force}}$$

This may be shown from the law of conservation of momentum Equation 1.3.11c which when integrated becomes Bernoullis' equation (Ref Sec 2.2)

$$P + \frac{1}{2}\rho V^2 = K \qquad \text{Equation 1.3.14}$$

where P = static pressure

$\frac{1}{2}\rho V^2$ = dynamic pressure, q

Since the static pressure can be neglected the dynamic or inertia force acting on the fluid is

$$F_{inertia} = q\, dA = \frac{1}{2}\rho V^2 \times l^2$$

where dA = cross sectional area of the stream = l^2

The shear stress was shown in equation 1.3.4 to be

$$\tau = \mu \frac{du}{dy} \qquad \text{Equation 1.3.12}$$

which in terms of this problem is

$$\tau = \frac{F_{friction}}{dA} = \mu \frac{V}{l}$$

so that the friction force is

$$F_{friction} = \frac{\mu V}{l}\, dA = \frac{\mu V}{l} \cdot l^2$$

$$F_{friction} = \mu V l$$

Taking the ratio of the inertia forces to the friction forces gives

$$R_e \approx \frac{F_{inertia}}{F_{friction}} = \frac{\frac{1}{2}\rho V^2 l^2}{\mu V l} = \frac{\rho V l}{2\mu}$$

$$\therefore R_e = \frac{\rho V l}{\mu} \approx \frac{\text{inertia forces}}{\text{friction forces}} \qquad \text{Equation 1.3.13}$$

If μ/ρ is defined as the kinematic coefficient of viscosity, ν , the Reynolds number is written as

$$R_e = \frac{V l}{\nu} \qquad \text{Equation 1.3.14b}$$

Relating this concept of the Reynolds number back to the transition from laminar to turbulent flow a better understanding of the role of the Reynolds number is obtained. It may be said that laminar flow results when the viscous forces are large enough to overcome or damp out the oscillations caused by the dynamic forces, that is, at low Reynolds number. Conversely, turbulent flow occurs when the dynamic force becomes so large as to overcome the viscous damping forces resulting in cross flow and rotational flow. Thus, transition is directly a function of the Reynolds number. If transition is found to occur at say R_e = 500,000 then by increasing the velocity or density or decreasing the viscosity will cause transition to occur earlier in the tube. That is, the critical Reynolds number (i.e., 500,000) occurs sooner. It should be noted that the critical Reynolds number for the system shown in Figure 1.3.13 depends on two independent variables namely the initial turbulence in the tank and the roughness of the tube walls.

1.3.L.4 BOUNDARY LAYER TRANSITION AND GROWTH

With this understanding of laminar and turbulent flow and the contribution of the Reynolds number to predicting transition let us return now to the discussion of the boundary layer. As might be anticipated from the Reynolds experiment, flow over a surface is initially laminar until the critical (transition) Reynolds number is reached after which the flow becomes turbulent. It is likewise found that the transition Reynolds number depends on the up-stream turbulence and the surface roughness.

The boundary layer was previously seen to grow as the flow passed over a surface and to have a characteristic velocity profile. The shape and rate of

growth of this velocity profile is dependent on whether the flow is laminar or turbulent.

As a uniform velocity flow approaches and passes over a smooth flat plate, Figure 1.3.9 a laminar boundary layer is developed which grows according to the equation

$$\delta_L = 4.9.4 \sqrt{\frac{\mu x}{\rho u}} = \frac{4.96\, x}{\sqrt{R_{e_x}}} \qquad \text{Equation 1.3.15}$$

where δ_L = laminar boundary layer thickness

x = the distance from the leading edge of the plate

u = free stream velocity

Re_x = is the Reynolds number base on x rather than 1

The velocity profile at any point is given by

$$\frac{u}{u_o} = \frac{3}{2}\left(\frac{y}{\delta_L}\right) - \frac{1}{2}\left(\frac{y}{\delta_L}\right) \qquad \text{Equation 1.3.16}$$

where Y = the vertical distance above the plane

u_o = the free stream velocity

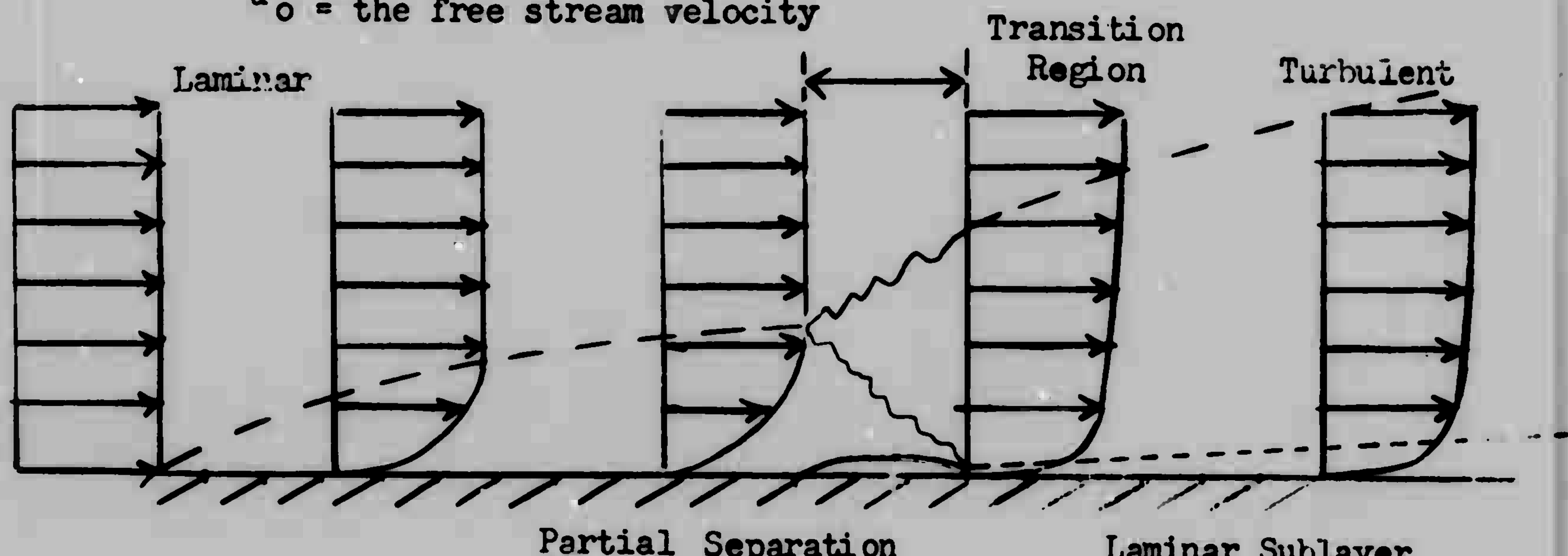


Figure 1.3.14

When the Reynolds number reaches the critical value transition begins. In the transition region the flow is neither laminar nor turbulent but is mixed with the lower portion being primarily laminar and the upper part being primarily turbulent. A region of partial separation sometimes occurs nearest the surface but it disappears once transition has been accomplished. In effect the transition region is one in which the turbulent boundary layer is born and the laminar layer is shrinking to a fraction of its original size. The laminar layer continues to exist, however, as a small sublayer next to the surface. After transition it grows very slowly in size under the now turbulent boundary layer according to the following relation.

$$\delta_s = \frac{72\, x^{0.1}}{(\rho u_0/\mu)^{.9}} = \frac{72\, x}{R_e^{0.9}} \qquad \text{Equation 1.3.17}$$

The velocity profile in the sublayer is very near linear (i.e., $u = k\, y$)

In contrast to the slow growth of the sublayer the turbulent boundary layer grows very rapidly. The expression for boundary layer growth is:

$$\delta_T = .37 \left(\frac{\mu\, x^4}{\rho\, u}\right)^{1/5} = \frac{.376\, x}{R_e^{1/5}} \qquad \text{Equation 1.3.18}$$

where δ_T = turbulent boundary layer thickness.

The velocity profile is given by

$$\frac{u}{u_o} = \left(\frac{y}{\delta}\right)^{1/7} \qquad \text{Equation 1.3.19}$$

A comparison of the rate of growth of the various types of boundary layers shows that the turbulent boundary layer grows roughly 100 times faster than the laminar sublayer and 10 times faster than the laminar boundary layer.

1.3.4.5 VELOCITY PROFILES

A comparison of the laminar and turbulent velocity profiles shows a very significant difference.

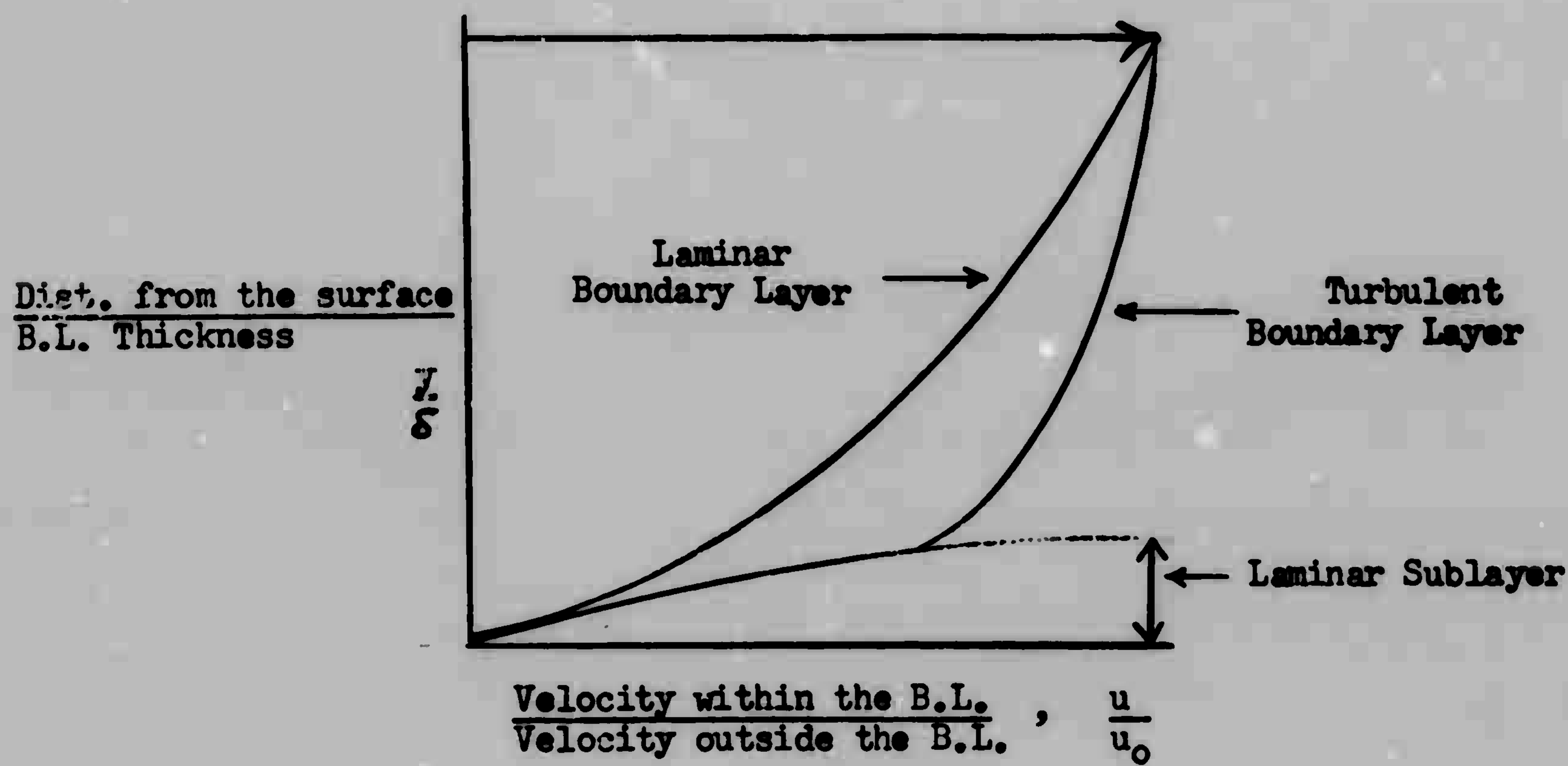


Figure 1.3.15

Plotted as a percentage of the boundary layer thickness and the free stream velocity it is seen in Figure 1.3.15 that the velocity in the turbulent boundary layer is much greater than in the laminar boundary layer. Also notice that the 1/7 relationship given in equation 1.3.19 holds only until it reaches the sublayer then the linear sublayer relationship is followed. From this it is seen that the turbulent boundary layer has considerably more energy in the lower

levels than a corresponding laminar layer. This occurs because the turbulence mixes more of the high energy air from the upper levels into the lower levels.

The velocity profile is the primary factor which determines the skin friction on the surface. Recalling that the shear stress for laminar flow is

$$\tau = \mu \frac{du}{dy}$$

and observing that the flow next to the surface is always laminar even when turbulent flow exists, the viscous resistance can be calculated. This is because shear stresses other than those caused by $\frac{du}{dy}$ at the surface can not be transmitted to the surface; they can only effect the layer adjacent to them which in turn eventually effects the one next to the surface. Thus, the velocity gradient at the surface is the only one that need be known to calculate the skin friction. Using the previous equation

$$\text{Skin Friction} = \tau \times A = \mu A \frac{du}{dy} \qquad \text{Equation 1.3.20}$$

From this and Figure 1.3.15 it is seen that more skin friction will be obtained for a turbulent boundary layer than for a laminar boundary layer.

While lower skin friction may be realized with a laminar boundary layer it may not be practical to utilize them because of separation problems or because of the difficulty in maintaining laminar flow without transition. Quite frequently separation and the resulting pressure drag causes a much greater increase in the total resistance than does the increase in skin friction.

1.3.4.6 SEPARATION

Like skin friction, separation originates in the boundary layer and is primarily a function of the velocity profile. Unlike skin friction, however, a steep velocity profile at the surface is desirable to prevent separation. Separation occurs when the fluid flow no longer follows the contour of the surface.

Separation may be caused by two things: 1) A pressure gradient along the direction of flow and/or 2) The inertia of the fluid carrying the stream away from the surface when it rounds a sharp curve. While the latter cause may be the most obvious from the point of view of the forces acting, it is not the primary cause of separation. Corners sharp enough to produce separation of this type are easily avoided on most aerodynamic shapes. It is the pressure gradient which is the primary cause of separation.

The velocity profile in Figure 1.3.11 is altered somewhat when a pressure gradient is defined as $\frac{dp}{dx}$; that is, the pressure changes a given amount dp as the stream moves a length, dx over a surface (dx is positive in the direction of flow). A positive pressure gradient is one in which the pressure increases as x increases. In a negative pressure gradient the pressure decreases with increased x. An example of a negative pressure gradient is found over the forward upper surface of a wing while a positive gradient exists over the aft upper surface, Figure 1.3.16.

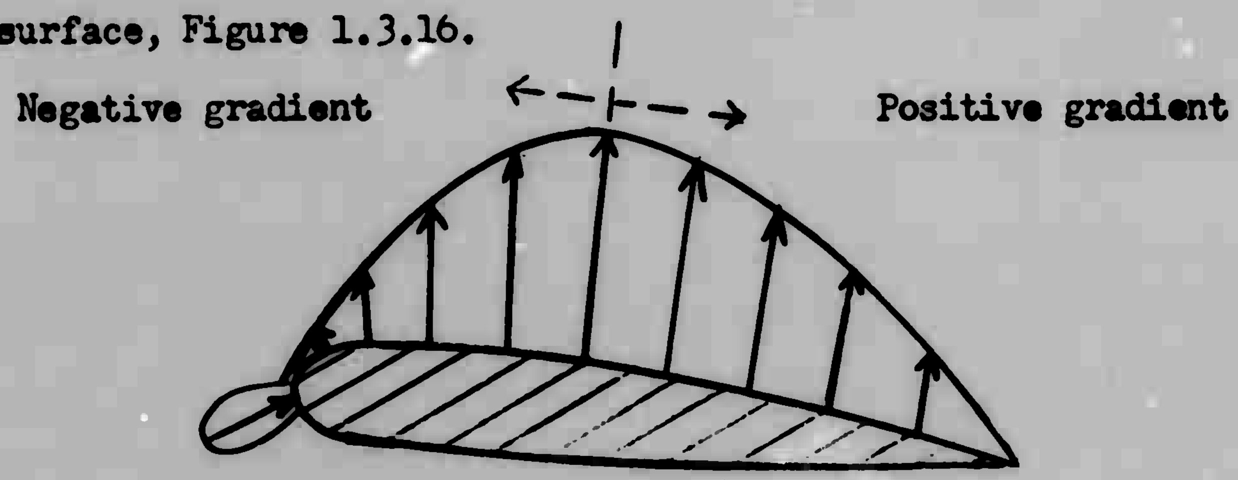
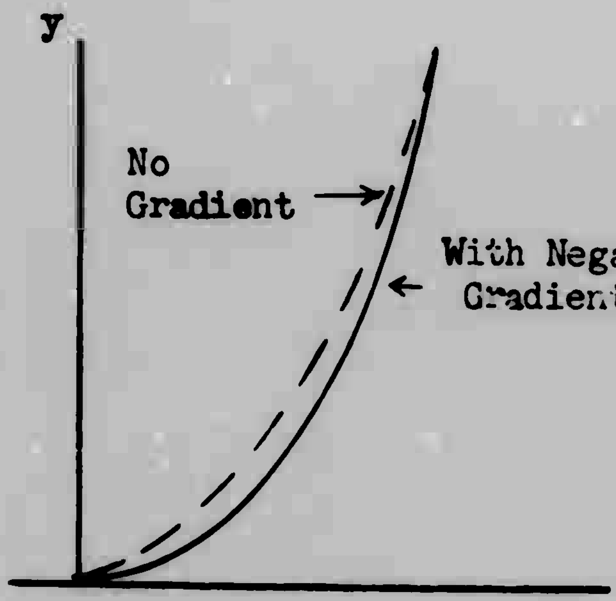


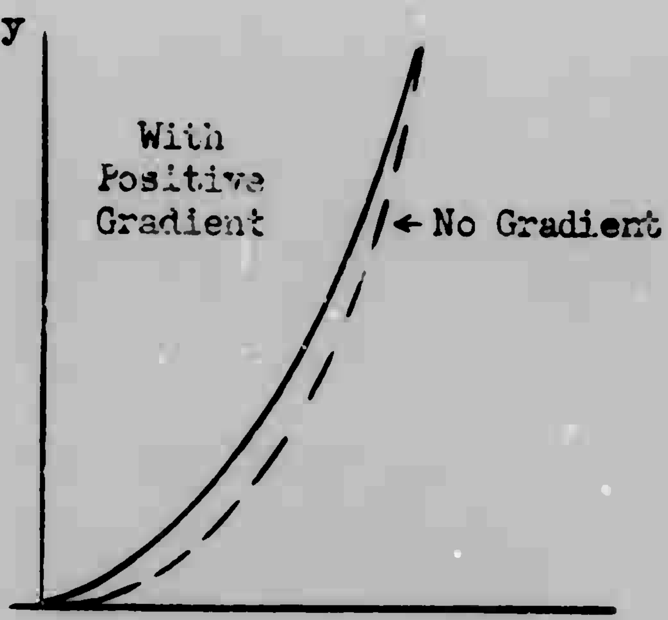
Figure 1.3.16

That is, the pressure gradient continually dissipates some of the energy of the boundary layer. The longer the flow acts against this positive gradient the thinner the profile becomes.

When a negative gradient exists it means that the fluid is flowing into a region of lower pressure; therefore, the fluid tends to accelerate causing the velocity profile to increase or become fuller, Figure 1.3.17a.



a) Negative Gradient



b) Positive Gradient

Figure 1.3.17

On the other hand a positive gradient means that the fluid is traveling into a region of higher pressure. For the flow to continue into this region it must do work against this increasing pressure. The only source of energy available is the kinetic energy of the fluid which therefore must be dissipated. Thus, the velocity in the boundary layer decreases when flowing into a positive pressure gradient until the flow reverses direction. This reversal of flow direction is called separation and is defined as the point at which the slope of velocity profile $\frac{du}{dy}$ at the surface equals zero. The effect of a positive pressure

gradient on a stream as it flows along a flat plate is shown in Figure 1.3.18

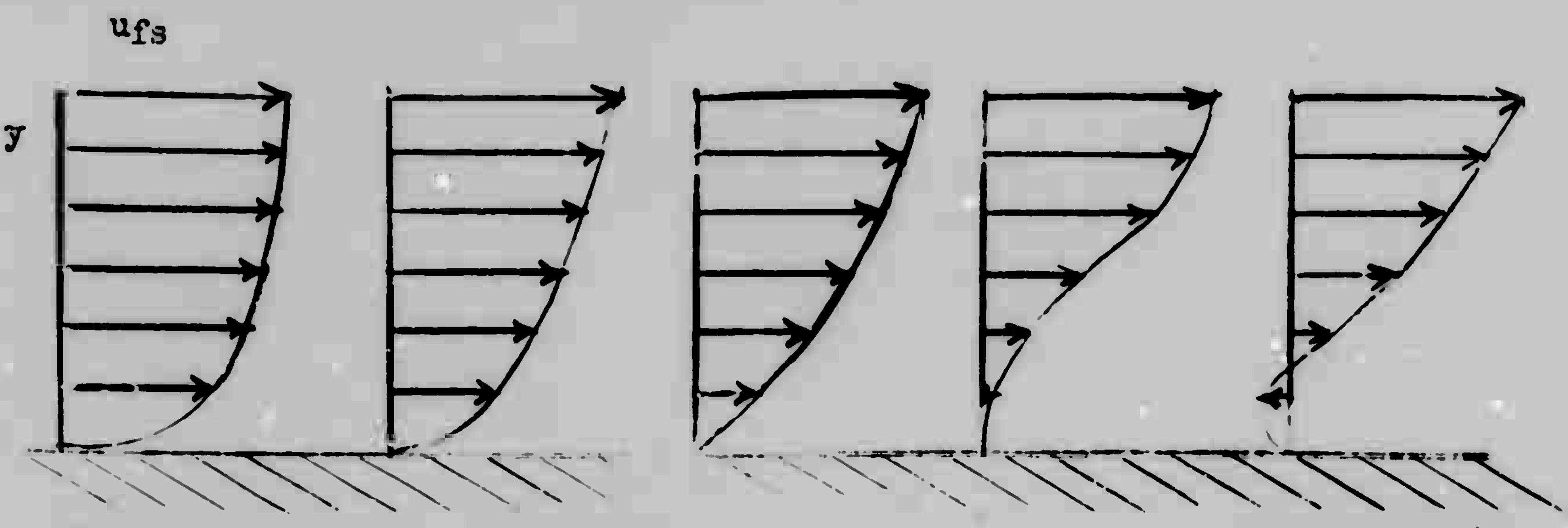


Figure 1.3.18

Thus, we see that separation occurs in the presence of a positive pressure gradient. It can be shown that separation can not occur in the presence of a negative gradient. Because separation occurs with a positive pressure gradient it is often referred to as an adverse pressure gradient since separation is generally an undesirable condition from an aerodynamic point of view. Thus, it is seen that a pressure gradient can cause or prevent separation. It is to this end, the prevention of separation, that boundary layer control systems are designed.

1.4 THERMODYNAMICS

The science of thermodynamics deals with relations between heat, work and energy. It is based on two general laws of nature called the first and second laws of thermodynamics. By logical reasoning from these laws it is possible to correlate many of the observable properties of matter. The principles of thermodynamics will be used in section two and five to study compressible flow and in section four in the study of engines. This section is not intended to be a thorough study of the science of thermodynamics but rather a review of the basic thermodynamic principles and concepts which may be helpful in studying and understanding later sections.

Thermodynamics is not concerned with the structure of matter but merely the transfer of energy without particular reference to the mechanism of this transfer. While this allows broad application of thermodynamic principles to a number of fields, it also limits its usefulness, in that, thermodynamics can predict many relationships between properties of matter but not the actual magnitude of these properties.

1.4.1 FUNDAMENTAL CONCEPTS AND DEFINITIONS

1.4.1.1 SYSTEM AND SURROUNDINGS

It has been said that thermodynamics is the study of the effect of heat, work and energy on matter. The term "system" is used to describe a given quantity of matter on which these effects are to be observed. The system is envisioned as being surrounded by a closed surface such as the walls of a tank; however, the system need not be surrounded by physical boundaries nor must the boundaries be fixed.

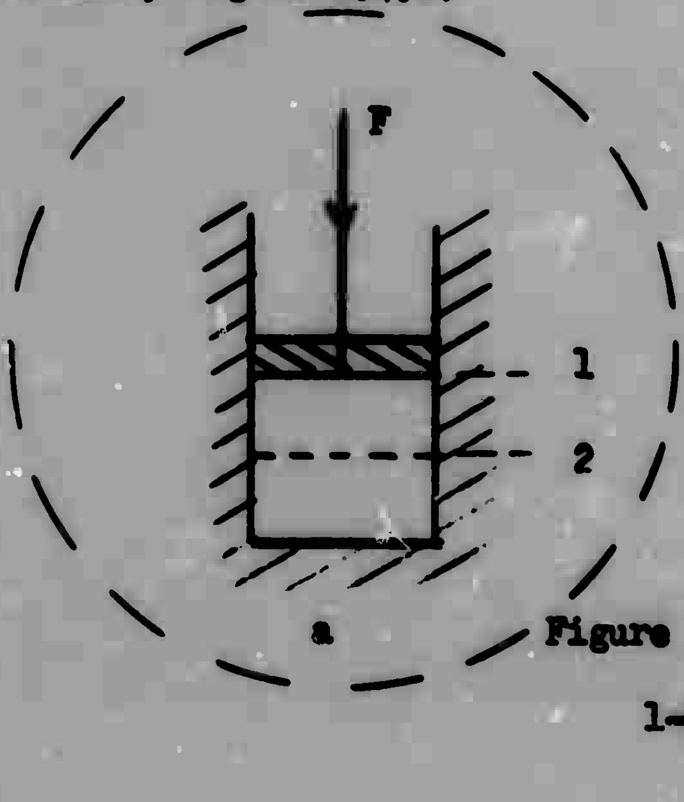
A system may be one of two types, either "closed" or "open". A "closed system" is one which encompasses a given mass of fluid and expands or contracts to maintain this condition and only energy is allowed to cross the boundary. An example of such systems is a cylinder equipped with a moving piston which expands and compacts, or the fluid element discussed in section 1.3. An "open system" is one which mass as well as energy crosses the boundary and the mass within the system need not be constant. An example of this type of system is the control volume of section 1.3 or a simple water pump.

Thus, in the most general terms the system is defined as the region where transfers of mass and energy are to be studied.

Anything not included within the boundaries of a system is referred to as the "surroundings". In most cases the atmosphere serves in this capacity.

A system can exchange energy with its surrounding by the performance of work or by the performance of work by the transfer of heat. If conditions are such that no energy transfer can take place the system is said to be "isolated". To be isolated, the system must be thermally insulated so that there is no flow of heat to or from the system and there must be no work done on or by the system.

As an example of the above concepts let us consider a movable piston in a cylinder, Figure 1.4.1.



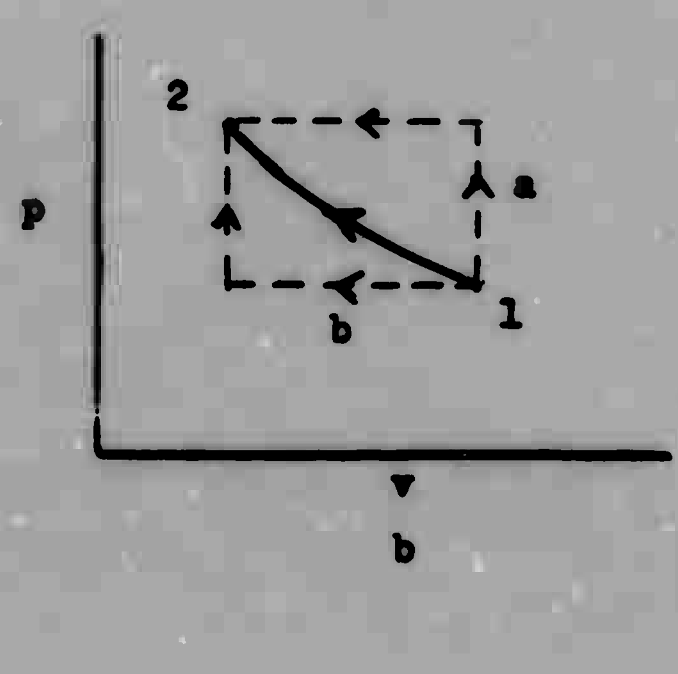


Figure 1.4.1

If the piston is pushed down to position (2) and the system is defined by the dotted line surrounding the piston, then the system is isolated since no heat or work is transferred across the boundary. However, if a new system is defined as just the fluid within the piston then it is no longer isolated since work is done by the surroundings and heat is generated by the friction and transferred to the system.

1.4.1.2 PROPERTIES AND STATE OF A SYSTEM

The condition of a system is defined in terms of recognizable features, such as, pressure, temperature, color, density and volume, which assume specific measurable values as the system is taken through a series of changes. If these features are uniquely identifiable and reproducible they can be used as dimensions to quantatively describe the condition of the system. Such features are called "<u>properties</u>" of the system. Thus, the properties define the condition, or "<u>state</u>", of a system. Conversely, the state of a system is defined in terms of a certain minimum number of properties required to completely describe the system. These properties are called the "<u>state variables</u>" or "<u>thermodynamic coordinates</u>". For a gas, the state variables are pressure, density and temperature. Their interrelationship is given by an equation of state whose form depends on the type of gas and the required accuracy.

For most diatomic gases such as those found in air, the equation of state for a perfect gas gives accurate results through a wide temperature range. Other equations of state are available for gases with more complicated molecular structures and for diatomic gases at extreme temperatures. The equation of

state for a perfect gas is:

$$\frac{p}{\rho} = p\,v = R\,T \qquad \text{Equation 1.4.1}$$

where p is the pressure $\sim$ lb/ft^2

$\rho = \frac{1}{v}$, is the density $\sim$ slugs/ft^3

v is the specific volume $\sim$ ft^3/slug

T is the temperature (degrees absolute)

R is the universal gas constant ft^2/sec^2 ^oK or $\frac{\text{ft lb}}{\text{slug } ^oK}$

Other equations of state may be found in thermodynamic texts. The most common of these is the Van der Waals equation. Others are the Dieterici equation, the Berthelot equation and the Beattie-Bridgeman equation.

While temperature, pressure volume and density are all properties, there are two types of properties and it is necessary to distinguish between them. It may be recognized by the most casual observer that some properties depend on the extent of the system, such as volume, mass, area and energy, while others such as pressure, density, velocity and temperature do not. Those properties which depend on the extent of a system are called extensive properties and those which are independent of the extent of the system are called intensive properties. Since for general application it is desirable to deal with variables which are independent of the extent of the system, extensive variables are frequently avoided by dividing by the extensive quantity, mass, volume or area. Thus, instead of considering the extensive variable, volume in the equation of state, the extensive variable is divided by the mass of the system and is called the specific volume or the volume per unit mass. In reality, all intensive variables

are specific values. For instance: Pressure is the force per unit area where area is the extensive variable; density is the reciprocal of the specific volume; and temperature is the molecular energy per unit mass of the molecules.

1.4.1.3 EQUILIBRIUM AND CHANGE OF STATE

In order to specify a property of a system it is necessary for it to be the same throughout. If this were not so, it would be possible only to specify the value of the property at each location in the system but it would not be possible to establish a value for the whole system. The condition where all of the properties are uniform throughout the system is called "equilibrium". Thus, equilibrium conditions are required before any property can be specified for a system. Considering the piston in Figure 1.4.1, it is obvious that as the piston is moved at a finite rate from (1) to (2) the state variables pressure and temperature will increase. It is not so obvious, however, that they will not increase uniformly throughout the volume. This is because the pressure rise caused by the piston movement is transmitted at a finite rate, namely, the speed of sound. Therefore, the pressure and temperature is higher at the face of the piston than it is at the opposite end of the cylinder. Thus, it is apparent that if equilibrium conditions are to be maintained the compression process must be accomplished very slowly. In fact, this is a general axiom for equilibrium: "Equilibrium may be maintained for a change in state only when the process takes place over an infinite length of time or through a very small increment." Thus, an equilibrium process of necessity is accomplished very slowly or at a finite rate through a very small change in thermodynamic coordinates.

Any change in coordinates that occur under anything but equilibrium conditions can not be traced as to path, Figure 1.4.1b. For instance, in the previous example the pressure and volume undergo a change but if this change occurs quickly it is not known whether the volume changed first and then the pressure changed (path a), or visa versa (path b), or whether it occurred by some combination of each such as the equilibrium line shown by the solid equilibrium line in Figure 1.4.1b. Since the equilibrium path is the only one which can be definitely specified for changes in coordinates it is extremely useful in the study of thermodynamic processes.

1.4.1.4 THERMODYNAMIC PROCESSES

Any change in the thermodynamic coordinates of a system is called a process. If a process is carried out in such a way that the pressure, temperature and density are essentially uniform at each instant, the process is said to be "reversible". Thus, a reversible process is made up of a succession of equilibrium states. Processes involving departures from uniformity will cause losses due to turbulence, friction, etc., and are said to be "irreversible".

The complete significance of reversibility and irreversibility can not be seen at this time but a qualitative feel for the principle can be gained from the previous example of the piston. Let us assume first, that the piston in Figure 1.4.1 is frictionless and that the compression process is carried out through a series of equilibrium steps. If the process is reversible the piston will return to its original position when the force is slowly released. This is because all of the energy exerted in pushing the piston down was stored in the gas in a recoverable form which could be converted back into work to return the piston to its original position. On the other hand, if the piston is not

frictionless or if the process is carried out very quickly there will be energy losses due to turbulence and/or friction, so that, there will be insufficient available energy to return the pistion to its original position.

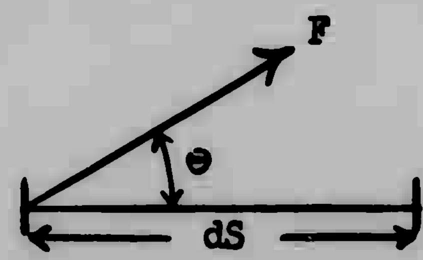
Almost all actual processes are irreversible because they take place at a finite rate with finite changes in temperature and pressure between the system and the surroundings. Nevertheless, the concept of a reversible process is a useful and important one in thermodynamics.

1.4.1.5 ENERGY, HEAT AND WORK

A system may possess energy by virtue of its temperature, velocity, position, chemistry, electrical potential or nuclear structure. The exact quantity of energy possessed above some reference level is a measure of its capacity for doing work or transferring heat. Heat and work are simply a measure of the transfer of energy across the boundary of the system in response to a driving potential, such as, temperature in the case of heat transfer and pressure in the case of work. Each are done without the transfer of mass. It is incorrect to speak of the heat or work of a system for these quantities are manifested only as they effect their surroundings. For instance, if we have a cup of coffee and a glass of beer sitting side by side, the coffee (system 1) because of its higher energy level transfers heat to the beer (system 2). The heat is only seen as it is transferred across the boundary. Now, if the beer and the coffee are isolated in one container, a new system is defined. Under the definition of heat there is no transfer of energy across the boundary and therefore, there is no transfer of heat. In this case the total energy of the system remained constant and even though the two elements within the system obviously changed temperature there was no exchange of energy with the surroundings.

Thus, heat and work refer to the interchange of energy between a system and its surroundings. Heat most generally refers to the exchange of thermal energy as transferred through the mechanisms of conduction, convection and radiation, while work most generally refers to physical force such as that caused by pressure, magnetism and electrical potential applied over a given distance. Work used for most thermodynamic applications results from pressure forces.

While the work can always be expressed as



$$\text{Work} = \int \bar{F} \cdot d\bar{S} = \int F \cos \theta \, dS$$

where θ is the angle between the force and the direction of motion. In most cases of practical interest in thermodynamics the work is associated with a change in volume and it is more conveniently expressed in these terms.

Consider a system of arbitrary shape such as might be enclosed by an uninflated balloon.

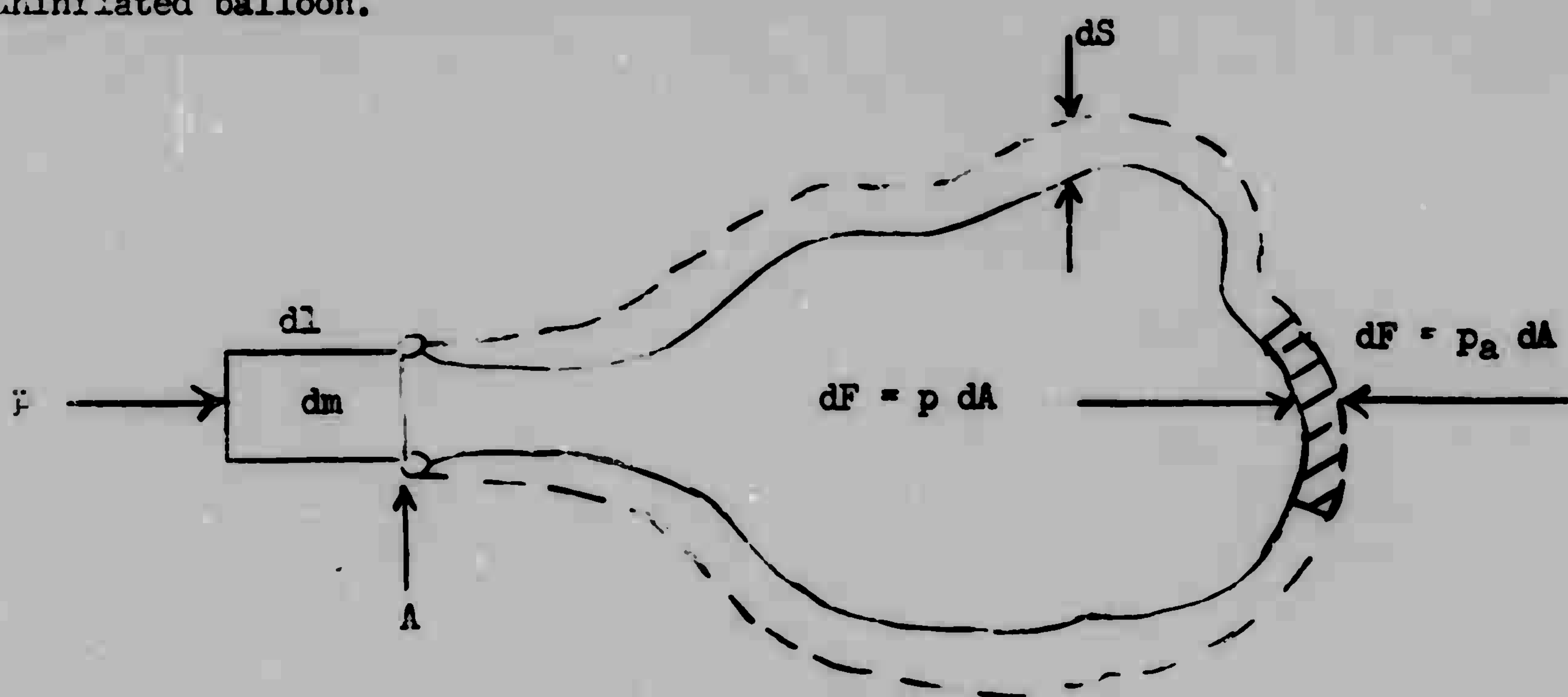


Figure 1.4.2

If the balloon is partially inflated, as shown by the dotted lines in Figure 1.4.2, work is done against the atmospheric pressure, which, if the balloon was reversibly inflated would equal the internal pressure, p, (neglect tension in the balloon). The increment of force acting on an element of the balloon is then

$$dF = p\ dA$$

The work done is

$$dW = dF \times dS = p\ dA\ dS$$

Integrating over the surface area gives the increment of work required to partially inflate the balloon

$$dW = p\ AdS$$

where A is the total surface area.

But AdS is the increase in the volume of the system, dV, so that, the work done is

$$dW = p\ dV$$

dividing by the mass of the system converts to the intensive properties of work per unit mass

$$dw = p\ dv \qquad \text{Equation 1.4.2}$$

If heat and work are restricted to transfers of energy across the boundary, what then is responsible for the quantity which is loosely referred to as "the heat" of a hot body? Matter, because of the vibratory and translational motion of its atoms and molecules has what is called INTERNAL ENERGY. That is the kinetic energy of the molecules impinging on the surface of a container

impart energy which is measured in terms of the temperature of the system. Thus, the temperature is a measure of the internal energy of a system, in fact, it is directly proportional to the internal energy, I.E.

$$I.E. = U = c\,T \qquad \text{Equation 1.4.3a}$$

where c is a constant of proportionality which will be discussed later.

A second type of energy that matter can possess is that of position relative to an attracting body. This type of energy is called POTENTIAL ENERGY. For the purposes of this writing potential energy will be restricted to gravitational attraction. This type of energy is given by

$$\text{Potential Energy} = P.E. = \text{gravitational force} \times \text{height}$$

$$P.E. = mgz = W_z \qquad \text{Equation 1.4.3b}$$

The kinetic energy is the energy resulting from the dynamics of matter and from basic physics is given as

$$\text{Kinetic Energy} = K.E. = \tfrac{1}{2} mu^2 = \tfrac{1}{2} \frac{W}{g} u^2 \qquad \text{Equation 1.4.3c}$$

The FLOW ENERGY or FLOW WORK as it is more correctly called, must be considered only in open system where a flow process is involved. It is the work required to force a given element of mass into or out of the system. For example, to inflate the balloon in Figure 1.4.2 work had to be done on each element of mass dM to force it into the system. The work done on each volumetric element containing dM is

$$dW = p\,A\,dl = p\,dV$$

Therefore, for each unit mass the required work is

$$w = p\ v$$

$$\text{F.E. or F.W.} = w = p\ v \qquad \text{Equation 1.4.3d}$$

Other forms of energy such as chemical, electrical and nuclear energy would normally be included in a complete discussion of the total energy, but they are not required for this simplified discussion.

The total energy of a system is given by the summation of all of the significant energies possessed by the system.

$$E = \text{I.E.} + \text{P.E.} + \text{K.E.} + \text{F.E.} \qquad \text{Equation 1.4.3}$$

$$E = U + M\ gz + \tfrac{1}{2}\ mu^2 + m\ p\ v +$$

or in specific values for a unit mass

$$E = e + gz + \tfrac{1}{2}\ u^2 + p\ v +$$

where e is the internal energy per unit mass

1.4.1.6 ENTHALPY

A system has many properties that help to identify the state. In some systems these properties appear consistently in specific combinations. It is sometimes convenient to express these combinations of properties as a single new property. Enthalpy, h, is such a composite term that appears frequently in equations for thermodynamic systems. It is defined as the sum of the internal energy, e and the product p v

$$h = e + p\ v \qquad \text{Equation 1.4.4}$$

Since e, p and v are properties then h must be a property. Because enthalpy is made up of the combination of internal and flow energies it is of particular significance in open systems where mass flow across the boundary must be considered.

The relative value of the enthalpy of a system with respect to the surroundings represents the systems capacity for doing work.

1.4.2 FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics is a statement of the principle of conservation of energy. In very general terms, it asserts that the net flow of energy across the boundary of a system by transferring heat and doing work is equal to the change in energy of the system. Or, in other works, energy is neither created nor destroyed but merely transferred or transformed. A general mathematical formulation of this principle is written as

$$Q - W = E_2 - E_1 = \Delta E \qquad \text{Equation 1.4.5}$$

That is, the heat transferred, Q, minus the work done, W, is equal to the change in energy of the system between the initial and final state, E_1 and E_2. As a matter of convention and consistency, heat added to a system and work done by the system on the surrounding are considered as being positive.

Since from their definition, heat and work represent increments of energy transferred across the boundary, the first law is often written in differential form

$$dQ - dW = dE$$

or when divided by the mass

$$dq - dw = dE/m \qquad \text{Equation 1.4.5a}$$

For a closed system no flow process is involved so the flow work and kinetic and potential energies need not be considered. Therefore, the first law can be written as

$$dq - dw = de \qquad \text{Equation 1.4.5b}$$

For an opened system, flow does take place so that the kinetic, potential and flow energies must be considered a typical steady flow system which might represent a pump, refrigerator or engine of the noninternal combustion type a shown in Figure 1.4.3

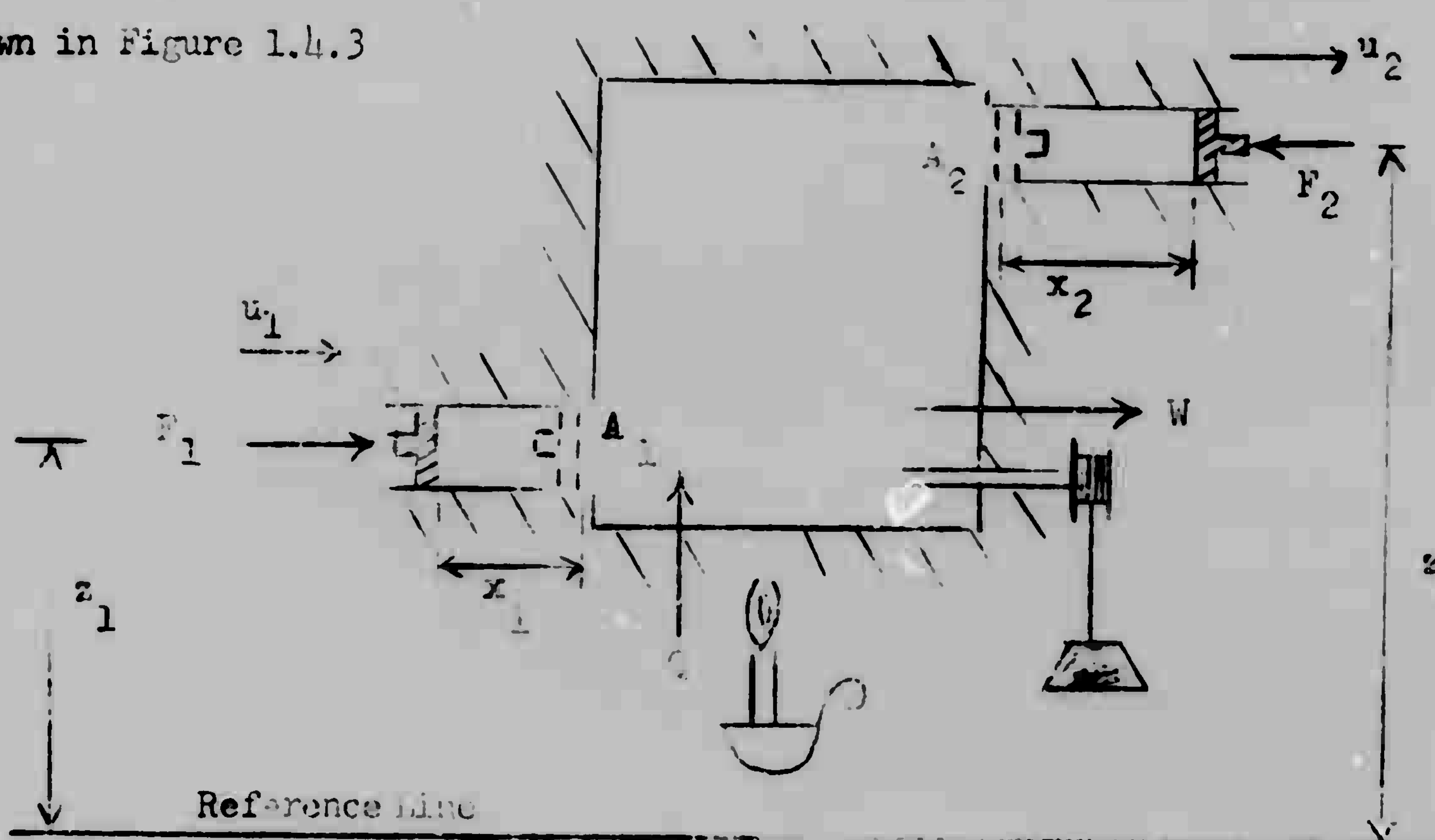


Figure 1.4.3

In this system fluid is entering the system through entrance A_1 and leaving through exit A_2 with entrance and exit conditions as shown. In addition, heat is being added to the system and work is extracted. It may be imagined that at some instant pistons are inserted in the inlet and exit pipes and that at an instant later they have moved to the positions indicated by the dotted lines. During this time an element of mass has been forced into the system while at the same time a corresponding element has been moved out as

required by the steady flow condition. The net work done by the system during this instant is

$$\text{Net work} = W + F_2 x_2 - F_1 x_1$$

Since A_1 and A_2 are the piston areas

$$F_1 x_1 = p_1 A_1 x_1 = p_1 V_1$$

and

$$F_2 x_2 = p_2 A_2 x_2 = p_2 V_2$$

so the net work is then

$$\text{Net work} = W + p_2 V_2 - p_1 V_1$$

where V_1 and V_2 are the volumes occupied by their respective masses.

The internal energies of the entering and leaving masses are

$$m_1 e_1 \text{ and } m_2 e_2$$

but since, $m_1 = m_2$ is required by the steady flow condition, the increase in internal energy of the system is

$$m (e_2 - e_1)$$

similarly the change in kinetic energy is

$$\tfrac{1}{2} m \; u_2^2 \; - u_1^2 \;)$$

and the potential energy change is

$$mg (z_2 - z_1)$$

Using the first law and equating the change in heat and work to change in total energy gives an expression of the first law for an opened system under steady flow conditions.

$$Q - W + p_2 V_2 - p_1 V_1 = m (e_2 - e_1) + \frac{1}{2} m (u_2^2 - u_1^2) + m g (z_2 - z_1)$$

by dividing through by m and rearranging

$$q - w = (e_2 + p_2 v_2 + \frac{1}{2}u_2^2 + g z_2) - (e_1 + p_1 v_1 + \frac{1}{2}u_1^2 + g z_1)$$

Equation 1.4.5c

In terms of enthalpies the equation is

$$q - w = (h_2 + \frac{1}{2} u_2^2 + g z_2) - (h_1 + \frac{1}{2} u_1^2 + g z_1) \qquad \text{Equation 1.4.5d}$$

In either form the statement of first law gives considerable insight into the workings of thermodynamic processes used in engine cycles. The so called adiabatic process is of particular importance in thermodynamics. An adiabatic process is defined as one in which no heat is transferred (i.e., $q = 0$). Thus, an isolated system is adiabatic. Since the primary function of most practical engines is to convert energy of some sort into work, they may be assumed to be quasi adiabatic. That is, the heat transfer is small compared with the total energy expended in the cycle. If this assumption is good at least to a first approximation then the first law becomes a simple relationship between the energy expended and the work done.

To allow further intrepretation of the first law it is convenient to define the enthalpy and internal energy in terms of measurable properties of the system namely the temperature.

1.4.2.1 SPECIFIC HEAT (HEAT CAPACITY) AND RATIO OF SPECIFIC HEATS

The heat capacity, c, of a gas is defined as the heat required to raise the temperature of a unit mass one degree or the heat per degree.

$$c = \frac{dq}{dT} \qquad \text{Equation 1.4.6}$$

From the first law it is seen that

$$dq - dw = de$$

$$dq = de + p\,d\,v \qquad \text{Equation 1.4.7a}$$

$$\frac{dq}{dT} = \frac{de}{dT} + p\frac{dv}{dT} \qquad \text{Equation 1.4.7b}$$

Thus, the value of c depends on whether there is a change in volume or not. Hence, defining the specific heat requires a definition of how the change is to occur (i.e., constant pressure, volume, etc.).

The specific heat at constant volume is obtained directly from equation 1.4.7b where $dv = 0$ and is written

$$c_v = \left.\frac{dq}{dT}\right|_v = \frac{de}{dT} \qquad \text{Equation 1.4.8}$$

This shows that under constant volume conditions the specific heat is equal to the rate of change of internal energy with respect to temperature. This quantity might be measured by the insulated system shown in Figure 1.4.4a. Measuring the temperature of the constant volume system under the influence of a known heat input and dividing the heat input by the temperature rise gives the specific heat at constant volume, c_v.

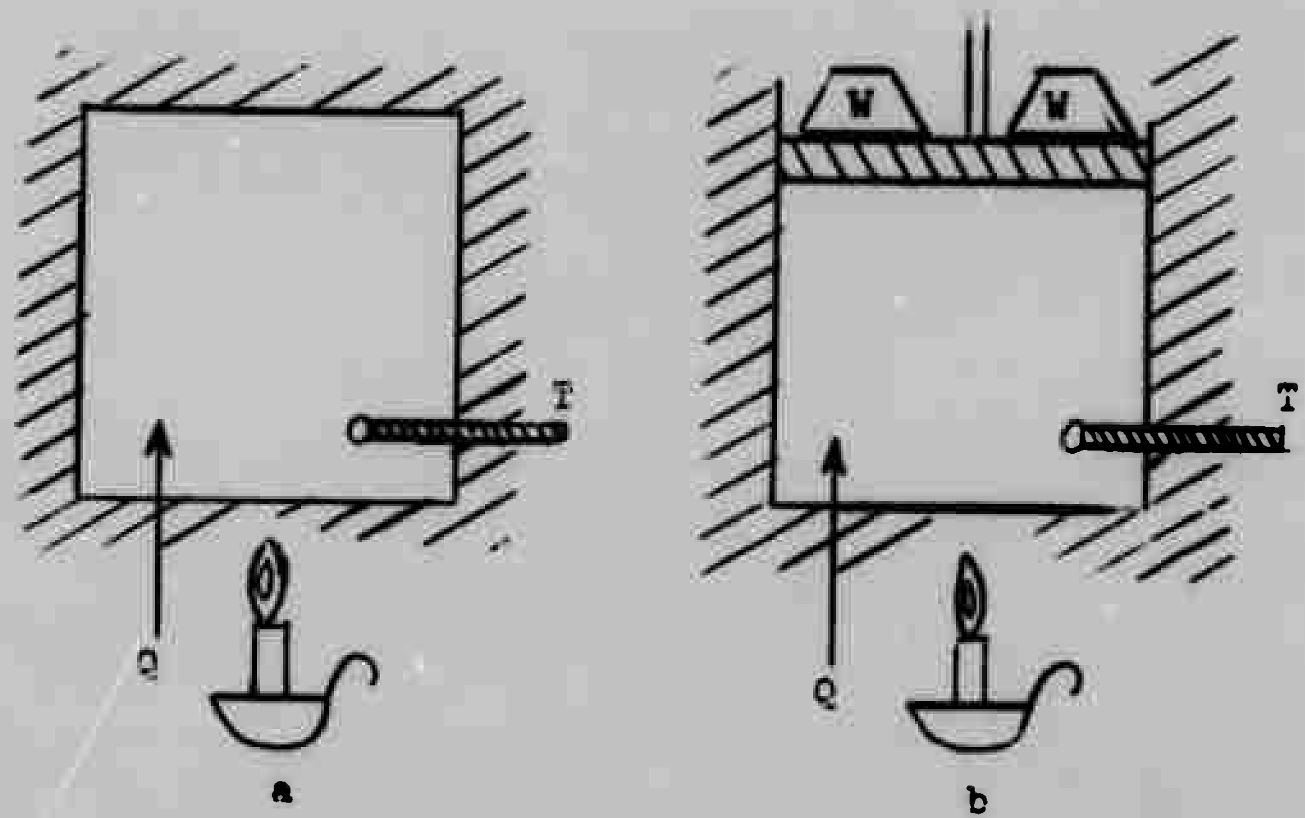


Figure 1.4.4

The specific heat at constant pressure may be obtained by modifying equation 1.4.7a

$$dq = de + p\,dv \qquad \text{Equation 1.4.7a}$$

by adding and subtracting v dp

$$dq = de + p\,dv + v\,dp - v\,dp$$

$$dq = dh - v\,dp$$

$$\frac{dq}{dT} = \frac{dh}{dT} - \frac{v\,dp}{dT} \qquad \text{Equation 1.4.9}$$

Therefore

$$cp = \left.\frac{dq}{dT}\right|_p = \frac{dh}{dT} \qquad \text{Equation 1.4.10}$$

Like c_v, c_p can be measured by a system such as that shown in Figure 1.4.4b where the constant volume container has now been equipped with a frictionless movable piston which is weighted to provide constant pressure during the heat addition. It is evident from Figure 1.4.4 that it takes less heat per

degree for the constant volume than for the constant pressure process. This is because the constant pressure system is allowed to expand during the heat addition thus lowering the temperature per unit of heat added. Thus it is, that c_p is always greater than c_v for any gas and that the ratio of specific heats, defined as

$$\gamma = \frac{c_p}{c_v} \qquad \text{Equation 1.4.11}$$

is always greater than 1.0.

From equation 1.4.8 and 1.4.10 it is evident that the specific heats can be used to determine the enthalpy and the internal energy

$$de = c_v \, dT \qquad \text{Equation 1.4.12a}$$

or

$$e = c_v T$$

and

$$dh = c_p \, dT$$

or

$$h = c_p T \qquad \text{Equation 1.4.12b}$$

Since R, c_v and c_p are physical constants for a given gas it might be suspected that they may be expressed in terms of each other.

This in fact is the case

$$h = e + pv$$

or from the equation of state for a perfect gas

$$h = e + RT$$

substituting equations 1.4.12

$$c_p T = c_v T + RT$$

or

$$R = c_p - c_v \qquad \text{Equation 1.4.13}$$

NOTE: For a perfect gas only.

1.4.3 THE SECOND LAW OF THERMODYNAMICS

The first law states without restriction the interchangability of work and thermal energy. However, certain observable but somewhat unexplainable occurrences in nature lead one to believe that this is not necessarily the whole story. It is to these unexplained facts that the second law is addressed.

Consider the following three processes:

1. Two blocks at different temperatures are brought together and are thermally insulated from their surroundings. When left to themselves they eventually come to the same temperature.

2. A paddle wheel is inserted in a jar of water and rotated at high speed so that the energy expended to turn the wheel is converted into heat.

3. A perfect gas in a tank at high pressure undergoes a free expansion through a valve into an evacuated tank. The temperature remains constant but the final pressure is less than the original pressure and the volume is greater.

All of the processess above take place as described. In each, the total energy of the system remains constant according to the first law.

Now let us suppose that starting with the end states the above processes are reversed. In the first, one of the blocks would become hotter while the other became cooler. In the second, heat from the water would cause the paddle wheel to rotate so that all of the work would be regained. In the third, all the gas would rush back through the valve and compress itself into the original tank. Obviously none of these reversed processes occur, but why? The total energy would remain constant in the reversed process just as it did in the original processes. There would be no violation of the first law. There

must therefore, be some natural principle in addition to the first law which determines the direction in which a process can take place. This priciple is the second law of thermodynamics. Like the first, it is a generalization from experience and is a statement that certain processes such as those described above, which are entirely consistent with the first law do not occur.

1.4.3.1 STATEMENT OF THE SECOND LAW

There are many different formalized statements of the second law. All of them in effect deny the possibility of the occurrence of some specified process. A few of the more general statements are:

"Energy can not progress from a lower to a higher potential." That is to say, heat can not be transferred from a cold to hot system; a gas can not flow from a low pressure vessel to a high pressure vessel; and thermal energy can not be entirely converted to work.

The last example, while not as obvious as the first two, is none the less profound. It implies that the shaft work and flow energies (kinetic and potential) can be converted at will to thermal energy but that thermal energy can not be entirely converted to work. This then establishes a ranking of work and thermal energy, the work and flow energies as being high grade energy and thermal energy as being low grade. This then leads to another statement of the second law.

"It is impossible to construct an engine that will convert all of the available thermal energy to work. This is because such an engine works between two thermal energy levels, one generally created by chemical or nuclear means and the other is determined by atmospheric conditions. Therefore, the maximum energy that

can be converted to useful work is the difference between these two levels. However, the total energy is the difference between a given temperature level and absolute zero. Therefore, only a portion of the total energy is available to do work. Similarly, the ocean by virtue of its absolute temperature and great volume has considerable thermal energy, but due to the lack of a similar body at a different thermal energy level its energy is unavailable.

1.4.3.2 ENTROPY

It is apparent that in each of the three processes originally considered that some physical characteristic of their end states must prevent reversal.

There must be some observable property of the system heretofore overlooked that determines the direction in which a process may take place. This property was found by Clausius to be the entropy, s, of the system. Like the internal energy, the entropy is a function of the state of the system only and for an isolated system it can increase or remain the same but never decrease. Thus, in terms of entropy the second law can be stated:

Processes in which the entropy of an isolated system decreases do not occur, or in more positive terms, the entropy of a process taking place in an isolated system increases or at best remains constant.

Furthermore, if the entropy of an isolated system is at a maximum no spontaneous change can occur since any change would cause a decrease in entropy. Therefore, the equilibrium condition of an isolated system is defined as the maximum entropy condition. Notice that the above comments apply to an isolated system only. The entropy of a nonisolated system can increase or decrease by doing external work or by transferring heat but it is also noted that if

another system causes a decrease in the entropy of the original system that its entropy is increased by at least the amount of the decrease in entropy of the original system.

1.4.3.3 MATHEMATICAL AND PHYSICAL INTERPRETATION OF ENTROPY

A physical interpretation of the entropy is not as easily obtained as are other properties. In a mathematical sense it is rather abstract; however, some attention must be given to its mathematical representation. It was previously seen that the work can be measured as a function of the two intensive properties p and v.

$$dW_{rev} = p\,dv \qquad \text{Equation 1.4.14}$$

where subscript rev is included as a reminder that this definition assumes a reversible process.

As yet no such representation for heat has been formulated since it is a function of the temperature only. This weakness is remedied by defining the heat in terms of two state variables namely the temperature and the entropy as follows.

$$dQ_{rev} = T\,dS \qquad \text{Equation 1.4.15}$$

As arbitrary as this definition may seem its use may be mathematically justified by showing that the quantity $dS = dQ/T$ is an exact differential but this is beyond the scope of this writing. It will suffice to accept the definition perse.

A physical interpretation of the entropy of a system is gained when it is realized that the entropy is simply a measure of the disorder of a system. This is apparent in the original example of the rapid expansion process of an isolated system. When the valve was opened, directed or ordered flow of the gas occurred. When the process was completed it was found that all that remained was the random chaotic motion of the molecules of the gas. From the

previous discussion the entropy of an isolated system was seen to be a maximum under equilibrium conditions. Therefore, the entropy of the more ordered initial condition must be less than the final condition in fact it is found that the initial entropy is a minimum. Thus it is seen that increased entropy means increased disorder. With this intrepretation the concept of the entropy may be generalized to almost any system. For instance, by this reasoning a pile of building blocks has maximum entropy but when they are piled in some ordered fashion the entropy becomes less. Likewise a ball sitting on the edge of a bowl represents the minimum entropy condition but when it is released and allowed to come to rest at the bottom of the bowl its entropy is a maximum. Thus the minimum energy condition is often associated with the maximum entropy. Since it is well known that a system left to itself will seek a minimum energy condition it is apparent the entropy is measuring the same sort of thing only in reverse magnitudes. Many examples of the physical significance of entropy can be visualized but it will suffice to say now that the entropy is a measure of the disorder of a system. It is also found to be a measure of the efficiency with which the process is carried out.

Since the entropy of an isolated system can not decrease the most efficient process is performed at constant entropy. Such a process is said to be <u>isentropic</u>.

An isentropic process is one in which $dS = 0$. From the basic definition, equation 1.4.15, it is apparent that an isentropic process must be both adiabatic and reversible. If a process is adiabatic but not reversible the entropy will increase and the process is not isentropic. Therefore, a reversible adiabatic porcess is isentropic.

1.4.4 COMBINED FIRST AND SECOND LAW

The real usefulness of thermodynamics is realized when the principles of the first and second laws are combined. By the first law

$$dq - dw = de$$

substituting from the second law

$$dq = T\,dS$$

and also substituting

$$dw = p\,dv$$

$$de = c_v\,dT$$

a basic expression of the combined first and second laws is obtained

$$T\,dS = c_v\,dT + p\,dv \qquad \text{Equation 1.4.16a}$$

The change in entropy is then given in terms of thermodynamic coordinates, p, T, and v

$$dS = c_v \frac{dT}{T} + \frac{p}{T}\,dv \qquad \text{Equation 1.4.16b}$$

Substituting $\frac{p}{T} = \frac{R}{v}$ from the equation

$$dS = c_v \frac{dT}{T} + R\,\frac{dv}{v}$$

and integrating from condition 1 to condition 2

$$S_2 - S_1 = \int_{S_1}^{S_2} dS = c_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{v_1}^{v_2} \frac{dv}{v}$$

$$S_2 - S_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \qquad \text{Equation 1.4.17a}$$

By putting equation 1.4.16a in terms of c_p another expression for the change in entropy is obtained.

$$T\,dS = c_v\,dp + p\,dv + v\,dp - v\,dp$$

$$T\,dS = dh = v\,dp = c_p\,dT - v\,dp$$

$$dS = c_p \frac{dT}{T} - \frac{v}{T}\,dp$$

substituting $\frac{v}{T} = \frac{R}{p}$ from the equation of state

$$dS = c_p \frac{dT}{T} - R\frac{dp}{p} \qquad \text{Equation 1.4.16c}$$

Integrating gives from 1 to 2

$$\int_{S_1}^{S_2} dS = c_p \int_{T_1}^{T_2} \frac{dT}{T} - R\int_{P_1}^{P_2} \frac{dp}{p}$$

$$S_2 - S_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \qquad \text{Equation 1.4.17b}$$

1.4.5 CHANGE IN STATE

When a particular process is specified such as constant temperature, pressure, volume and entropy, the expression for a change in state can be simplified. The expression for the first three conditions above can be obtained by simplifying the equation of state which applies to any process.

Constant Temperature

$$\frac{p_1}{p_2} = \frac{\rho_1}{\rho_2} = \frac{v_2}{v_1} \qquad \text{Equation 1.4.14a}$$

Constant Pressure

$$\frac{v_1}{v_2} = \frac{T_1}{T_2} = \frac{\rho_2}{\rho_1} \qquad \text{Equation 1.4.14b}$$

Constant Volume

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} \qquad \text{Equation 1.4.14c}$$

Still more important than the above relations is the equation for the change in state when the process is isentropic ($dS = 0$). This relationship is obtained by combining the equation of state and the first and second laws. Starting with equation 1.4.16c

$$dS = c_p \frac{dT}{T} - R \frac{dp}{p} = 0 \qquad \text{Equation 1.4.16c}$$

From the equation of state

$$T = \frac{pv}{R}$$

$$dT = \frac{p\,dv + v\,dp}{R}$$

$$\frac{dT}{T} = \frac{dv}{v} + \frac{dp}{p}$$

substituting this in equation 1.4.16c and simplifying

$$dS = c_p \left(\frac{dv}{v} + \frac{dp}{p}\right) - R \frac{dp}{p} = 0$$

$$c_p \frac{dv}{v} + c_p \frac{dp}{p} - (c_p - c_v) \frac{dp}{p} = 0$$

$$c_p \frac{dv}{v} + c_v \frac{dp}{p} = 0$$

$$\gamma \frac{dv}{v} + \frac{dp}{p} = 0$$

Integration gives

$$\gamma \ln v + \ln p + \ln K = 0$$

where $\ln K$ is the constant of integration. This expression may be rewritten as

$$\ln p + \ln v^{\gamma} = -\ln K$$

or $$p\,v^{\gamma} = K \qquad \text{Equation 1.4.18a}$$

where $-\ln K = K$

A similar expression in terms of temperature is obtained by integrating equation 1.4.16c directly

$$c_p \ln T - (c_p - c_v) \ln p + \ln K = 0$$

or

$$\ln p - \frac{c_p}{c_p - c_v} \ln T = \frac{1}{R} \ln K$$

$$\ln p - \frac{\gamma}{\gamma - 1} \ln T = \frac{1}{R} \ln K$$

$$\frac{p}{T^{\gamma/\gamma-1}} = c \qquad \text{Equation 1.4.18b}$$

where $c = K^{1/R}$, another constant of integration.

Equation 1.4.18 are the equations for the change in state for an isentropic process.

BLANK PAGE

SECTION II

BASIC AERODYNAMIC SUMMARY

2.1 THE ATMOSPHERE

2.1.1 INTRODUCTION:

The most important single item affecting the performance characteristics of an aircraft is the nature of the atmosphere through which it flies. Since this is the case, it is most important that the test pilot and engineer have an understanding of the atmosphere and the capabilities and limitations of the instruments by which the nature of this atmosphere is measured. In addition, there are certain definitions, basic assumptions, and technical notations, which the new test pilot must become familiar with in order to effectively carry out his mission.

2.1.2 THE ATMOSPHERE: DIVISIONS, LIMITS

As a matter of interest the chemical composition of the earth's atmosphere consists of 78% by volume of nitorgen, 21% by volume of oxygen, and 1% by volume of water vapor, carbon monoxide and other rare gases.

There is no well defined limit to this atmosphere. The density decreases slowly from the surface of the earth until at a distance of 500 miles the density is apporximately four molecules of air per cubic mile. This is normally taken as the upper limit.

The atmosphere is broken up into three major divisions which can be associated with certain physical characteristics. The division closest to the

earth's surface is called the troposphere. It's height varies from approximately 28,000 feet and -50°F at the poles to 56,000' and -110°F at the equator. These heights and temperatures vary from day to day and with the seasons of the year. This necessitates that a standard be established for comparison purposes. These standards will be discussed in latter sections.

In the troposphere the temperature decreases with altitude. All turbulence caused by convection or vertical currents and all weather conditions are contained in this division.

The second major division of the atmosphere is the stratosphere. This layer of air extends from the troposphere outward to a distance of approximately fifty miles. The boundry between the stratosphere and troposphere is called the tropopause. In the stratosphere all motion of the air is mainly horizontal and little turbulence is found. In addition, temperature is constant.

The third major division is the ionesphere. It is the outermost layer of the earth's atmosphere. It extends from approximately 50 miles to 500 miles. Large numbers of free ions are present in this area and all electrical phenomena occur here. This layer of the atmosphere is becoming more and more important with the development of the nation's space flight programs.

2.1.3 STANDARD ATMOSPHERE

As mentioned previously, the physical characteristics or nature of the atmosphere is not constant but changes from day to day and with the seasons of the year. Since the performance of an aircraft is a function of the physical characteristics of the air mass through which it flies, it also will vary as physical characteristics of the air mass varies. Thus, some standard air mass

conditions must be established in order that performance data can have some meaning when used for comparison purposes. As will be shown later, in the case of the altimeter the setting up of a standard will allow us to design an instrument for the measuring of altitude.

At the present time there are several atmosphere standards which have been established. The most common one in this country is the NASA standard atmosphere. A more recent one is the United States standard atmosphere. The European nations use the new ICAO (International Civil Aviation Organization) standard atmosphere. All of these standard atmospheres are basically the same, differing very little. All approximate the standard average day existing at 40° N latitude. The basic assumptions establishing the NASA standard atmosphere will be discussed, followed by a listing of the differences between the three different standards.

In the NASA standard atmosphere it is assumed that:

a. The atmosphere is perfect gas.
(Obeys the equation of state: $P/\rho = g RT$)

b. The air is dry. (No water vapor present).

c. The standard sea level conditions are:
$T_o = 15°C$; $P_o = 29.921$ " Hg = 760mm Hg
(Note: The subscript $_o$ denotes standard sea level conditions).

d. The tropopause occurs at 35,332 feet.

e. The temperature decrease with altitude is linear to the tropopause:
$T_a = T_o - k h$
where: K = lapse rate = .00356616° F/foot

f. The gravitational field is a constant 32.17405 ft/sec^2.

g. The stratosphere has a constant temperature of - 55° C or - 67° F.

Using these assumptions NASA compiled a set of tables which list the physical characteristics of the atmosphere in terms of pressure, temperature, and density for increments of altitude up to 60,000 feet. These tables are readily available in many different references and were the ones commonly used in this country for many years.

As previously stated the new ICAO and United States standard atmospheres use essentially the same basic assumtions as the NASA standard atmosphere. The difference between the three are best shown in the table below:

	NASA	NEW ICAO	U.S.
Acceleration of gravity	Constant	Variable	Variable
Sea level conditions	same as given	same except T_o=288.16 °K, P_o=29.92126	same as ICAO
Tropopause	35,332ft	36,089.24	36,089.24
Temperature lapse rate °C	$1.98225 \times 10^{-3}h$	$1.98120 \times 10^{-3}h$	same as ICAO
Temp. at Tropopause	-55°C	-56.34°C	-56.34°C
Defined upper limit (Based on the establishment of a standard temperature)	60,000 ft.	65,800 ft.	100,000 ft tentative to 246,000 ft.
Independent variable	tapeline altitude	geopotential	geopotential

Note: The word geopotential is best defined by the differential equation:

$$G\, dH = g\, dZ$$

where: g = absolute numerical value of the acceleration of gravity at true altitude. (The acceleration of gravity decreases as the distance from the earth increases.)

Z = true altitude

G = the dimensional constant 32.17405 ft/sec^2

For all practical purposes in the troposphere H = Z. Making this assumption there is actually only a 2% difference between the two at 400,000 ft.

2.1.4 TECHNICAL NOTATIONS AND DEFINITIONS:

Before proceeding further, let us review some of the basic concepts of Physics and some of the notations used in the field of Aerodynamics. The notations are essentially a means of expressing the technical language required in the engineering field and one must have an understanding of them in order to understand the ideas and theory written in the literature.

Temperature is a measure of molecular motion. At absolute zero temperature there would be no molecular motion. The symbol for temperature is a capital T. There are four units of temperature: degrees centigrade (^{o}C); degrees fahrenheit (^{o}F); degrees Rankin (^{o}R); and degrees Kelvin (^{o}K).

Pressure is a measure of both the speed and the number of molecules per unit volume, or density. In other words pressure is the net result of all molecular motions or bombardment. If a gas is heated in a closed container the speed of the random motion of the molecules increases, giving rise to an increase in temperature and pressure. When a gas is compressed with no addition or substraction of heat, there is an increase in both temperature and density. The notation for pressure is P or p. Pressure is normally thought of as weight or force per unit area.

Mass is weight divided by the acceleration of gravity (W/g). The unit of mass in the English system is slug, (#-ft/sec^2).

Density is mass per unit volume, (slugs/ft^3). The notation for density is the Greek letter rho, (ρ).

The most common subscript notations used are as follows:

a = ambient conditions.

t = test conditions or total conditions.

o = standard sea level conditions.

1,2,3, etc = specific conditions

Other common notations and relationships are as follows:

w = specific weight = weight/unit volume.

v = specific volume = volume/unit weight = $1/w$ = $\rho . g$

Boyle's Law

$p_1 v_1 = p_2 v_2 = C$ (constant temp)

Charle's Law

$v_1/T_1 = v_2 / T_2$ (constant pressure)

Equation of State

$p/\rho = g R T$

where: R = gas constant = 53.3 ft/°R or 96.04 ft/°K

a = linear acceleration (ft/sec^2)

F = force = mass x acceleration = ma

δ = delta = P_a / P_o

σ = sigma = ρ_a / ρ_o

θ = theta = T_a / T_o

Absolute zero temperature = 0°K = -273°C
= 0°R = -460°F

Ta (°K) = Ta (°C) + 273
Ta (°R) = Ta (°F) + 460 Absolute Temperature

°F = 9/5 °C + 32
°C = 5/9 (°F - 32)

If the student will become familiar with the above relationships and notations at this time he will find the following discussions much easier to follow and to understand. In addition, a complete list of the standard notations and symbols used at the Flight Test Center is included at the beginning of this handbook.

2.1.5 THE MEASUREMENT OF ALTITUDE

With the establishment of a set of standards for the atmosphere, and considering the problem of determining ones altitude above the ground, it is immediately apparent that there are several different means by which this can be done. This also leads to defining the type of altitude as determined by the means employed.

Tape line or true altitude is the actual linear distance above sea level. This could be determined by triangulation, radar, or actually using a long enough tape. In actual practice this is only important in climb performance tests and for ballistic purposes.

Since a set of standards have been established which denote the standard pressure, temperature, and density at each altitude, we now have means by which one's altitude may be determine.

A temperature altitude can be obtained by taking a temperature gage and modifying it to read in feet for a corresponding temperature as determinined from the standard tables. Since inversions are common and the temperature changes greatly with the seasons of the year and the latitude, such a technique would be impractical.

If some instrument were available to measure density the same type of technique could be employed and density altitude determined.

A third technique, and a much more practical one, is bases on pressure measurement. A pressure gage is used to sense the ambient pressure. Instead of reading pounds per square inch, or millimeters of mercury, it indicates the corresponding standard altitude for the particular pressure sensed. This altitude is called pressure altitude and is the one on which all flight testing is based today.

2.1.6 PRESSURE VARIATION WITH ALTITUDE

This third technique is the one on which present day type altimeters are designed. It should be obvious that the instrument will not give a true reading except when the pressure at altitude is the same as that for a standard day. This will only be true if the pressure and temperature at sea level is standard and the temperature lapse rate is also standard. Except under these conditions, the altitude indicated on the altimeter will be incorrect.

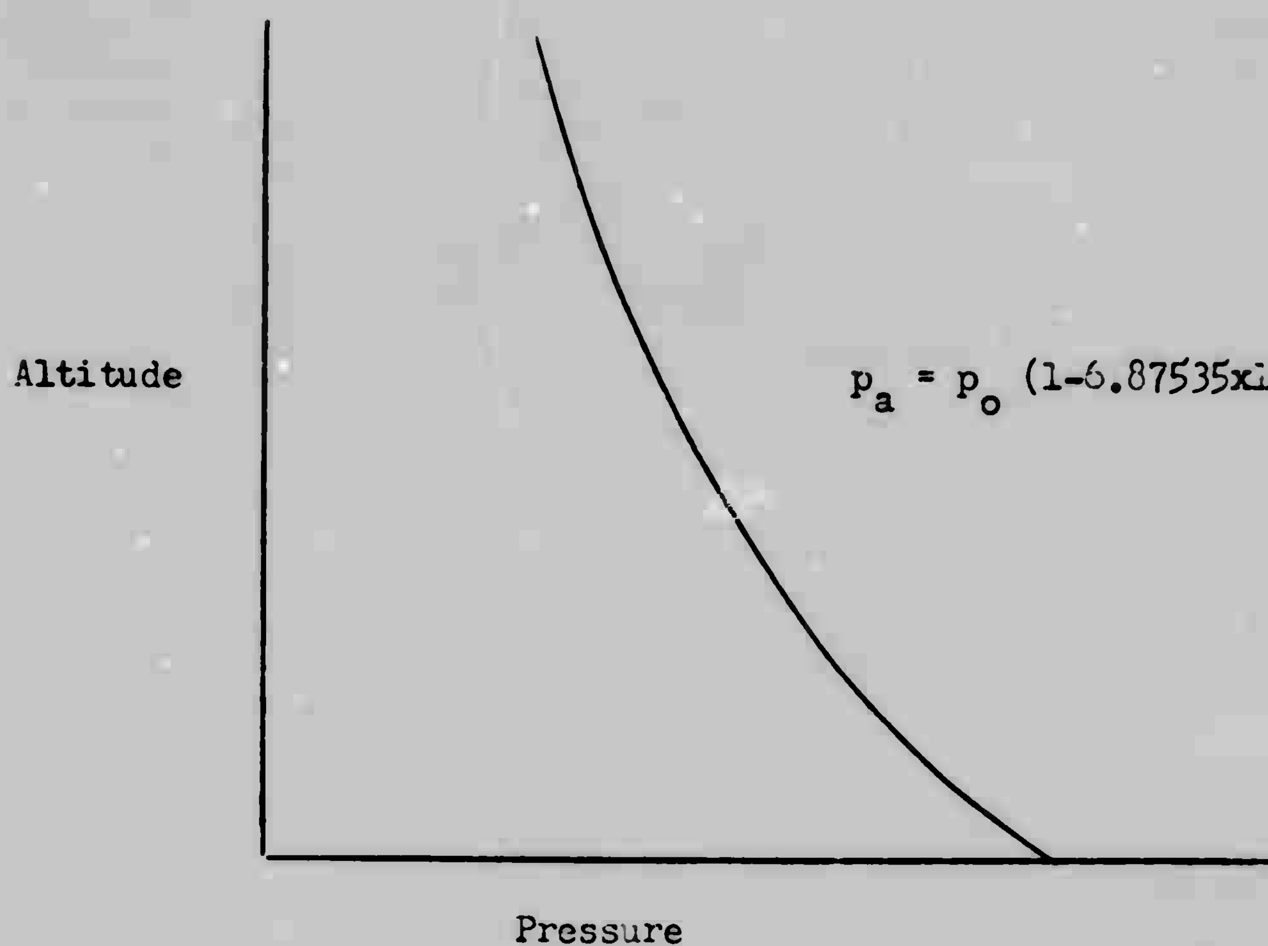
To further our understanding of the atmosphere let us consider the pressure variation with altitude as applied to the operation of the altimeter.

Using the equation of state and the U.S. standard atmosphere assumptions, the pressure lapse rate equation:

$$P_a = P_c \left(1 - 6.87535 \times 10^{-6}\, h\right)^{5.2561} \qquad \text{Equation 2.1.1}$$

can be derived and used to determine the standard variation of pressure with altitude below the tropopause. An example of this variation is plotted in Figure 2.1.1

BLANK PAGE

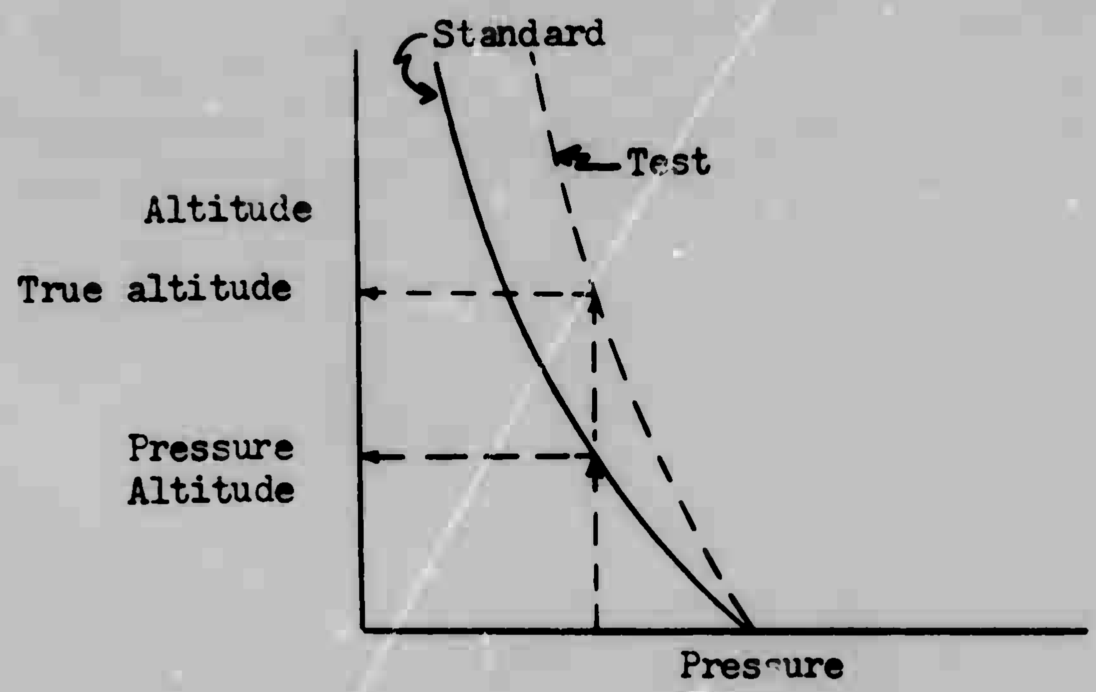


$$p_a = p_o \, (1-6.87535\text{x}10^{-6}h)^{5.2561}$$

Pressure Variation with Altitude Fig. 2.1.1

It is this curve which the altimeter presents as a reading of altitude. If the pressure does not vary exactly as prescribed by this curve then the altimeter will be in error. A provision is made in the construction of the altimeter in the form of an altimeter setting, whereby the scale reading can be adjusted up or down (the same as moving the curve of Figure 2.1.1 vertically) so that the altimeter will read the true elevation of the ground if the aircraft descends to ground level.

Figure 2.1.2 shows the pressure variation with altitude for a standard day and for a non-standard day called a test day.



Standard Day and Test Day Pressure Variation with Altitude Fig. 2.1.2

If the aircraft is flying at some true altitude on a test day the altimeter will sense the corresponding pressure as shown by the intersection of the long and short dashed lines. However, since the altimeter is built around the standard curve (shown by the solid curve) the instrument will indicate the altitude labeled "pressure altitude".

In order to see how temperature affects the pressure lapse rate, consider the forces acting on a unit element of the atmosphere.

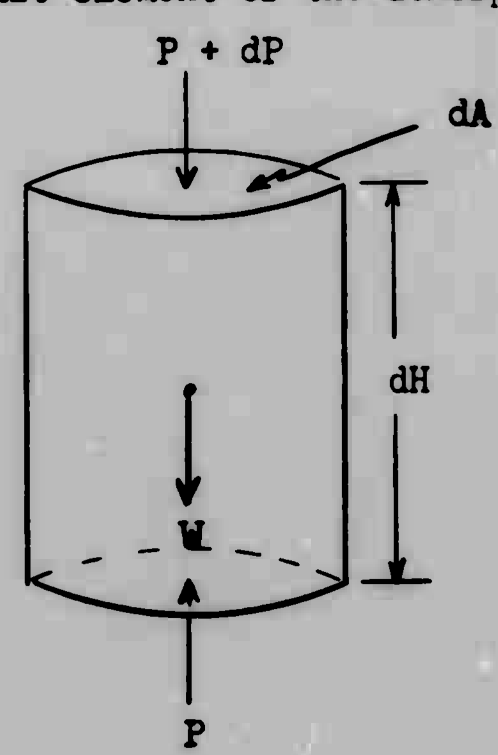


Fig. 2.1.3

Where $W = w \times volume = \rho g \times dHdA$

$\Sigma F = 0$

$F = PA$

$PdA - (P + dP)\, dA - wdH\, dA = 0$

$PdA - (P + dP)\, dA - wdH\, dA = 0$

$- dP = wd\, H = \rho g\, d\, H$

$dP/dH = - \rho g$ (Slope of the curve Fig's. 2.1.1 & 2.1.2)

Equation 2.1.2

and from the equation of state

$\frac{P}{\rho} = g\, RT$ or $\rho g = \frac{P}{RT}$ Equation 2.1.3

Therefore the slope of the curves in Figures 2.1.1 and 2.1.2

$(dH/dP) = - f\, (T)$ Equation 2.1.4

This means that the greater the temperature the greater the negative slope of the curve. Inspecting Figure 2.1.2, one can note that for every constant pressure the slope of the test day curve is greater than that of the standard day curve. Thus, the temperature for the test day is warmer than that for the standard day.

This variance between true altitude and pressure altitude is not as important as it might seem at first. As mentioned previously, true altitude is only important in climb performance tests and for certain ballistic purposes. In later sections, climb performance will be discussed and the technique by which pressure altitude may be corrected to give true altitude will be explained.

The forces acting on an aircraft in flight are not directly dependent on the ambient or absolute air pressure, but they are directly dependent upon the air density. Thus, in theory, density altitude is the independent variable which should be used for aircraft performance comparisons. However, from a practical standpoint since density is determined by pressure and temperature through the equation of state relationship, pressure altitude is used as the independent variable with test day data corrected for non standard temperature. This greatly facilitates flight testing since the test pilot can always hold a given pressure altitude regardless of what the test day conditions are. Twenty thousand feet pressure altitude is the same from one day to the next and from one aircraft to another. The parameter that does vary will be the temperature. By applying a correction for non-standard temperature to performance data as will be explained later, the data can be presented for any pressure altitude under standard day conditions. Presented in this form, pressure, density, and tape line altitudes are all equal.

2.2 AIRSPEED SYSTEM THEORY

2.2.1 INTRODUCTION

The fundamental fact to understand in the theory of airspeed systems is that the airspeed indicator is a pressure measuring instrument, not an instrument which measures the speed at which an airplane moves through the air. Actually the airspeed indicator senses the difference between two pressures, the total pressure (P_t) and the static pressure (P_s). These pressures are usually sensed in the pitot tube although some airplanes have the static source pick-up located on the fuselage.

It is desirable to have the airspeed indicator register true airspeed for use in navigation, but this is not possible for all conditions of altitude and temperature if we are to have a simple instrument which only senses a pressure differential. This fact will be discussed in detail later. The purpose of this discussion is to derive the relationship between the difference in pressures ($\Delta P = P_t - P_a$) and the quantity which is read from the airspeed indicator (assuming no instrument or position errors). In other words, we desire an equation of the form, $V = f\ (\Delta P)$.

This relationship is necessary to mark or calibrate the dial in speed units rather than in differential pressure which is actually sensed by the airspeed indicator. This relationship can be derived from Bernoulli's equation, which relates velocity with the pressure and density along a streamline. Two different equations will be obtained. One assumes the air to be an incompressible fluid; that is, the density remains constant. The

other considers the density of the air to change when coming to rest in the pitot tube. The first equation will give nearly correct results for low airspeeds (below 200 - 250 knots) but becomes increasingly inaccurate as the airspeed is increased. The second equation gives accurate results since it allows the density to change as it actually does when the velocity and pressure changes.

This discussion will not consider the effects of shock waves on the pressures sensed by the pitot - static system.

2.2.2 EULER'S EQUATION

First the differential equation for fluid motion known as Euler's equation will be derived. From this equation Bernoulli's equation is obtained. The derivation of Euler's equation is simplified, but the results obtained are identical to that obtained using a rigorous derivation.

Consider a non-viscous steady horizontal flow of a fluid through a streamtube as shown in Figure. No. 2.2.1. Imagine within the fluid a very small rectangular boxshaped boundary with a lenght of dL and an area dA as shown. The fluid enters the boundary at the front face (face 1) and leaves at the rear face (face 2). The velocity and pressure of the fluid changes in going from face 1 to face 2. The force acting on the front face is P_1 dA and the force acting on the rear face is P_2 dA where: $\Delta P = P_2 - P_1$ and

$$P_2 \, dA = (P_1 + \Delta P) \;\; dA$$

$$P_2 \, dA = P_1 \, (dp/dL) \, (dL) \, dA$$

dP/dL **is** the rate at which the pressure changes in the direction of flow.

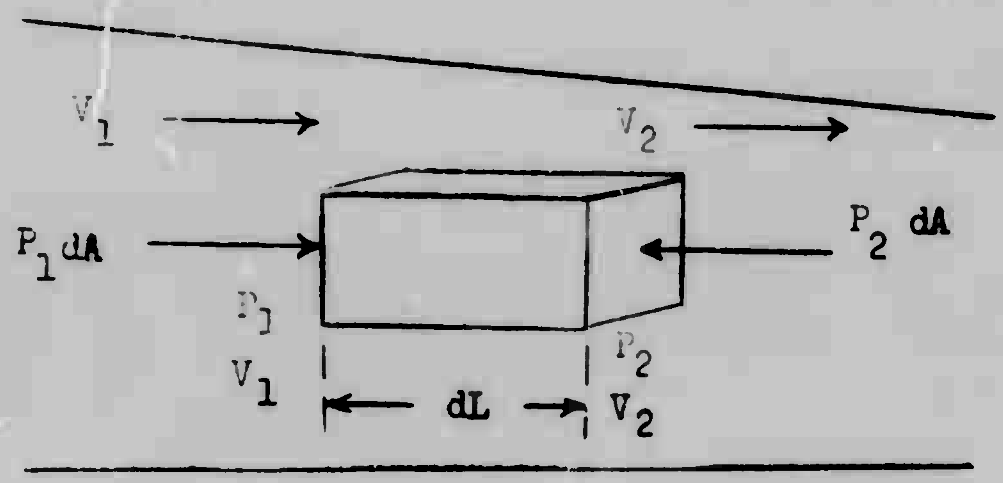


Figure 2.2.1

The net force acting on the fluid in the boundry is:

$$F = P_1 dA - P_2 dA$$

$$F = P_1 dA - (P_1 + dP/dL\,(dL))\,dA$$

$$F = -\,dP/dL\,(dL\,dA)$$

Newton's Second Law of Motion states that an unbalanced or net force is equal to the time rate of change of momentum or:

$$F = \frac{d}{dt}\,(mV)$$

It is assumed that the flow is steady so that, at any time, the same quantity of mass is within the boundary. Therefore, Newton's Second Law may be written as:

$$F = m\left(\frac{dV}{dt}\right) = ma$$

The net force will cause an acceleration of the fluid in the boundary. Thus if the pressure is greater at (1) than at (2) the velocity will increase from (1) to (2). Using Newton's Second Law:

$$F = -\frac{dP}{dL}\,dL\,dA = ma = m\left(\frac{dV}{dt}\right)$$

The mass (m) of the fluid within the boundary can be determined from the density and volume.

$$m = \rho \times \text{Vol.} = \rho\, dL\, dA$$

so that $$F = -\frac{dP}{dL}\, dL \quad dA = \rho \; dA \quad dL \; \frac{dV}{dt}$$

simplifying:

$$-\frac{dP}{dL} = \rho \frac{dV}{dt}$$

$$-\, dP = \rho \frac{dL}{dt} \quad dV$$

but $$\frac{dL}{dt} = V \quad \text{(instantaneous velocity of fluid)}$$

therefore $$-\, dP = \rho V \quad dV$$

$$\rho V\; dV + dP = 0 \; \text{(Euler's Equation)}$$ Equation 2.2.1

2.2.3 INCOMPRESSIBLE CASE

Now consider the fluid as being incompressible so that the density remains constant. Euler's differential equation with the density constant can now be solved.

$$\int \rho V dV + \int \rho dP = \int 0 = C$$

$$\rho \int V dV + \int dP = C$$

$$\rho \frac{V^2}{2} + P = C$$ Equation 2.2.2

This equation is Bernoulli's incompressible flow equation and applies whenever the density of the fluid does not change. The term $\frac{\rho V^2}{2}$ is called the dynamic pressure and is denoted by the symbol q.

$$q + P = C$$

The value of the constant C, called the Head, is a measure of the energy in the flow. The value of C for all points along a streamline will remain constant providing no energy is added or removed from the flow between the points in question. Therefore, Bernoulli's equation can be used for two points on the same streamline. Conditions at points A and B will be denoted by subscripts a and b.

$$\frac{\rho_a V_a^2}{2} + P_a = \frac{\rho_b V_b^2}{2} + P_b = C$$

(Since $\rho_a = \rho_b = \rho$)

$$\frac{\rho V_a^2}{2} + P_a = \frac{\rho V_b^2}{2} + P_b$$

Bernoulli's equation can be applied to the pitot tube. Consider the air approaching the airplane at a velocity V_T (equal and opposite in direction to the true airspeed of the airplane). The air slows as it approaches the pitot tube and finally comes to rest relative to the airplane in the pitot tube. Refer to Figure 2.2.2.

Free Stream Conditions

Point A

$\rho_a = \rho$

$P_a = P_s$

$V_a = V_t$

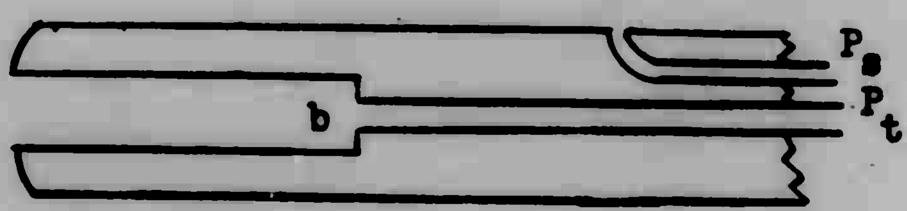
$V_t \longrightarrow$

Conditions at Point B

$\rho_b = \rho$

$P_b = P_t$

$V_b = 0$



Pitot - Static Tube Figure 2.2.2

The pressure of the air far ahead of the airplane is P_a and is equal to the pressure sensed by the static port (assuming no position error). The total pressure, P_t, is sensed by the total head port. The equation now becomes:

$$\frac{\rho V_t^2}{2} + P_a = P_t$$

$$\Delta P = P_t - P_a = \frac{\rho V_t^2}{2}$$

Solving for V_t

$$V_t = \sqrt{\frac{2\Delta P}{\rho}} \qquad \text{Equation 2.2.3}$$

This equation shows that if the airspeed indicator sensed density as well as differential pressure true airspeed could be obtained. Since the airspeed indicator only senses differential pressure the equation must be modified before the dial of the indicator can be calibrated.

Multiply the right hand side of the equation by $\sqrt{\frac{\rho_o}{\rho_o}}$

$$V_t = \sqrt{\frac{2\Delta P}{\rho} \cdot \frac{\rho_o}{\rho_o}}$$

$$V_t = \sqrt{\frac{2\Delta P}{\rho_o} \cdot \frac{1}{\sigma}}$$

or

$$V_e = V_t\sqrt{\sigma} = \sqrt{\frac{2\Delta P}{\rho_o}} \qquad \text{Equation 2.2.4}$$

Equivalent airspeed, V_e, by definition is equal to $V_t \sqrt{\sigma}$. This equation is the desired equation for the incompressible case. Since ρ_o is a constant, the equation may be simplified to:

$$V_e = 29 \sqrt{\Delta P}$$

(with V_e in ft/sec)

ΔP in lb/ft^2

Note that this equation is of the form, $V = f(\Delta P)$, where the V is V_e for the incompressible case. In order to determine V_t for this airspeed system one need only to know the density of the air and V_e since:

$$V_t = \frac{V_e}{\sqrt{\sigma}}$$

The old E6B computer performed this operation by setting in the temperature and pressure altitude (the only two variables necessary to determine the density).

This equation gives accurate enough results at low speeds (below 250 knots) and was used to calibrate airspeed indicators until the advent of the high speed airplanes of World War II and later. It is based on an assumption that is not exactly true; namely, that the density of the air remains constant.

2.2.4 COMPRESSIBLE CASE

Euler's equation is more difficult to solve when considering the density to change as the pressure changes. If the changes to the properties of the air along a streamline occur adiabatically (no gain or loss of energy) and are continuous, pressure and density may be related by the isentropic relationship:

$$PV^{\gamma} = K \text{ (constant)}$$

where

$$\gamma = \frac{C_P}{C_v} \text{, a constant (for air equal to 1.4).}$$

Since

$$v = \frac{1}{\rho g} \quad \text{then: } \frac{P}{(g\rho)^{\gamma}} = K$$

Substituting these terms in Euler's equation and intergrating we arrive at a new form of Bernoulli's flow equation.

$$\frac{V^2}{2} + \frac{\gamma}{\gamma - 1} \cdot \frac{P}{\rho} = C \qquad \text{Equation 2.2.5}$$

This is Bernoulli's equation for a compressible fluid. Note the differences when compared with Bernoulli's equation for an incompressible fluid.

Next, Bernoulli's compressible flow equation will be applied to the pitot tube as was done for the incompressible case. In the total head port the velocity is zero, pressure is the total pressure, P_t and the density is the total density, ρ_t.

$$\frac{V_t^2}{2} + \frac{\gamma}{\gamma - 1} \cdot \frac{P_a}{\rho_a} = \frac{\gamma}{\gamma - 1} \cdot \frac{P_t}{\rho_t}$$

From this equation a relationship for V_t in terms of P_t , P_a , and T_a can be derived.

The following equations for compressible flow are derived in Section 5 and presented in this section to complete the investigation of airspeed instruments designed to measure speed in the compressible flow regions of flight.

$$a = \sqrt{\gamma g R T_a} = \sqrt{\gamma P_a / \rho_a} \qquad \text{Equation 2.2.6}$$

$$M = V_t / a \qquad \text{Equation 2.2.7}$$

$$\frac{P_t}{P_a} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma - 1)} \qquad \text{Equation 2.2.8}$$

$$\frac{P_t}{P_a} = (1 + .2\, M^2)^{3.5}$$

Equation 2.2.8 is very useful in the study of aerodynamics. It is used to determine the total pressure of a moving fluid if the Mach number and static pressure are known.

An equation is now desired where $V = f(\Delta P)$. Using equation 2.2.8 an expression for ΔP will be determined. Substracting 1 from each side of equation 2.2.8 we have

$$\frac{P_t}{P_a} - 1 = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma - 1)} - 1$$

$$\frac{P_t}{P_a} - \frac{P_a}{P_a} = \frac{P_t - P_a}{P_a} = \frac{\Delta P}{P_a} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma - 1)} - 1$$

$$\frac{\Delta P}{P_a} + 1 = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma - 1)}$$

$$\left(\frac{\Delta P}{P_a} + 1\right)^{(\gamma - 1)/\gamma} = 1 + \frac{\gamma - 1}{2} M^2$$

Soving for M^2

$$M^2 = \frac{2}{\gamma - 1}\left[\left(\frac{\Delta P}{P_a} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]$$

$$M = \sqrt{\frac{2}{(\gamma - 1)}\left[\left(\frac{\Delta P}{P_a} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]} \qquad \text{Equation 2.2.9}$$

This equation shows that Mach number can be determined by having an instrument which measures ΔP and P_a. This is the way the Machmeter is is designed. It is essentially a combination of an airspeed indicator which senses ΔP and an altimeter which senses P_a.

An equation for V_t is still desired and it can be easily obtained from equation 2.2.9

Since $M = \frac{V_t}{a}$ or $M a = V_t$

Then

$$M a = \sqrt{\gamma g R T_a} \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{\Delta P}{P_a} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]}$$

$$\therefore V_t = \sqrt{\frac{2\gamma g \; RT_a}{(\gamma - 1)} \left[\left(\frac{P_t - P_a}{P_a} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]} \qquad \text{Equation 2.2.10}$$

Note, that to have an airspeed indicator give true airspeed directly it must sense three variables, ΔP, P_a and T_a. This is a complicated instrument, although such airspeed indicators have been built. If this equation is modified by using sea level standard day temperature (T_o) and pressure (P_o) instead of the free stream temperature and pressure it then becomes:

$$V_t \text{ (S.L. St'd Day)} = \sqrt{\frac{2\gamma g \; RT_o}{(\gamma - 1)} \left[\left(\frac{\Delta P}{P_o} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]}$$

An airspeed indicator calibrated from this equation will only indicate true airspeed when the airplane is flying at sea level under standard temperature conditions. Note, however, that this equation is of the form desired; that is, the velocity is a function of only one variable, ΔP, or $P_t - P_a$. This is the equation used to calibrate the present day airspeed indicator but it will only register V_t under special conditions. The quantity which it indicates is called calibrated airspeed (V_c). (Instrument and position errors assumed zero.)

Thus:

$$V_c = \sqrt{\frac{2\gamma g \; RT_o}{(\gamma - 1)} \left[\left(\frac{\Delta P}{P_o} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]}$$

Or

$$V_c = \sqrt{\frac{2\gamma}{(\gamma - 1)} \cdot \frac{P_o}{\rho_o} \left[\left(\frac{\Delta P}{P_o} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]} \qquad \text{Equation 2.2.11}$$

(Since $\frac{P}{\rho} = g\,RT$)

Introducing the constant values simplifies the equation to:

$$V_c \text{ (in ft/sec)} = 2495\sqrt{\left(\frac{\Delta P}{2116} + 1\right)^{.286} - 1}$$

We therefore read calibrated airspeed directly from the airspeed indicator (neglecting instrument and position errors) but it is still desired to know the true airspeed of an airplane. The procedure to determine true airspeed is not as obvious as it was for the incompressible equation in which we only had to correct the indicated reading for density. Our problem would be simplified if we could determine the equivalent airspeed from the calibrated airspeed since:

$$V_e = V_t\sqrt{\sigma}$$

Then we could easily determine true airspeed by making a correction for density. We will now derive an equation for V_e for the compressible case.

$$V_t = \sqrt{\frac{2\gamma P_a}{(\gamma - 1)\rho_a}\left[\left(\frac{\Delta P}{P_a} + 1\right)^{(\gamma-1)/\gamma} - 1\right]}$$

But $$V_t = \frac{V_e}{\sqrt{\sigma}} = \sqrt{\frac{\rho_o}{\rho_a}} \cdot V_e$$

So that $$\sqrt{\frac{\rho_o}{\rho_a}}\, V_e = \sqrt{\frac{2\gamma P_a}{(\gamma - 1)\rho_a}\left[\left(\frac{\Delta P}{P_a} + 1\right)^{(\gamma-1)/\gamma} - 1\right]}$$

$$\therefore V_e = \sqrt{\frac{2\gamma P_a}{(\gamma - 1)\rho_o}\left[\left(\frac{\Delta P}{P_a} + 1\right)^{(\gamma-1)/\gamma} - 1\right]}$$ Equation 2.212

Note that this equation is the same as the equation for V_c except that P_a replaces P_c. The difference between V_c and V_e is denoted as ΔV_c and can be written as:

$$\Delta V_c = V_c - V_e$$

Therefore

$$\Delta V_c = \sqrt{\frac{2\gamma P_o}{(\gamma - 1)\rho_o}\left[\left(\frac{\Delta P}{P_o} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]} - \sqrt{\frac{2\gamma P_a}{(\gamma - 1)\rho_o}\left[\left(\frac{\Delta P}{P_a} + 1\right)^{(\gamma - 1)/\gamma} - 1\right]}$$

Equation 2.2.13

Note that if $P_a = P_o$ (pressure altitude of S.L.) V_c will equal V_e so ΔV_c is zero regardless of the magnitude of ΔP. Also, we can calculate ΔV_c for any altitude and any values of V_c (since a value of V_c determines ΔP). The calculated values of ΔV_c will apply to all airplanes, and a graph or table of ΔV_c vs V_c and pressure altitude are presented in the Flight Handbooks for all aircraft. The quantity ΔV_c is called the compressibility correction to the airspeed indicator. This term is sometimes misunderstood to mean that this correction is only necessary when the airplane is going supersonic or generating shock waves. At a pressure altitude of sea level this correction remains zero even at supersonic speeds, and the correction may be substantial even at low calibrated airspeeds at high altitude. Refer to the graphs of ΔV_c vs V_c. AFFTC TN-59-46 Charts A-67-70.

In order to determine true airspeed from calibrated airspeed, first find the equivalent airspeed using the compressibility correction. Then the true airspeed may be easily determined by applying the density correction in the same manner as described in the section on incompressible flow.

2.2.5 FUNCTIONAL RELATIONSHIPS

From the previous paragraphs we can now make functional relationships for various types of airspeed and Mach number.

$$V_t = f(P_t, P_a, \rho_a) = f(P_t, P_a, T_a)$$

$$V_e = f(P_t, P_a)$$

$$V_c = f(P_t, P_o) = f(\Delta P)$$

$$\Delta V_c = f(P_t, P_a)$$

$M = f(P_t, P_a)$ but since $V_c = f(P_t)$ and $H_c = f(P_a)$ then

$$M = f(V_c, H_c)$$

2.2.6 USE OF COMPUTERS

It should be mentioned again that the computer face of the E6B dead reckoning computer only determines true airspeed from equivalent airspeed and does not include the compressibility correction (ΔV_c). The same applies to the high speed version of the E6B type computers (Type C-10) except that a correction factor table is presented on the sliding card as a means of obtaining true airspeed correctly. The old D-5 circular slide rule-type computer (if any are still in use) takes into account the compressibility correction (ΔV_c) in determining true airspeed from calibrated airspeed. The D-4 computer is inaccurate (errors of 2½% or more) when determining true airspeed at high altitudes. The type E-11 dead reckoning computer (circular slide rule type) considers the compressibility correction in determining true airspeed. In using the E-11 computer the ambient temperature must not be used directly to determine true airspeed but the indicated temperature from the free air temperature gage must be used if one is installed in the airplane. The indicated temperatures used are based on a temperature recovery factor of 0.80 for the true airspeed; only the pressure altitude and calibrated airspeed are required. The Mach number scale of the computer is determined from equation 2.2.10 and gives accurate results.

2.3 AERODYNAMIC FORCES

2.3.1 INTRODUCTION

The performance of an aircraft is directly related to the forces acting on it. An examination of the primary forces acting on the aircraft shows that there are four: lift, weight, thrust and drag. For performance analysis all are considered to act through the center of gravity as shown in Figure 2.3.1 below.

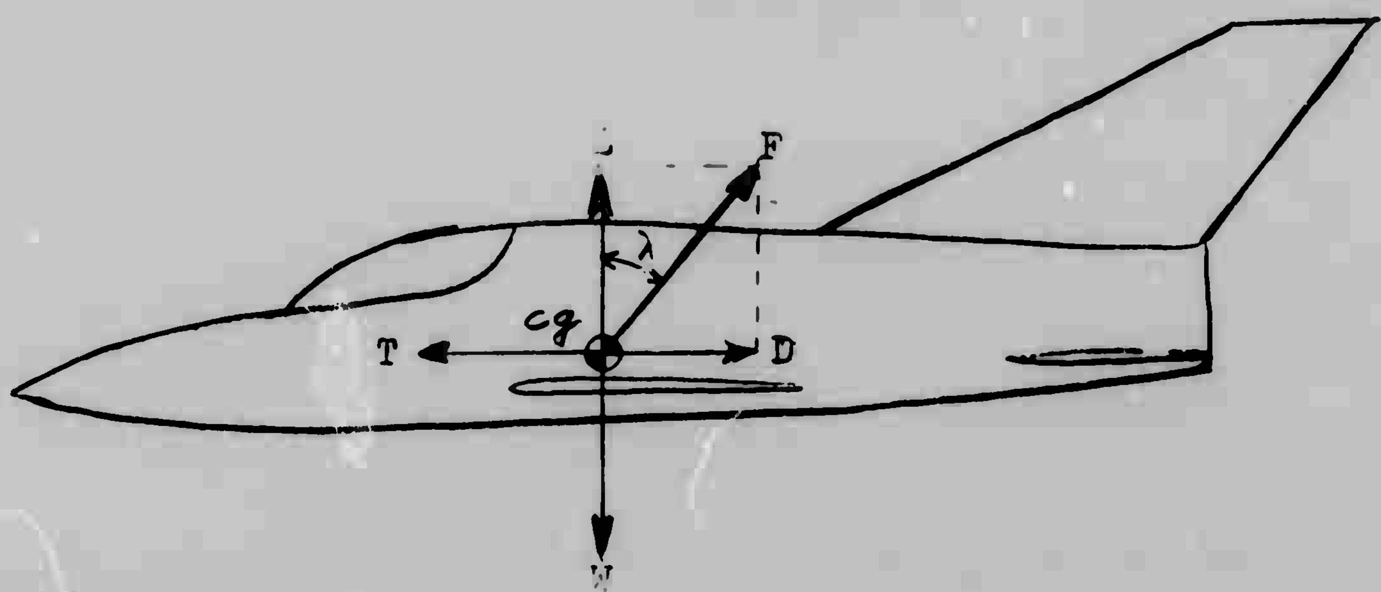


Figure 2.3.1

If the thrust exceeds the drag the aircraft accelerates along the flight path and is said to have excess thrust, $T_{ex} = T - D$. If the lift is greater than the weight the aircraft is said to be maneuvering and has a normal acceleration which is equal to the ratio of the lift to the weight component.

$$n = \frac{L}{W}$$

or

$$L = nW \qquad \textbf{Equation 2.3.1}$$

Thrust is directly attributable to the power plant while lift and drag are aerodynamic forces which result from the reaction of the air flowing over the aircraft. It is the aerodynamic forces which we wish to investigate in this section.

In order to properly orient ourselves to the language used in aerodynamics, let us put forth a few commonly used definitions.

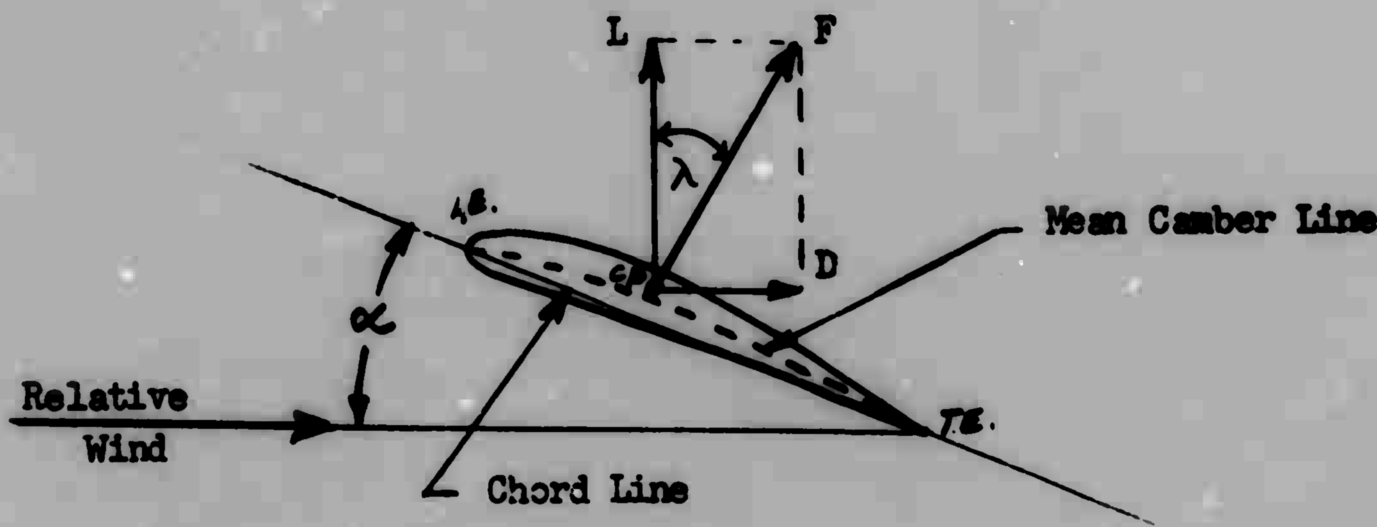


Figure 2.3.2

Chord-Line is the straight line between the leading edge and trailing edge of the wing or of the mean camber line end points.

Mean-Line or Mean-Camber Line is the line described by points which are [illegible] from the upper and lower surfaces.

Camber is a measure of the curvature of an airfoil as evaluated by the height of the mean camber line above or below the chord line.

Relative Wind refers to the motion of the air relative to the aircraft and is equal and opposite to the velocity of the aircraft.

Angle of Attack is the angle between the relative wind and the chord line.

Center of Pressure (C.P.) is point on the airfoil through which all of the aerodynamic forces may act without the creation of a moment.

Resultant Aerodynamic Force is the summation of all of the aerodynamic forces acting on an airfoil. Its point of application is at the center of pressure.

Lift is the component of the resultant aerodynamic force which is perpendicular to the relative wind.

Drag is the component of the resultant aerodynamic force which is parallel to the relative wind.

Aerodynamic Center is the point on the wing through which the aerodynamic forces may act such that the moment coefficient is constant for all angles of attack.

The angle λ is the angle between the lift vector and the resultant aerodynamic force.

Aerodynamics is a specialized branch of fluid mechanics. While it is possible to consider lift and drag from a fluid mechanical view point it is not necessary for the development of the basic relations. An involved development of fluid mechanical relationships will be deferred until later.

2.3.2 AERODYNAMIC FORCE

2.3.2.1 GENERAL

Prior to launching into a mathematical dissertation let us qualitatively consider some of the physical concepts of fluid flow which result in the development of aerodynamic lift and drag.

From the definition it is evident that the aerodynamic lift and drag can be resolved into one resultant aerodynamic force, F, acting through the center of pressure as shown in Figure 2.3.2. It is this resultant force with which we shall first be concerned.

There are two basic forces which act on a body in fluid flow. Frictional forces arise because of the viscous or adhesive qualities of the fluid. That is, the fluid is sticky and tends to cling to the surface like cold molasses to a spoon. While the viscosity of air is considerably less than molasses, viscous effects are still present. Pressure forces are particularly significant since they are the ones which produce lift. A body in a fluid is acted upon by two pressure forces: The static or ambient pressure which is the same over all of the body and the dynamic pressure or pressure due to motion which produces regions of high and low pressure over a body. We are all familiar with static pressure. It is the pressure acting on the surface of your body right now. We are also familiar with dynamic pressure. It is the pressure we feel when we put our hand outside of a moving vehicle. In the case of the moving vehicle, we find that the static pressure continues to act uniformly over the surface but that the dynamic pressure is superimposed on it to produce a resultant

force which must be resisted by the muscles of our arm. If we represent our hand by a flat plate we can pictorially show the effects of static and dynamic pressure.

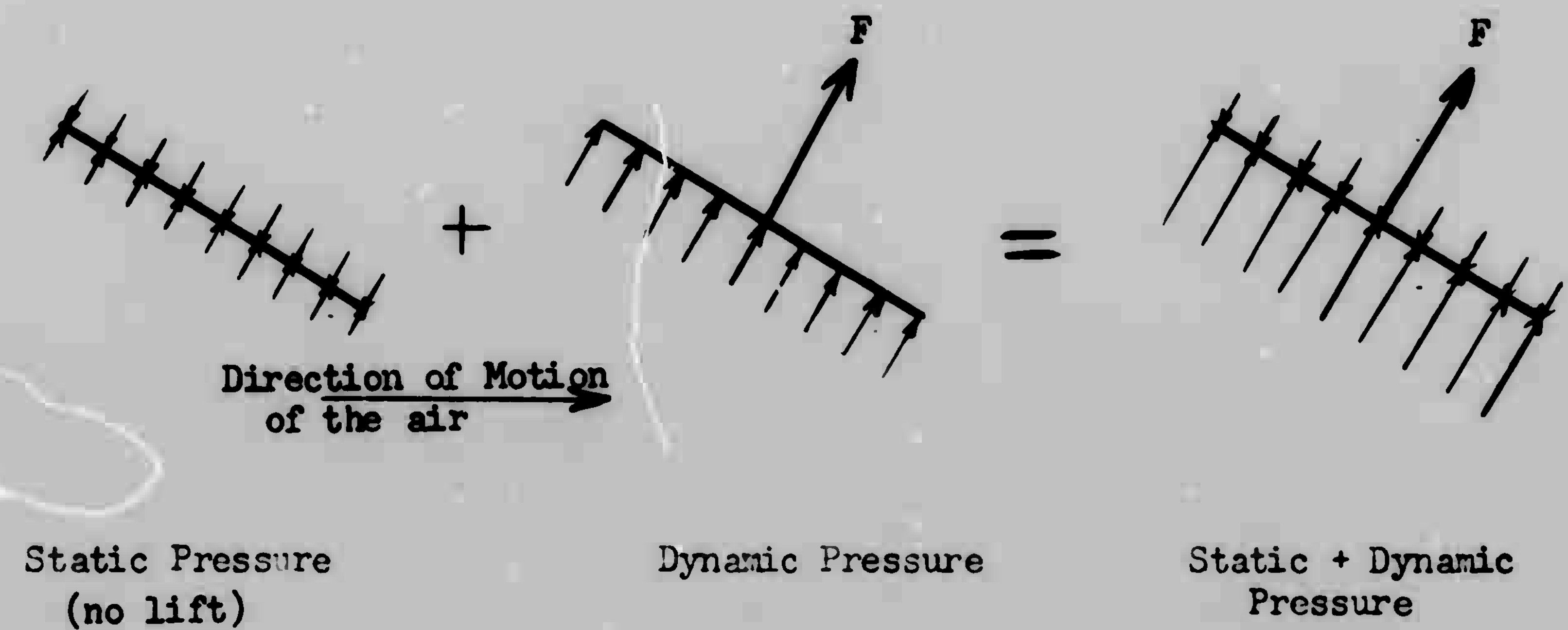


Figure 2.3.3

Note from the above sketch that pressure always acts perpendicular to the surface and that the resultant force is produced only by the dynamic pressure. From the latter observation we might guess that the force, F, is given by the product of the dynamic pressure and the area of the plate, S.

$$F = qS \qquad \text{Equation 2.3.2}$$

where $q = \frac{1}{2} \rho V_t^2$

The result unfortunately is not this simple since it is possible to develop considerably larger forces from an airfoil than is predicted by this equation. While q and S are definitely factors in the aerodynamic problem they are not the only variables to be considered as will be shown.

Consider the forces acting on the turning vane shown below:

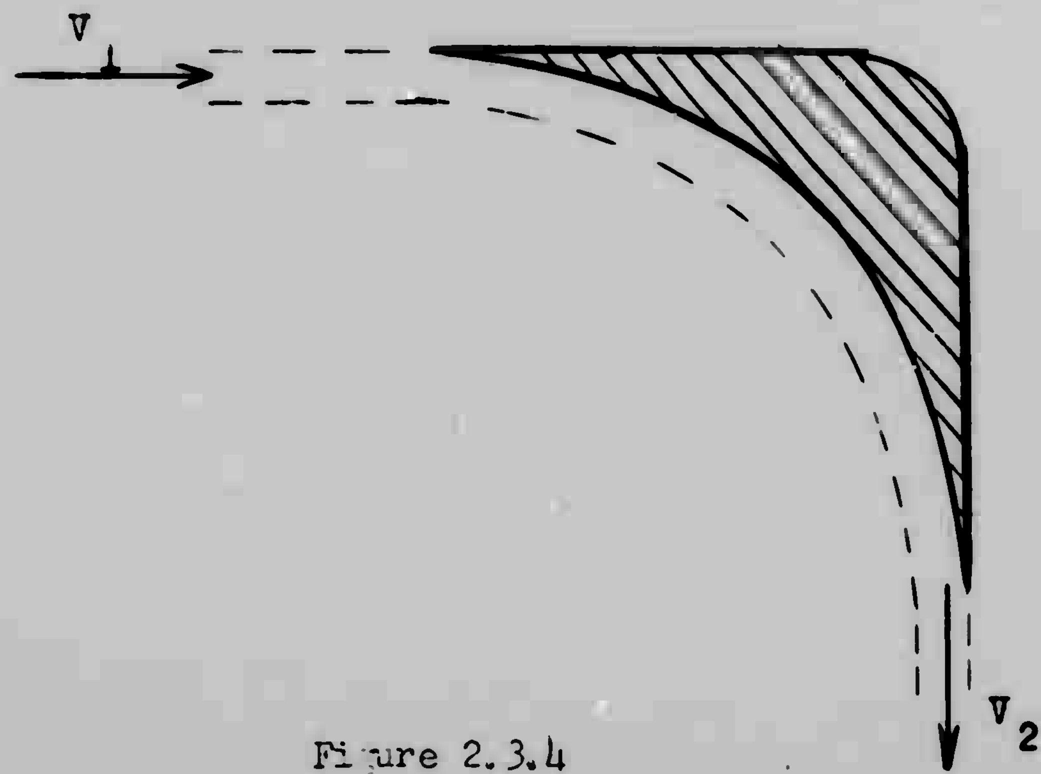


Figure 2.3.4

To simplify the problem, let us assume that the fluid is nonviscous and incompressible. Hence, we have no compressibility effects to consider and no frictional forces to slow down the flow as it passes over the vane. We can calculate the force acting on this vane by the application of Newton's second law.

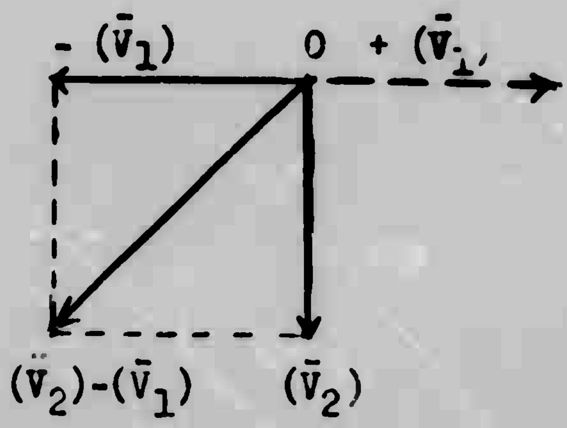
$$F = ma = \frac{d(mV)}{dt}$$

This states that the force is equal to the change in momentum which when differentiated gives

$$F = \frac{d(mV)}{dt} = m\frac{dV}{dt} + V\frac{dm}{dt}$$

The second term is zero since there is no change in the mass flow as it passes over the vane. Also, when the flow is steady and frictionless there is no change in velocity with the passage of time. Therefore, the above equation becomes

$$F = \dot{m}\, dV = \dot{m} \Delta \bar{V}$$



where: $\dot{m}$ = mass flow rate = ρVA = constant

$\Delta \bar{V}$ = vector increment = $\bar{V}_2 - \bar{V}_1$

substituting these, the above equation becomes

$$F = \rho VA \Delta \bar{V} = \rho VA (V_2 - V_1)$$

where ρ = density

A = cross-sectional area of the stream

Since velocity is a vector quantity we must take the vector difference by considering the changes in both magnitude and direction. Since the flow is steady and frictionless there is no change in the magnitude of the velocity; however, the stream is deflected through 90 degrees. Performing the vector subtraction yields the following result

$$\Delta \bar{V} = \bar{V}_2 - \bar{V}_1 = \frac{V}{\sin 45^o} = \frac{V}{.707} = 1.414V$$

Substituting this result into the force equation gives

$$F = (\rho VA)\ 1.414V = 1.414A\rho V^2$$

Multiplying and dividing by 2 gives a more familiar form to the force equation

$$F = 2.828 \frac{\rho V^2}{2} A = 2.828\ q\ A$$

Thus, we see that the momentum analysis used on the turning vane has provided an unexpected increase in force over that predicted by the simple energy analysis. While a turning vane differs from an airfoil, in that the airfoil is immersed in the fluid and the turning vane is not, they are similar in function in that both cause a deflection of the flow.

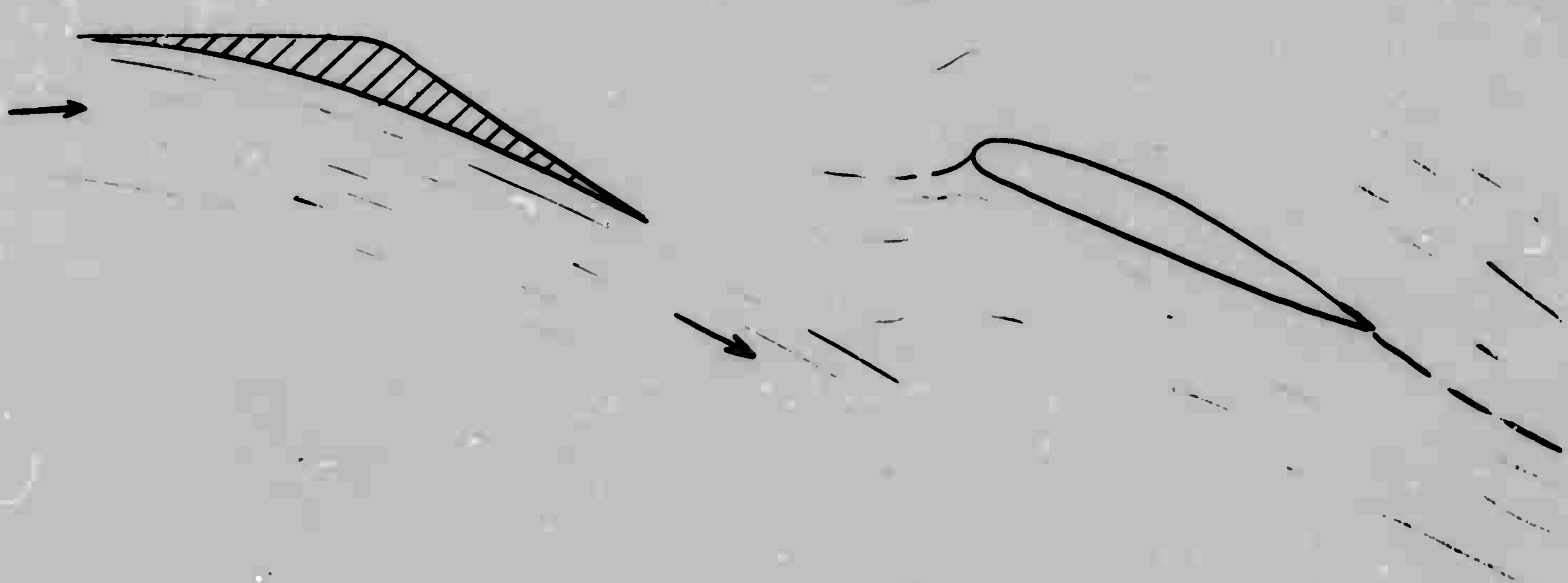


Figure 2.3.5

Another means of visualizing aerodynamic lift is to consider what happens in a venturi in steady incompressible flow (ρ = const)

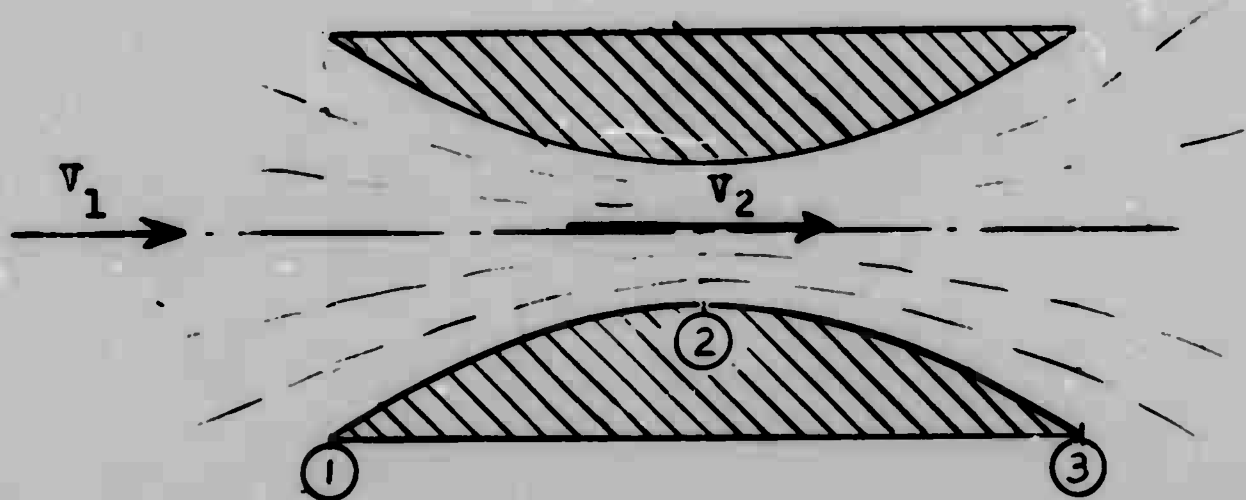


Figure 2.3.6

In steady flow the amount of fluid going past station (1) equals the amount past station (2). Because the cross sectional area is smaller at station (2) the velocity must be faster to get the same amount of fluid past station (2) as it is going past station (1). This is shown by the following equation

$$\rho V_1 A_1 = \rho V_2 A_2$$

since density is constant in incompressible flow

$$V_1 A_1 = V_2 A_2$$

$$V_2 = \frac{A_1}{A_2} V_1$$

For the venturi shown A_1 is greater than A_2; therefore, V_2 is greater than V_1 by the ratio of A_1/A_2.

Now referring to the incompressible Bernoulli equation,

$$P + \frac{1}{2}\rho V^2 = \text{constant}$$

writing this for station 1 and 2

$$P_1 + \frac{\rho V_1^2}{2} = P_2 + \frac{\rho V_2^2}{2}$$

and solving for, P_2

$$P_2 = P_1 - \frac{\rho}{2}(V_2^2 - V_1^2)$$

We have already seen that V_2 is greater than V_1 so that the quantity in the parenthesis is positive. This positive number is subtracted from P_1. This, then shows that P_2 is less than P_1 or that the pressure is lower in the throat of a venturi.

From this we can draw an analogy between an airfoil and the venturi. Let us examine the flow through the ventui, Figure 2.3.6. A particle starting on the center line at station 1 stays on the center line through to station 3 while particles starting on either side of the center line move toward the center line as they approach station 2 and away from the center line as they go from station 2 to 3. The farther away from the center line the more pronounced is this effect.

Now let us look at the flow over the upper portion of a flat bottomed airfoil.

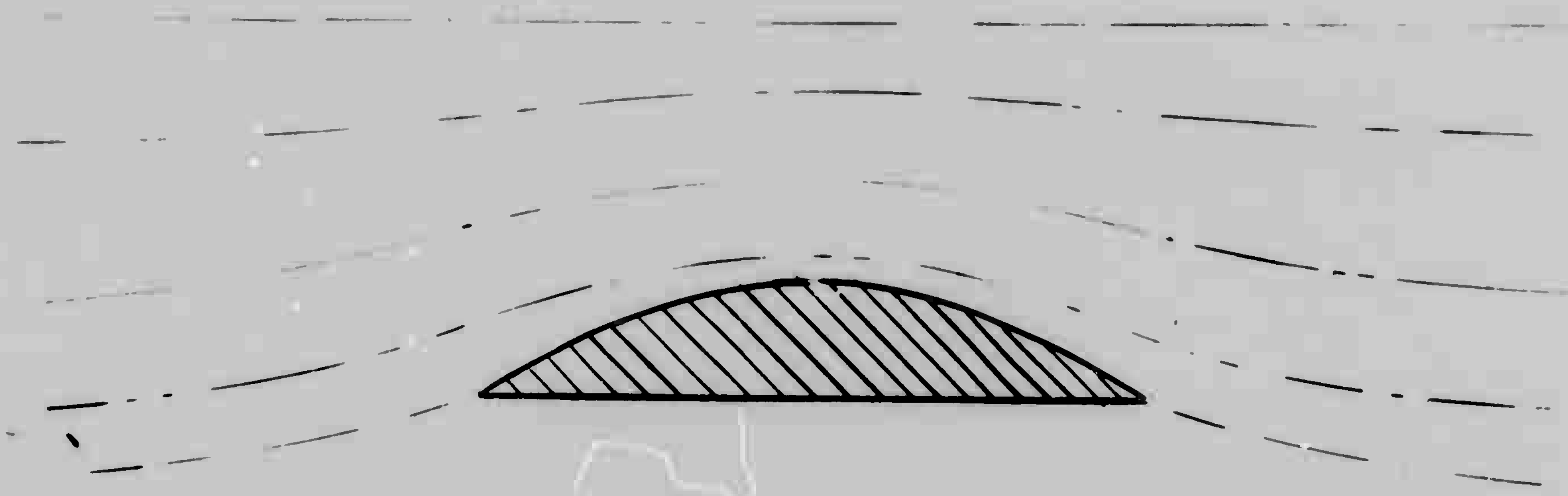


Figure 2.3.7

Note that the flow moves up and over the wing in much the same way as it does over the lower half of the venturi and that the farther away from the wing stream lines are, the less they are deflected. Theoretically the streamlines at an infinite distance from the wing are undeflected. From this it is obvious that the flow over the wing is very similar to the flow over the lower half of a venturi, in which case, low pressures are created over the upper surface of the wing while ambient pressure or even some dynamic pressure acts on the underside. Thus, an aerodynamic force is created due to this pressure difference. It should be noted that as the airfoil becomes thicker the pressure over the upper surface decreases, thus increasing the lift. Thus, camber increases the lifting capability of a wing.

Aerodynamic lift can be easily demonstrated by blowing over a curved piece of notebook paper. As you blow over the paper it will tend to rise and roll toward you rather than away as you might expect. Try it!

2.3.2.2 DIMENSIONAL ANALYSIS

Thus far we have seen that the aerodynamic force varies with the surface area, S, and the dynamic pressure, q, which in turn is a function of the

density, ρ, and the true speed, V_t. In our simple analogy we ignored the effects of vicosity and compressiblility. The viscosity tends to slow the flow next to the surface causing skin friction drag while compressiblity gives rise to wave drag and other effects associated with transonic and supersonic flight. The latter is associated with the speed of sound, a. Thus, we can then write a functional relationship for the resultant aerodynamic force, F.

$$F = f\,(\rho,\ V_t,\ S,\ \mu,\ a)$$

μ, (mu) = the coefficient of viscosity

Utilizing this functional relationship and the technique of dimensional analysis (Ref Sec 1.2) an equation can be obtained for the resultant aerodynamic force which in turn can be resolved into aerodynamic lift and drag. Expressing the above functional relationship in exponential form gives

$$F = C\left[\rho^a \ V_t^{\,b} \ S^c \ \mu^d \ a^e\right]$$

which F, L, t units is

$$F = C\left[\left(\frac{F_t^{\,2}}{L^4}\right)^a \left(\frac{L}{t}\right)^b \left(L^2\right)^c \left(\frac{F_t}{L^2}\right)^d \left(\frac{L}{t}\right)^e\right]$$

setting up the simultaneous equations

$$\begin{aligned}
F:&\quad 1 = a + d\\
L:&\quad 0 = -4a + b + 2c - 2d + e\\
t:&\quad 0 = 2a - b + d - e
\end{aligned}$$

solving in terms of d and e gives

$$a = 1 - d$$

$$b = 2 - d - e$$

$$c = 1 - d/2$$

substituting these into the exponential equation

$$F = C\left[\rho^{(1-d)}\, V^{(2-d-e)}\, S^{(1-d/2)}\, \mu^{d}\, a^{e}\right]$$

$$F = C\left[\rho \cdot \rho^{-d}\, V^{2} \cdot V^{-d} \cdot V^{-e}\, S \cdot S^{-d/2}\, \mu^{d}\, a^{e}\right]$$

$$F = C\left[\left(\rho V^{2} S\right)\left(\frac{\mu}{\rho V S^{\frac{1}{2}}}\right)^{d}\left(\frac{a}{V}\right)^{e}\right]$$

The term $\dfrac{\mu}{\rho V \ell}$ is defined as the Reynolds number and V/a is the Mach number.

The term V^2 is twice the dynamic pressure, q, so multiplying and dividing by 2 gives

$$\rho V^{2} = 2\left(\tfrac{1}{2}\rho V^{2}\right) = 2q$$

With the above definitions the aerodynamic force equation may be written as

$$F = \left[2C\left(\frac{1}{R_e^{d}}\right) \times \left(\frac{1}{M^{e}}\right)\right]\frac{1}{2}\rho V_t^{2} S$$

or

$$F = \left[2C\,\frac{1}{R_e^{d}}\,\frac{1}{M^{e}}\right] q S$$

The variables in the bracket remain relatively constant at low speeds and low angles of attack and may be referred to as the aerodynamic force coefficient. While this expression is correct we would really want to find the lift and drag. The obvious way to do this would be to resolve the aerodynamic force into components normal to and parallel to the relative wind; however, the angle λ is not usually known.

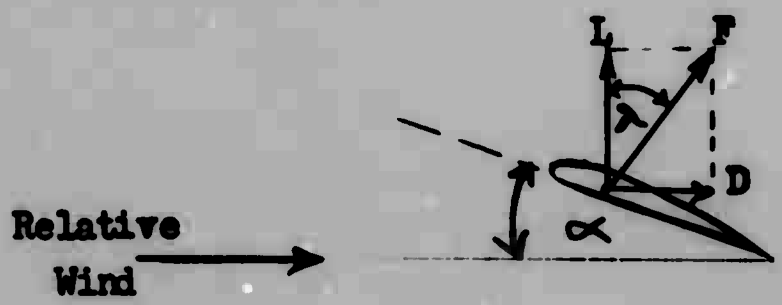


Figure 2.3.8

Assuming that λ is known, then the lift and drag coefficients can be expressed as

$$C_L = \left[2C \, \frac{1}{R_e^d} \times \frac{1}{M^e} \right] \cos \lambda$$

$$C_D = \left[2C \, \frac{1}{R_e^d} \times \frac{1}{M_e} \right] \sin \lambda$$

and the equations for the lift and drag are

$$L = C_L \; q \; S \qquad \text{Equation 2.3.3}$$

$$D = C_D \; q \; S \qquad \text{Equation 2.3.4}$$

In order to avoid the problem of finding λ and resolving the resultant force, the lift and drag coefficients are directly determined by experimental means through the above equations. Hence we bypass the determination of the resultant aerodynamic force and seldom refer to it.

Since at low speed $C_L = f(R_e, \alpha)$ the data is normally obtained in wind tunnels by holding Reynolds number constant and varying angle of attack. The data is then presented in the following form

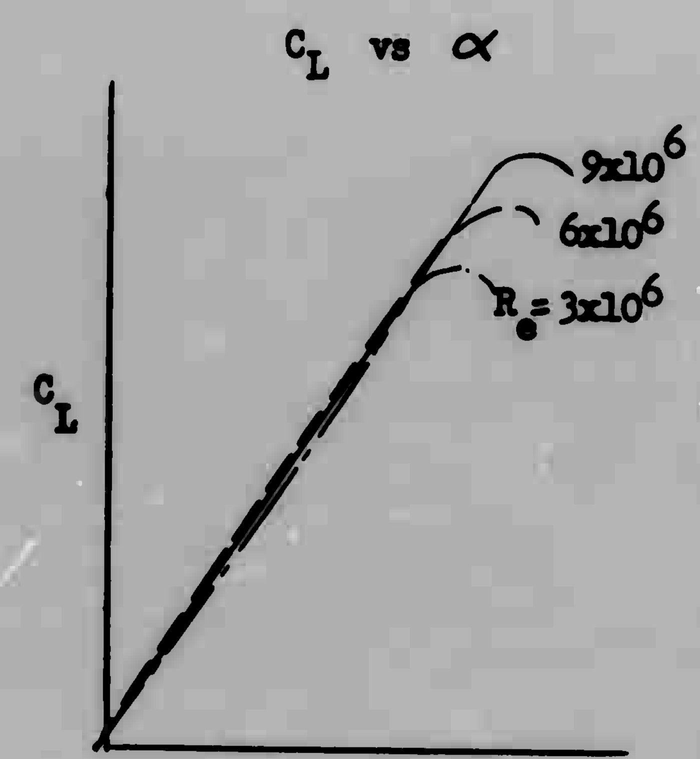


Figure 2.3.9a

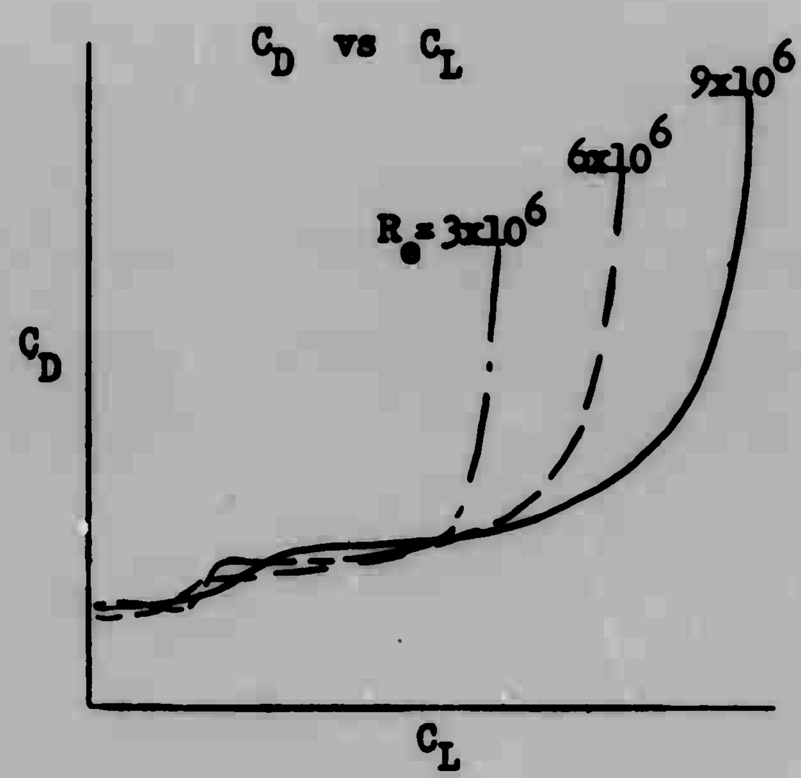


Figure 2.3.9b

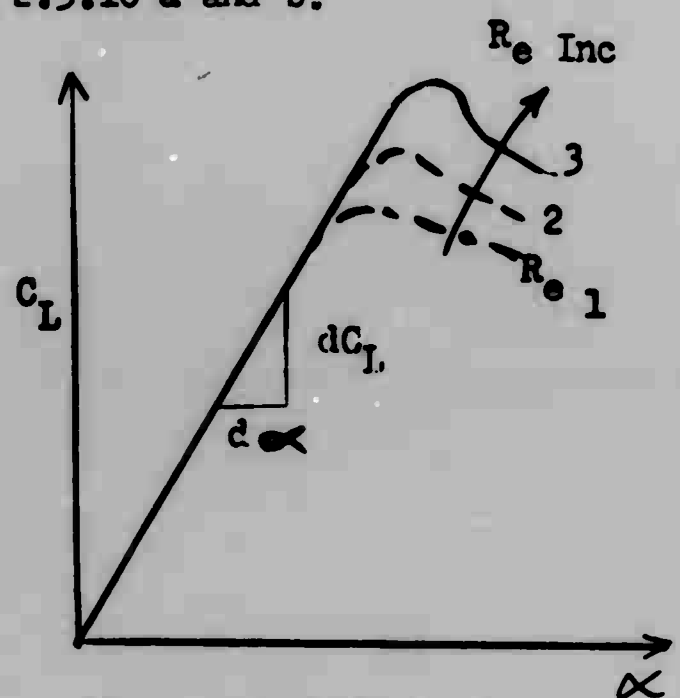
Examples of these can be seen in any NASA airfoil Report or in most aerodynamics book. (See Dommasch, Sherby and Connolly Airplane Aerodynamics pages 490-505)

2.3.3 AERODYNAMIC LIFT

While we have seen that the aerodynamic coefficients are affected by Reynolds' number and Mach number we shall consider their effects as small for the following discussion except in the region of high lift. Reynolds' number and Mach number effects will be discussed later but first let us discuss the variation of the lift coefficient with angle of attack for some typical airfoil section and planforms.

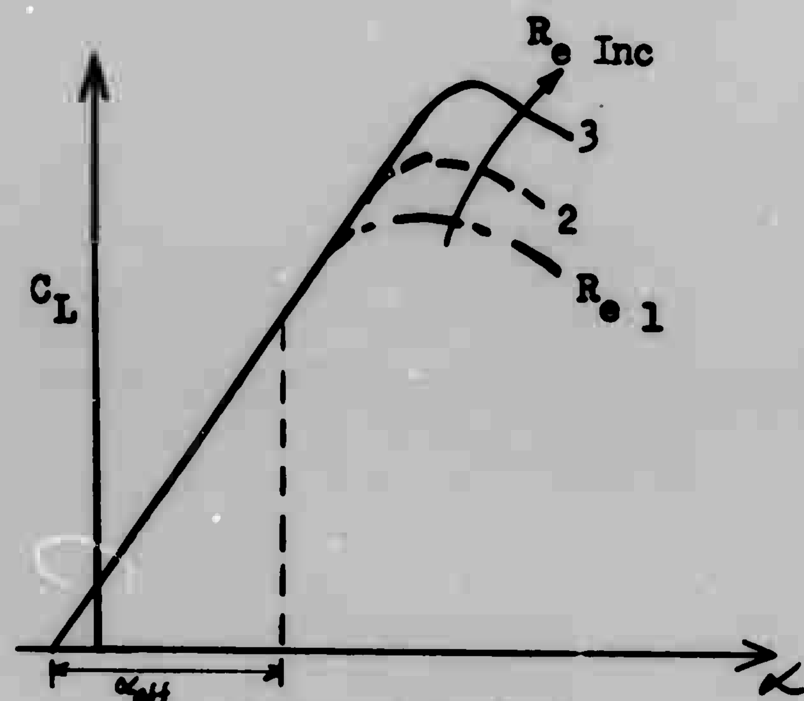
2.3.3.1 C_L VERSUS α CURVES

Some typical plots of C_L versus angle of attack are shown in Figure 2.3.10 a and b.



Symmetrical Airfoil

Figure 2.3.10a



Cambered Airfoil

Figure 2.3.10b

Because of its symmetry, a symmetrical airfoil at a zero angle of attack produces as much upward forces as it does downward so that the $C_L - \alpha$ curve for all symmetrical airfoils produce zero lift at zero angle of attack.

A cambered airfoil, however, has no such symmetry at zero angle of attack and therefore produces lift. Consequently the angle of attack required to produce zero lift is at some negative angle which depends on the shape and amount of camber.

An important characteristic of the lift curve is that it is linear through most of its range except in the region of stall at maximum lift coefficient. Since it is linear the slope is constant and is equal to $\frac{dC_L}{d\alpha} = a$. In the region of constant slope the lift coefficient is

$$C_L = a\alpha \text{ for a symetrical airfoil}$$

or in general

$$C_L = a\,(\alpha - \alpha_{oL}) \text{ for any airfoil} \qquad \text{Equation 2.3.5}$$

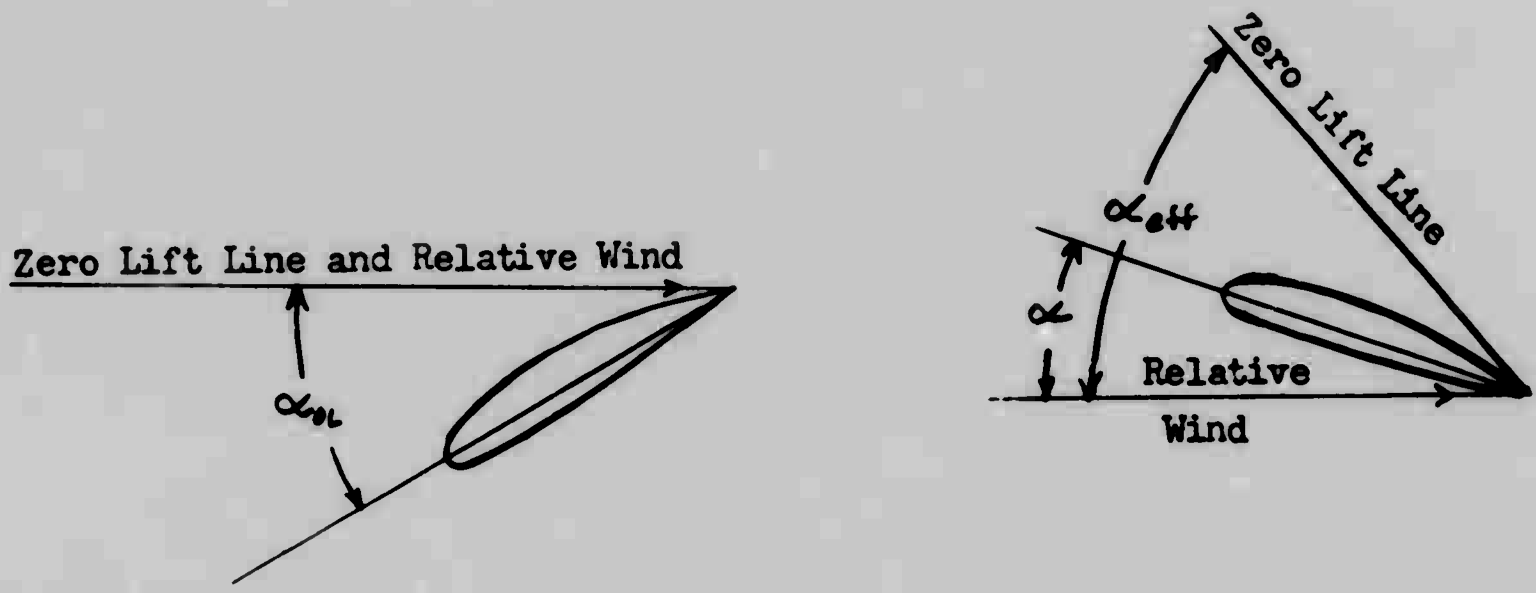
For example, if $a = .12$ $\alpha_{oL} = -2^{\circ}$

What is the C_L at $\alpha = 8^{\circ}$?

$$C_L = .12\,(8 - (-2)) = 1.2$$

Zero Lift Line and Effective Angle of Attack.

Frequently it is more convenient to reference angle of attack to some line other than the chord line. Such a reference is the zero lift line which is frequently used for stability work. The zero lift line is defined as a line parallel to the relative wind and passing thru the trailing edge when the airfoil is at zero lift.



Airfoil at Zero Lift

Airfoil Developing Lift

Figure 2.3.10c

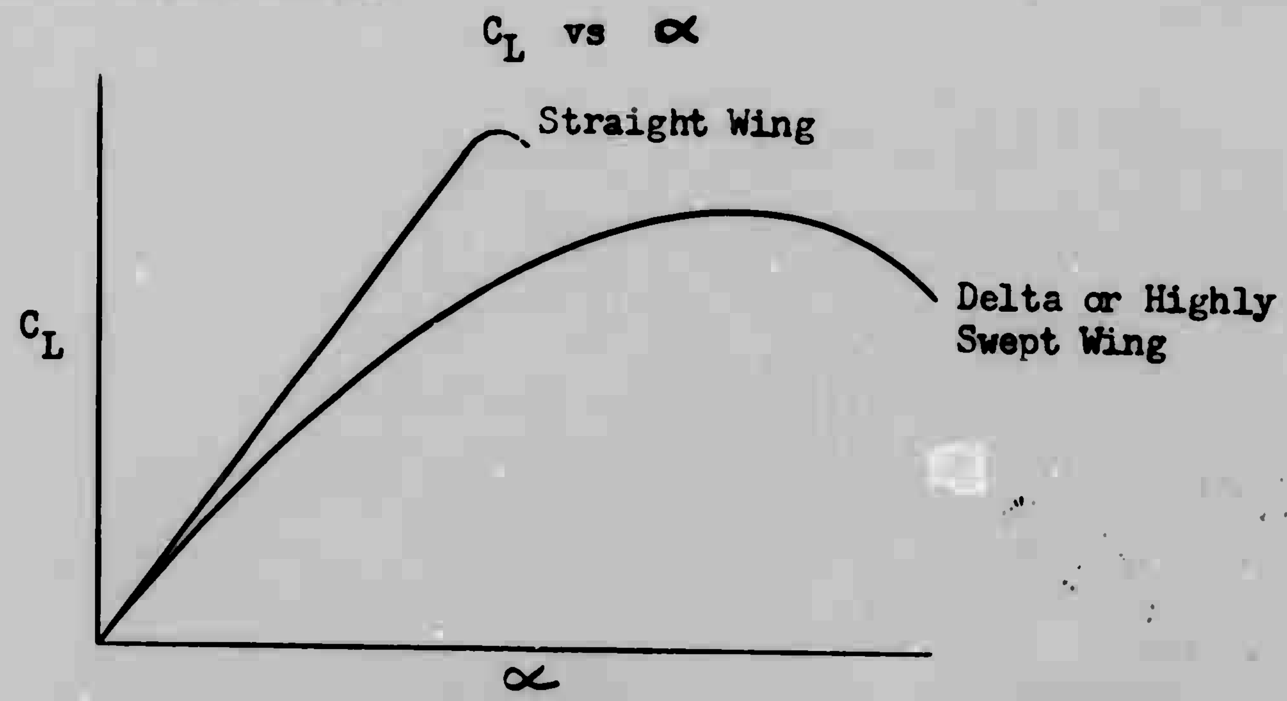
The effective angle of attack, α_{eff}, is defined as the angle of attack of the zero lift line. Thus, at zero effective angle of attack the lift is zero, so that the coefficient of lift is given by

$$C_L = a\,\alpha_{eff} \qquad \text{Equation 2.3.6}$$

for both symetrical and unsymetrical airfoils.

2.3.3.2 PLANFORM EFFECTS

It should be noted that all aircraft do not have linear lift curve slopes. That is, the planform has a great effect on the slope of the curve. For instance, a delta wing or a highly swept wing will have a lift curve similar to that shown in Figure 2.3.11



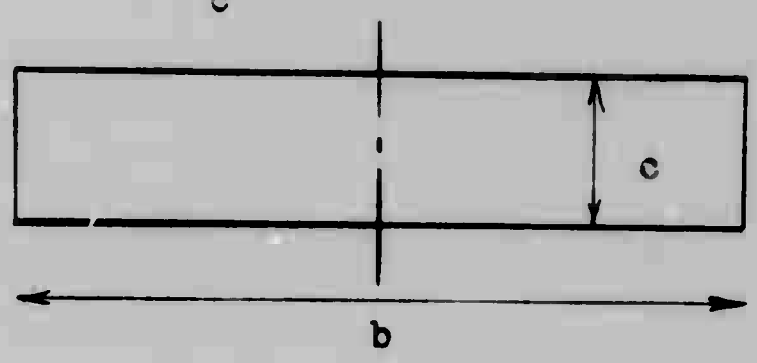
Effect of wing planform on a given airfoil section.

Figure 2.3.11

2.3.3.3 ASPECT RATIO EFFECTS

Aspect ratio also effects the lift curve. Aspect ratio (AR) is defined as the span divided by the chord length of a straight wing

$$AR = \frac{b}{c} \qquad \text{Equation 2.3.7}$$

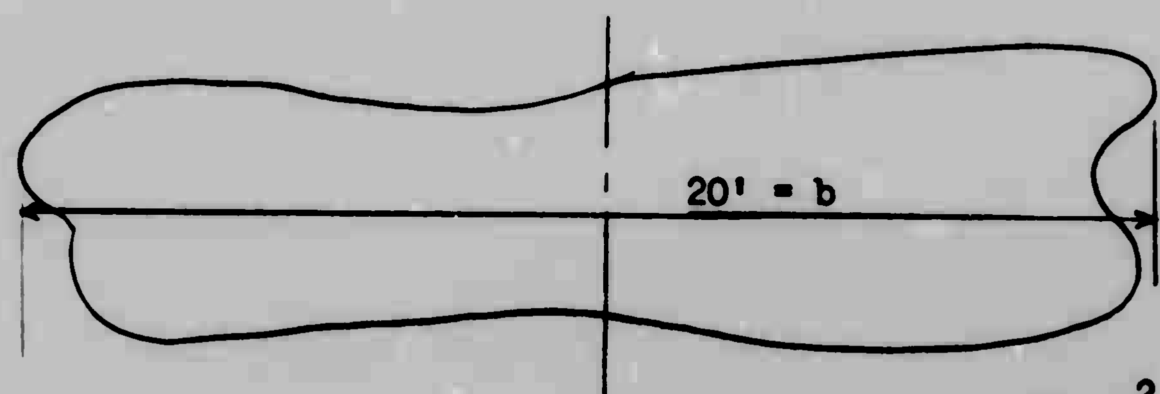


Not all wings are straight with a constant chord, so another consistant definition applicable to any planform is required. This is obtained by the following procedure

$$AR = \frac{b}{c} \cdot \frac{b}{b} = \frac{b^2}{S} \qquad \text{Equation 2.3.8}$$

where S is the wing area.

With this definition the aspect ratio of any wing can be determined if the wing area and span are known. For example, the aspect ratio of the wing shown is



$$AR = \frac{(20)^2}{150} = \frac{400}{150} = 2.67$$

A wing of finite span does not develop the maximum possible lift for any given angle of attack due to tip losses caused by the leakage of air from the bottom of the wing to the low pressure area on the top. The shorter the span the more area is effected by these losses. Therefore, a low aspect ratio wing would have to be at a higher angle of attack to develop the same lift coefficient as a high aspect ratio wing of the same wing area. The effects of aspect ratio on the $C_L - \alpha$ curve is shown in Figure 2.3.12

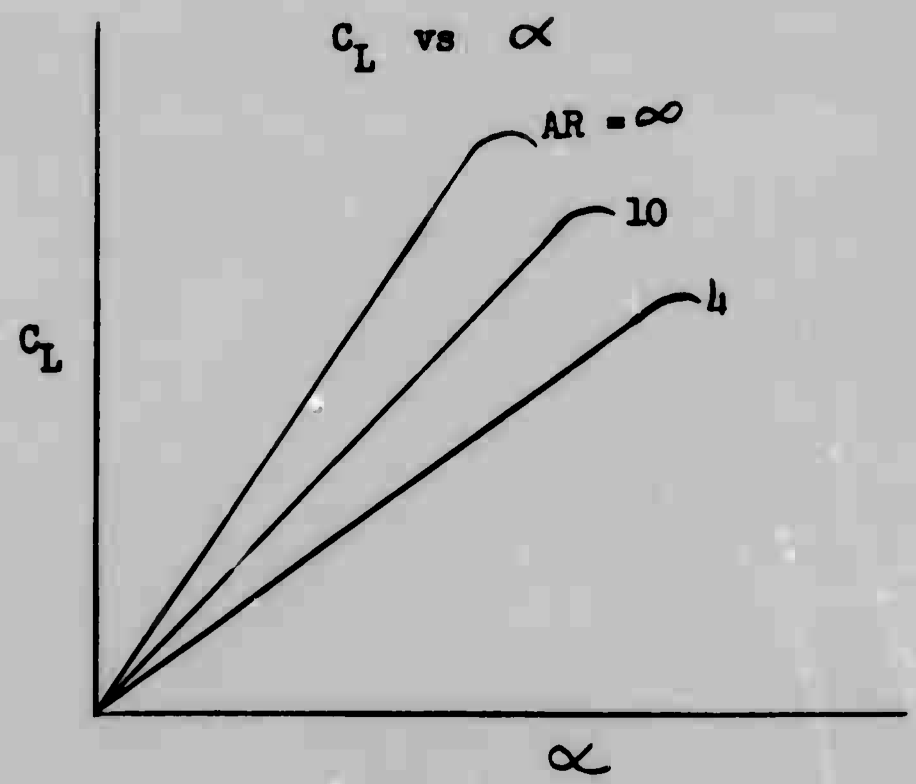


Figure 2.3.12

2.3.3.4 OTHER FORMS OF THE LIFT EQUATION

From dimensional analysis the lift equation was obtained as

$$L = C_L\, q\, S$$

or $$L = C_L \tfrac{1}{2} \rho V^2 S = nW$$

where n is the load factor and W is the weight of the aircraft.

Several other equivalent forms of this equation can be obtained and may be more useful depending on the information that is available.

The basic difference between these equations is the definition of q in terms of V_e or Mach number.

$$q = \tfrac{1}{2} \rho_a V_t^2$$

<u>In terms of V_e</u>

$$V_e = \sqrt{\sigma}\, V_t = \sqrt{\rho_a/\rho_o}\, V_t$$

$$V_t^2 = \frac{\rho_o}{\rho_a} V_e^2$$

$$\therefore q = \tfrac{1}{2} \rho_o V_e^2$$ **Equation 2.3.9**

or $$\boxed{C_L = \frac{nW}{\frac{1}{2}\rho_o V_e^2 S}}$$ **Equation 2.3.10**

<u>In terms of Mach number</u>

$$M = \frac{V_t}{a}$$

$$a^2 = \frac{\gamma P}{\rho_a}$$

$$V_t^2 = M^2 a^2 = M^2 \frac{\gamma P}{\rho_a}$$

$$q = \frac{1}{2}\rho_a M^2 \frac{\gamma P}{\rho_a} = \frac{1}{2}\gamma P M^2$$

or $$\underline{q = \frac{1}{2}\gamma P_o \delta M^2} = \underline{1481 \delta M^2}$$ **Equation 2.3.11**

where $\gamma = 1.4$ $P_c = 29.92$ "Hg = 14.7 lb/in^2

$$\boxed{C_L = \frac{nW}{1481 \delta M^2 S}}$$ **Equation 2.3.12**

2.3.4 **REYNOLDS NUMBER AND MACH NUMBER** EFFECTS

In section 2.3.2.2 it was seen that the aerodynamic coefficients vary with Reynolds number and Mach number, therefore, a brief discussion of these effects is in order. While both effects may occur at the same time in actual practice we shall discuss each separately for the sake of clarity.

2.3.4.1 REYNOLDS NUMBER EFFECTS

Figure 2.3.9 shows that the effects of Reynolds number become significant at high lift coefficients and that higher maximum lift coefficients can be attained by increasing the Reynolds number. In section 1.3 it was seen that the Reynolds number determines whether a flow will be laminar or turbulent and that the pressure gradient is the prime factor responsible for separation.

Further, it was seen that turbulent boundary layers resist separation better than laminar but create more skin friction. Separation on the other hand is equally undesirable because it increases the pressure drag. Therefore, some compromise must be reached between laminar and turbulent boundary layers on an airfoil. The boundary layer on a typical airfoil is laminar over the forward portion and becomes turbulent and eventually separates toward the trailing edge. Let us consider the flow over an airfoil at a constant Reynolds number and varying angle of attack. As the angle of attack increases the local velocity and hence the local Reynolds number increases over the upper surface. The increase in velocity decreases the pressure on the upper surface causing a larger adverse pressure gradient on the aft portion. The increased local Reynolds number and pressure gradient causes transition and separation to occur farther forward on the wing. This forward movement proceeds slowly as the angle of attack is increased through moderate angles (less than 8 to 10 degrees), but increases rapidly at higher angles. Thus, for any given Reynolds number, drag increases slowly at low angles of attack and rapidly at the higher angles.

Now let us consider what happens to the lift and drag at moderate to high angles of attack when the free stream Reynolds number is increased. An increase in the free stream Reynolds number represents an increase in the energy of the flow. Because of this additional energy the boundary layer becomes turbulent farther forward on the surface and is able to remain attached longer, separating nearer the trailing edge. Because there is less separation the airfoil has lower drag and a higher $C_{L_{max}}$.

Since Reynolds number effects are most apparent at high lift coefficients, we would expect it to have a great effect on the stalling speed of an aircraft. We can best evaluate the effects on stalling speed if Reynold's number and $C_{L_{max}}$ are defined in terms of equivalent airspeed

$$R_e = \frac{\rho V_t l}{\mu} = \frac{\rho \frac{V_e l}{\sqrt{\sigma}}}{\mu} = \frac{V_e \sqrt{\rho}\, l}{\sqrt{\rho_o}\, \mu}$$

The coefficient of viscosity, μ , varies little with pressure and temperature (negligable within $\pm$ 70°C).

Considering viscosity as constant along with ρ_o and l we see that for the same V_e Reynolds number decreases with altitude.

The lift equation in terms of V_e is

$$C_{L_{max}} = \frac{nW}{\frac{1}{2} \rho_o V_e^2 S}$$

or

$$V_e = \sqrt{\frac{2\, nW}{\rho_o\, C_{L_{max}}\, S}} \qquad \textbf{Equation 2.3.13}$$

From this we see that as Reynolds number decreases, causing a decreased $C_{L_{max}}$, the stall speed increases. Note: Since equivalent airspeed and calibrated airspeed are approximately the same this trend can be noted in flight from cockpit instruments on stalls performed at low and high altitude.

2.3.4.2 MACH EFFECTS

Mach effects result because air is a compressible fluid. While the compressiblility of air is negligable at low speed it becomes very important at speeds approaching Mach 1.0.

As the velocity of the air increases over an airfoil at low speed there is virtually no change in the density; however, at high speed the same change in velocity causes a large change in the density. This effect can be seen from the following equation

$$\frac{d\rho}{\rho} = - M^2 \frac{dV}{V}$$

At low speed, say M = .2, a 10 percent increase in velocity over the wing causes a .4 percent decrease in the density. However, at high speed (M = .8) a 10 percent increase in velocity causes a 6.4 percent decrease in the density. Thus, the compressibility of the air becomes significant as the Mach number increases. Since Mach number is the controlling factor in determining the amount of compressibility that occurs, the terms compressibility effects and Mach effects are used interchangably.

The effect of compressibility on the lift coefficient may be seen from an equation proposed by Glauert for thin airfoils.

$$C_L = \frac{C_{L\rho}}{\sqrt{1 - M^2}}$$ where $C_{L\rho}$ is the incompressible C_L **Equation 2.3.14**

In the linear range this may be written

$$C_L = \frac{(\alpha)\, a_\rho}{\sqrt{1 - M^2}}$$

where a_ρ is the slope of the $C_L - \alpha$ curve for incompressible flow (Fig. 2.3.10) and α is the angle of attack referred to the zero lift line rather than the chord.

Differentiating this equation with respect to α gives

$$\frac{dC_L}{d\alpha} = \frac{a_\rho}{\sqrt{1 - M^2}}$$

From this we see that the slope of the lift curve increases with increased Mach number. From the previous equation it is seen that all of the curves pass through the angle attack for zero lift as shown in Figure 2.3.13.

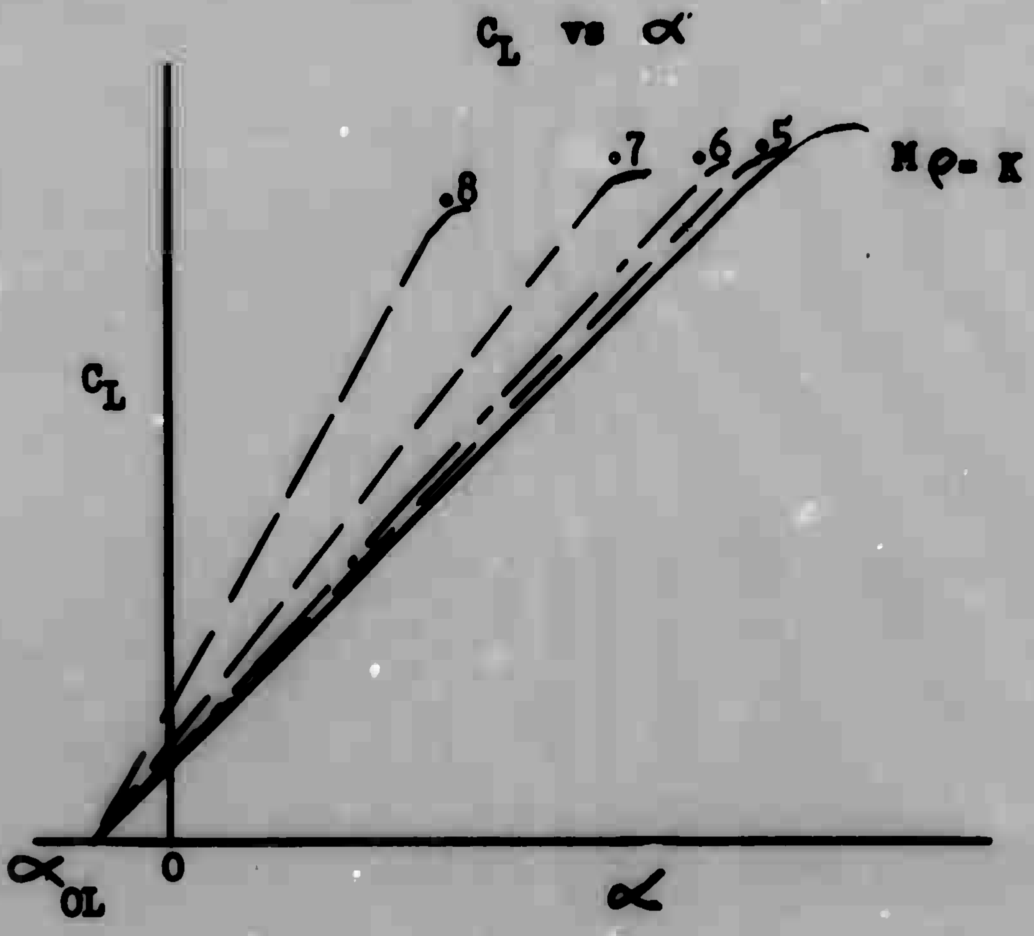


Figure 2.3.13

This figure shows that the lift coefficient increases for a given angle of attack as the Mach number is increased. This effect does not continue indefinitely because the equation on which it was based is not good above the critical Mach number * where shock waves begin to form on the surface, in fact, in the supersonic range the trend is reversed.

* Critical Mach number is the aircraft speed at which the local flow over the surface just becomes sonic.

The drag coefficient is likewise increased in high speed flight; however, its effects are restricted to speeds greater than critical Mach number, M_{cr}. A typical plot of the drag coefficient versus Mach number at constant lift coefficient is shown in Figure 2.3.14.

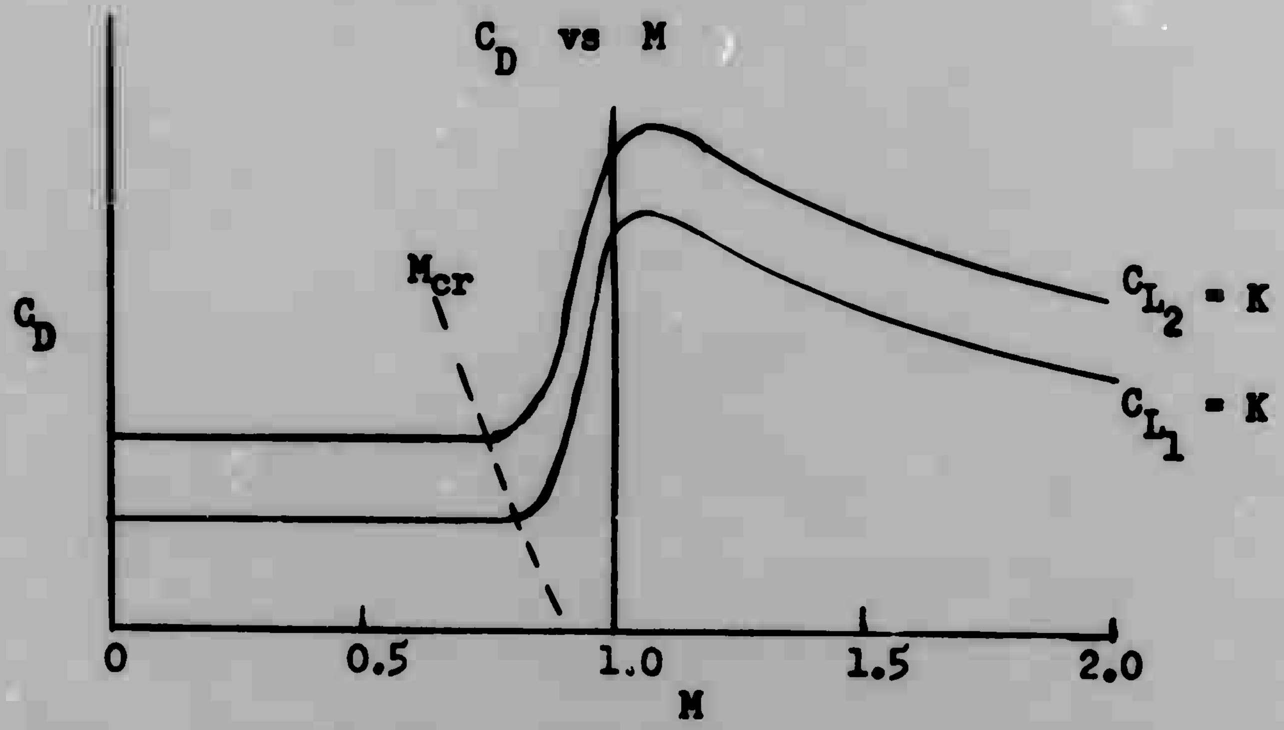


Figure 2.3.14

This shows no increase in C_D until after the critical Mach number is passed, then a sudden increase in drag divergence is experienced. This is caused by the formation of shocks on the aircraft. Detailed discussion of these effects will be delayed until Section 5.

2.3.5 HIGH LIFT DEVICES

Maneuvering and slow speed flight and landing require high lift coefficients. This is apparent from the following equation.

$$C_L = \frac{nW}{(\frac{1}{2}\rho V^2 S)}$$

If the load factor, n, is to be high or the velocity, V, is to be low, a large lift coefficient is required. In order to provide lift coefficient greater than the maximum lift coefficient of a given airfoil it is necessary

to resort to some sort of special hardware known as high lift devices. Devices of this type are slots, slats, flaps (both leading and trailing edge) and boundary layer control (BLC). Some of these devices are characteristically low speed devices such as flaps while others are suitable for both high and low speed applications such as slots, slats and BLC. Numerous variations on these devices have been proposed and used on operational aircraft. It is beyond the scope of this writing to consider in detail the various arrangements which can be used. We will merely define each type and briefly discuss their effects on the aircraft.

Increasing lift of an airfoil can be accomplished by any one or combination of three methods. The first is to increase the wing area. The second is to increase the camber of the wing. The third is to delay separation through some means of boundary layer control.

A slot is basically a boundary layer control device since it takes high energy air from the lower surface of the wing and ducts it through the wing into the low energy boundary layer of the upper surface, see Figure 2.3.15a. In doing so it delays separation and allows higher lift coefficients to be developed. A slot is relatively ineffective at low angles of attack but becomes very effective at high angles thus improving the high lift characteristics without significantly compromising the low lift characteristics.

A slat operates on the same principle as a slot except that it is located near the leading edge and represents an additional airfoil in front of the basic airfoil. Its function is to direct the flow over the leading edge of the airfoil. See Figure 2.3.15b. A slat may also be of the movable type which

remains retracted at high speed and extends at low speeds and high angles of attack. Figure 2.3.15c. This increases the wing area slightly as well as increasing the flow over the upper surface of the wing.

Slats and slots are an aerodynamic means of effecting the boundary layer control. Boundary layer control may also be accomplished by artificial means such as blowing air provided by a compressor over the wing or sucking the low energy boundary layer through the surface of the wing. Figure 2.3.15d, e.

Flaps provide increased lift by increasing the camber of the wing. Several arrangements are commonly used. The simple flap is a simple hinged leading or trailing edge (Figure 2.3.15f). A split flap is so named because the trailing edge is split with only the lower half being hinged. Figure 2.3.15g. This causes high drag when lowered. A Fowler flap increases the lift by increasing the wing area as well as by increasing the camber. In the retracted position it looks just like the split flap; however, when it is extended it lowers and translates aft, thus increasing the wing area and camber. Figure 2.3.15h. Because of the unique movement of this type of flap the mechanism is quite heavy and complicated, and therefore may not be practical for all applications.

A Fowler flap is capable of producing a 100% or more increase in maximum lift coefficient. A split flap may produce up to 80 or 90% increase while slots and slats produce up to 60% increase.

Slots, slats and flaps effect the aerodynamic characteristics of an airfoil similiar to that shown in Figure 2.3.16.

Figure 2.3.15a Slot

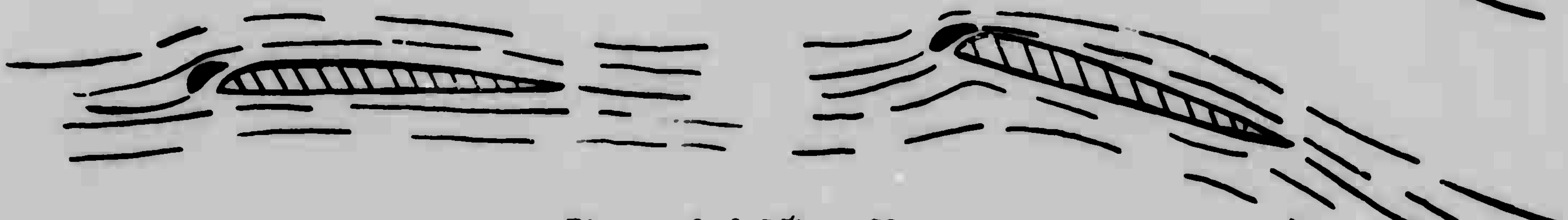


Figure 2.3.15b Slat

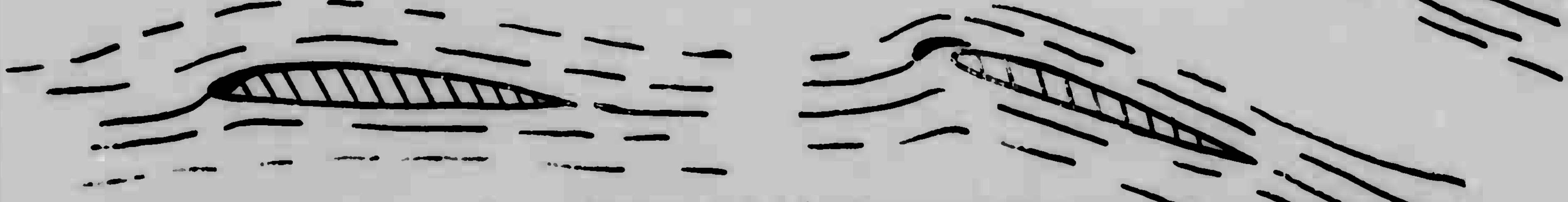


Figure 2.3.15c Movable Slat

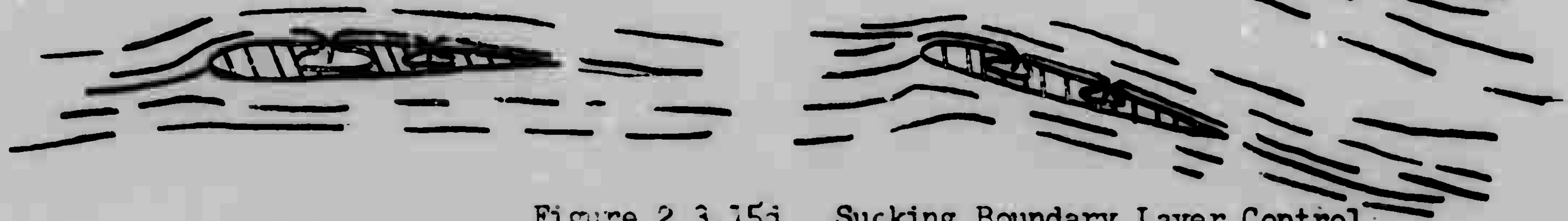


Figure 2.3.15i Sucking Boundary Layer Control

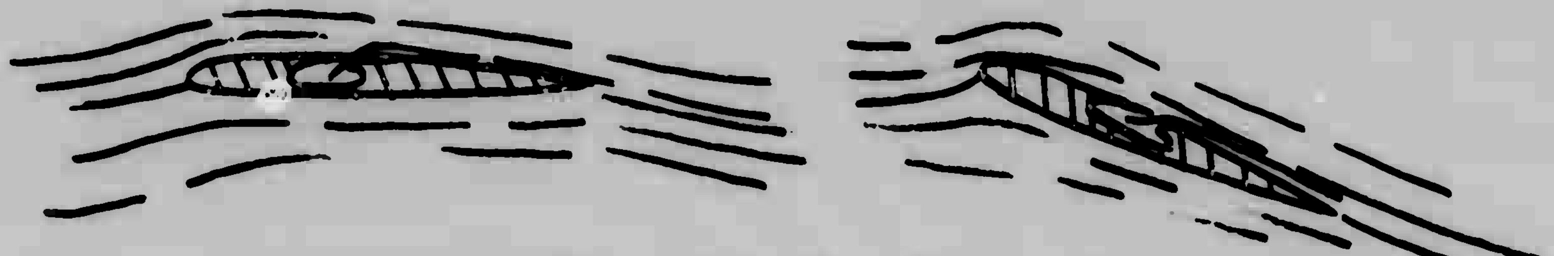


Figure 2.3.15e Blowing Boundary Layer Control

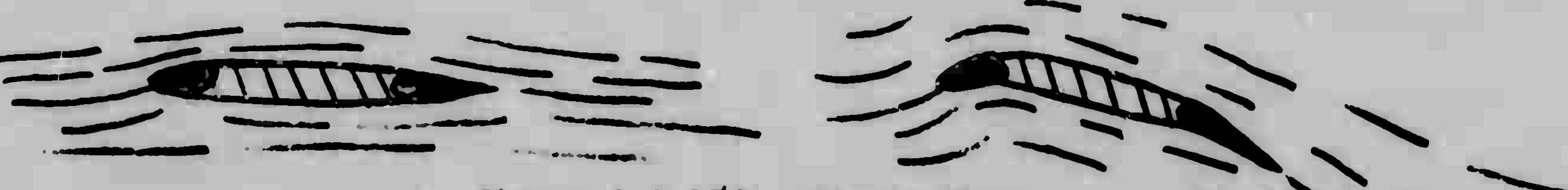


Figure 2.3.15f Simple Flap

Figure 2.3.15g Split Flap

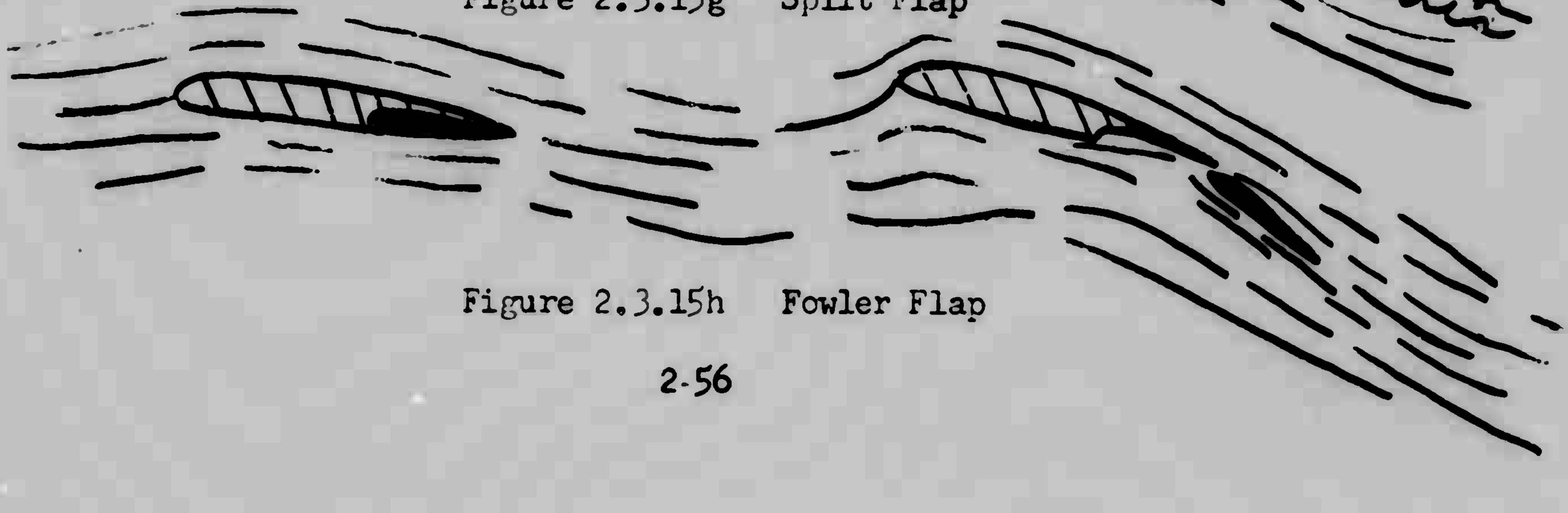


Figure 2.3.15h Fowler Flap

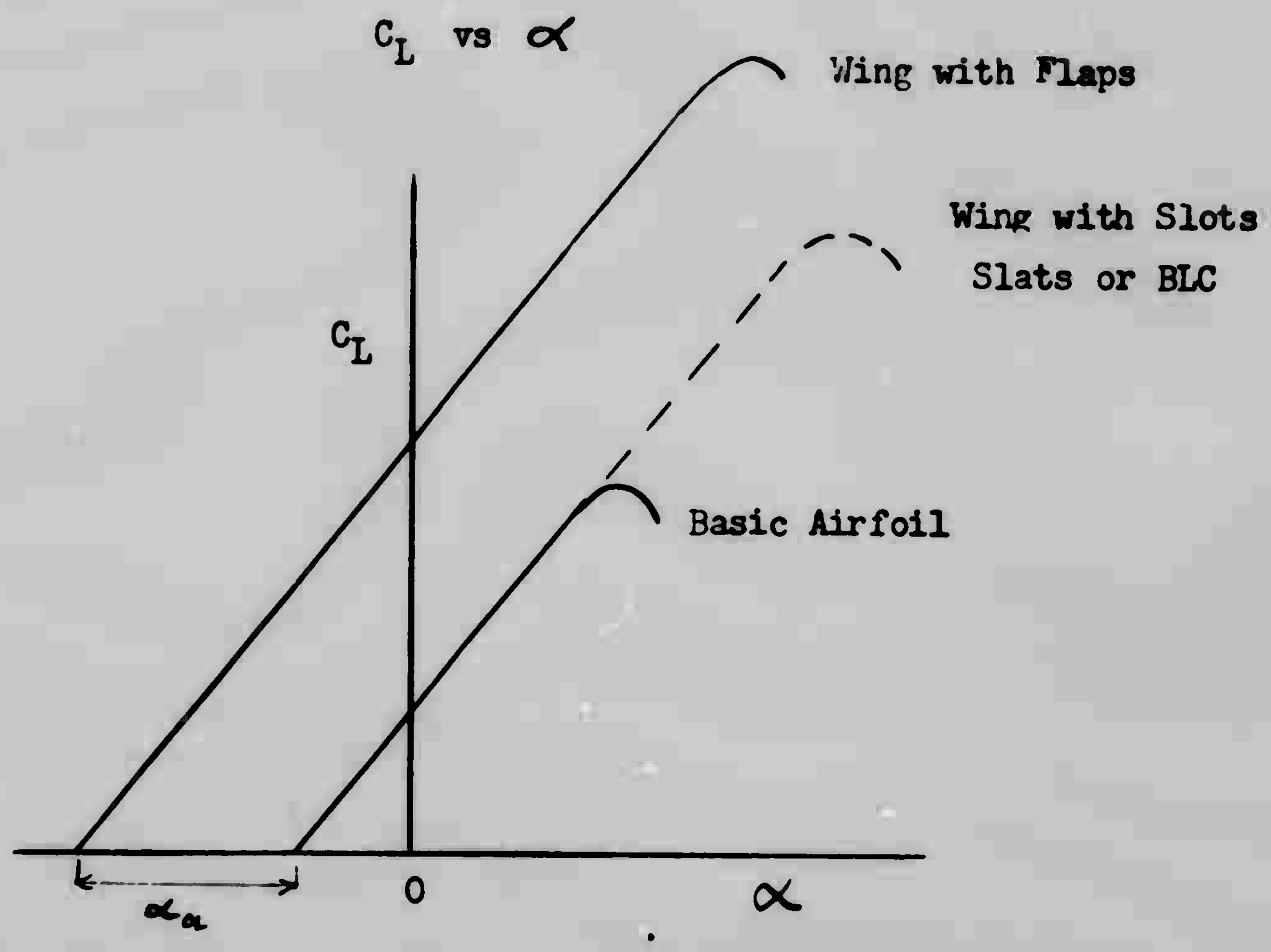


Figure 2.3.16

Basically slots, slats and BLC do not effect the C_L - α relationship significantly at low angles of attack, they only serve to extend the curve to higher lift coefficients. Flaps on the other hand increase the camber of the airfoil, thus changing the angle of zero lift and pitching moment of the aircraft. The primary effect of flaps is to shift the C_L - α curve parallel and to the left in proportion to the camber with a sizable increase in maximum lift coefficient.

2.3.6 AERODYNAMIC DRAG

The drag of an aircraft is the total resistance to its passage through the air and acts opposite to the direction of flight. In section 2.3.2 the equation for drag was given as

$$D = C_D \, q \, S = C_D \, \tfrac{1}{2} \rho_a V_t^{\,2} \, S \qquad \text{Equation 2.3.15}$$

As done in section 2.3.3.4 for C_L, C_D can be expressed in terms of V_e and M

$$C_D = \frac{D}{\tfrac{1}{2} \rho_o V_e^{\,2} S} \qquad \text{Equation 2.3.16}$$

or

$$C_D = \frac{1}{1481 \; M^2 S} \cdot \frac{D}{\delta} \qquad \text{Equation 2.3.17}$$

2.3.6.1 TYPES OF DRAG

The total drag of an aircraft is made up of many component drags; that is, it is the sum of the drags caused by the wing fuselage, empanage, interference, cowl flaps, etc. Drag may be broken down in many ways. A designer for instance is interested in the drag of the individual components of the aircraft while a flight test engineer is interested in the total drag and its major components. The basic types of drag are pressure drag, skin friction drag, induced drag and compressibility drag.

Pressure Drag arises because of the pressure distribution over a body. High pressures on the forward portion and low pressure on the aft section of a body cause drag.

Skin Friction is the drag caused by the viscosity of the air flowing over the body and is proportional to the shear stress in the fluid. (see section 1.3 and 2.4.1)

Profile Drag is the pure resistance type of drag which does not contribute to lift. It is the sum of the pressure drag and skin friction drag.

Induced Drag is the drag due to lift and is caused by the aft component of the resultant aerodynamic force.

Compressibility Drag or Wave Drag is the drag which results when the flow over the surfaces of the aircraft exceeds Mach 1.0. Supersonic flow over wing and fuselage surfaces result in the formation of shocks which cause a sizable increase in drag.

In assessing component drags the designer evaluates each of the above types of drag for each component and sums them to obtain the total drag. In doing this he must add an additional component called interference drag.

Interference Drag results because the normal flow over the wing or other components does not merge smoothly with the flow over the fuselage. As a result a region of turbulence and otherwise disturbed flow exists which causes additional drag.

As a result of the component system of analysing drag many types of drag have been defined in order to distinguish between the various drags of each component. In some cases, the same name is attached to the drag of different components. Therefore, when using component drag data one must be certain of the definition of the various drag terms.

For flight test purposes it is not necessary to make a detailed breakdown of the total drag. It is conventional to make the following breakdown. All drag which is not a function of lift is called parasite drag * and all drag which is a function of lift is called induced drag. If the speed is greater than the critical Mach number (i.e., where sonic flow first occurs over the aircraft)

* Parasite drag includes, profile and interference drag.

an additional drag is added to account for the losses due to shock waves which are caused by the compressibility of the air. The total drag is then written as:

$$D_{total} = D_p + D_i + D_M \qquad \text{Equation 2.3.18}$$

where

D_p is the parasite drag = $C_{D_p}\, q\, S$

D_i is the induced drag = $C_{D_i}\, q\, S$

D_M is the wave drag = $C_{D_M}\, q\, S$

since

$$D = C_D\, q\, S$$

$$D_{tot} = C_{D_{tot}}\, q\, S = C_{D_p}\, q\, S + C_{D_i}\, q\, S + C_{D_M}\, q\, S$$

or

$$C_{D_{tot}} = C_{D_p} + C_{D_i} + C_{D_M} \qquad \text{Equation 2.3.19}$$

2.3.6.2 DRAG COEFFICIENTS

<u>Induced Drag</u>

Induced drag was defined in the previous section as the drag due to lift. By circulation and down wash theory it can be shown that for an elliptical lift distribution the induced drag coefficient is given by

$$C_{D_i} = \frac{C_L^2}{\pi \mathit{AR}} \qquad \text{(elliptical lift distribution)}$$

In order to account for deviations from the theoretical elliptical lift distribution the efficiency factor, e, is included in the denominator.

$$C_{D_i} = \frac{C_L^2}{\pi \text{AR} e} \qquad \text{(for any lift distribution)} \qquad \text{Equation 2.3.20}$$

The efficiency factor is constant at subsonic speeds but tends to decrease at speeds in excess of the critical Mach number. For the elliptical lift distribution the efficiency factor is 1.0. For all other lift distributions it is less than 1.0, thus giving a higher induced drag for non-elliptical lift distributions. It is apparent then that the minimum induced drag is obtained from an elliptical lift distribution.

While an efficiency factor of one is not obtained in practice, it may be approached by using an airfoil whose planform is an ellipse by changing the airfoil section along the span or by warping the wing to different angles of attack along the span. Any combination of the above may be used but it should be noted that if the warping technique is used an elliptical lift distribution will generally occur only at one speed. The other methods approach the optimum throughout the subsonic speed range.

With the new expression for C_{D_i} the total drag coefficient is

$$C_D = C_{D_p} + \frac{C_L^2}{\pi \text{AR} e} \qquad \text{for subsonic flight} \qquad \text{Equation 2.3.21}$$

$$C_D = C_{D_p} + \frac{C_L^2}{\pi \text{AR} e} + C_{D_M} \qquad \text{for transonic and supersonic flight} \qquad \text{Equation 2.3.22}$$

Another form of the induced drag which is sometimes used is given by the following:

$$D_i = C_{D_i} q S = \frac{C_L^2}{\pi \text{AR} e} q S$$

where

$$C_L = \frac{L}{qS}$$

$$D_i = \left(\frac{L}{qS}\right)^2 \cdot \frac{qS}{\pi \mathcal{R} e} = \frac{L^2}{qS} \cdot \frac{1}{\pi \mathcal{R} e}$$

but

$$\mathcal{R} = b^2/S$$

$$D_i = \frac{L^2}{qS} \cdot \frac{S}{\pi b^2 e} = \frac{(L/b)^2}{\pi q e} \qquad \text{Equation 2.3.23}$$

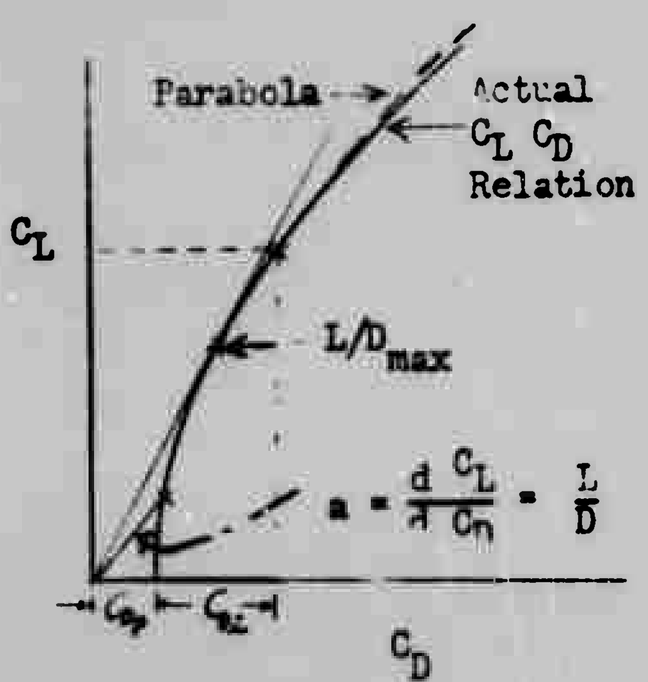
Note that L/b is the span wise loading and that it is not necessary to know the aspect ratio or the induced drag coefficient.

Parasite Drag

The parasite drag of an aircraft is the total drag other than that caused by lift and compressibility effects. It represents the minimum drag of an aircraft, and is a constant. The minimum drag is that which exists at zero lift and can only be obtained in flight by flying at "zero g".

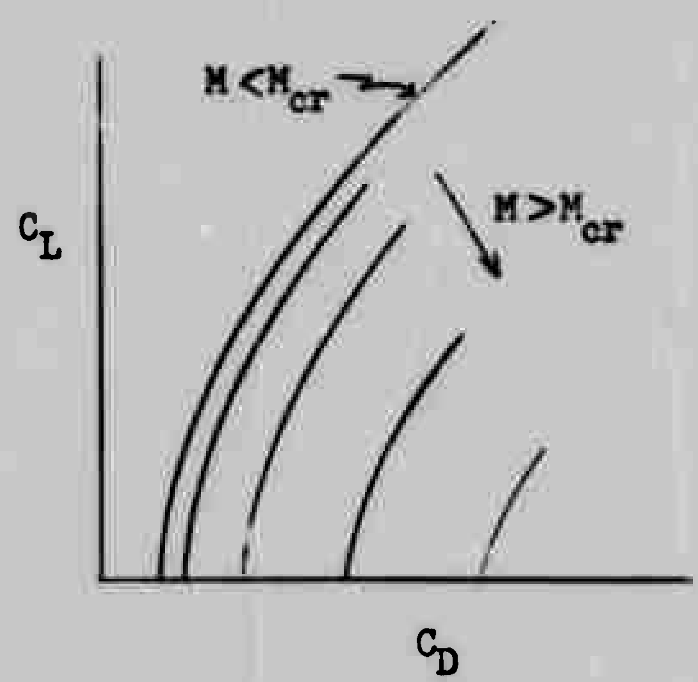
Drag Polar

The relationship between lift and drag is normally shown on a plot of C_L versus C_D which is known as the drag polar, Figure 2.3.16.



Subsonic Polar

Figure 2.3.16a



Transonic Polar

Figure 2.3.16b

Since in the subsonic region

$$C_D = C_{D_p} + \frac{C_L^2}{\pi AR e} \qquad \text{Equation 2.3.24}$$

and C_{D_p} and $1/\pi AR e$ are constant, the curve is a parabola. This theoretical result holds in practice at all but the extreme angles of attack (high C_L). The minimum parasite drag coefficient is seen at $C_L = 0$ and C_{D_i} is seen to vary with C_L as shown.

When the flight speed is greater than the critical Mach number an additional increment is added as discussed previously. The basic low speed polar is unchanged but the curves shift to the right as shown in Figure 2.3.16b. The slopes of the higher Mach number curve will change depending on the variation of C_{D_M} with C_L.

Since the relationship between C_L and C_D closely approximates a parabola, plotting C_L^2 versus C_D is a straight line as shown in Figure 2.3.17

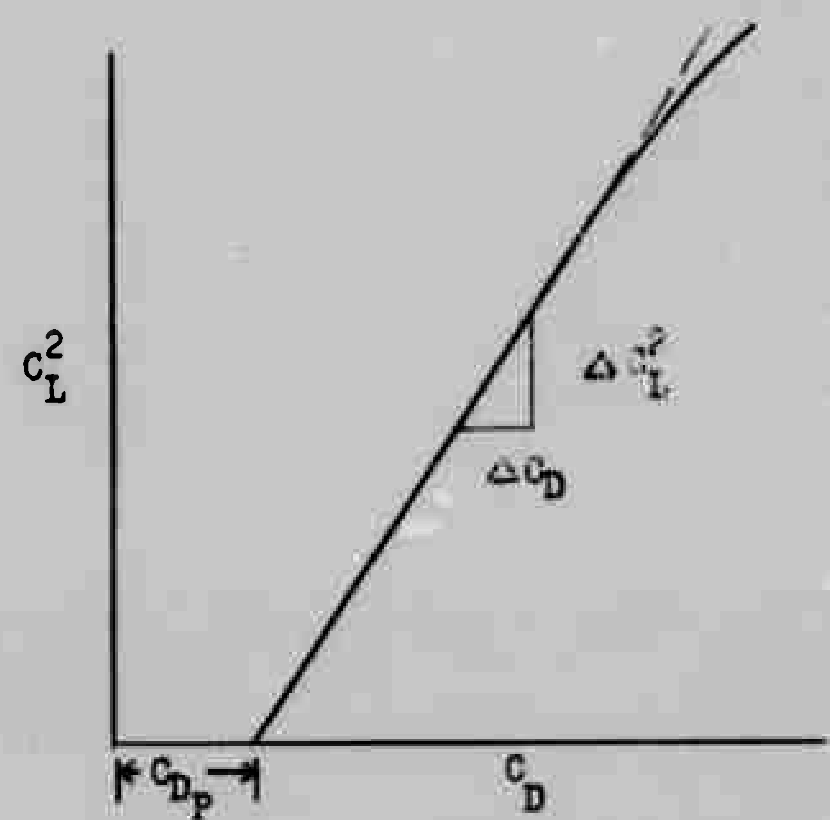


Figure 2.3.17

The general equation of a straight line is

$$y = m x - b$$

where m is the slope $= dy/dx$

and b is a constant

By analogy to the drag equation

$$x = C_L^2 \; ; \; y = C_D$$

$$m = \frac{1}{\left(\frac{dC_L^2}{dC_D}\right)} = \frac{1}{\pi \mathit{AR} e} \; ; \; b = C_{D_p}$$

From this we see that the slope

$$\frac{dC_L^2}{dC_D} = \pi \mathit{AR} e$$

and the efficiency factor, e, can be determined for any aircraft by measuring the slope and applying the above equation.

Equation 2.3.20 indicates that induced drag increases with decreases in both efficiency factor and aspect ratio. These effects are shown in Figure 2.3.8a and b.

C_L vs C_D

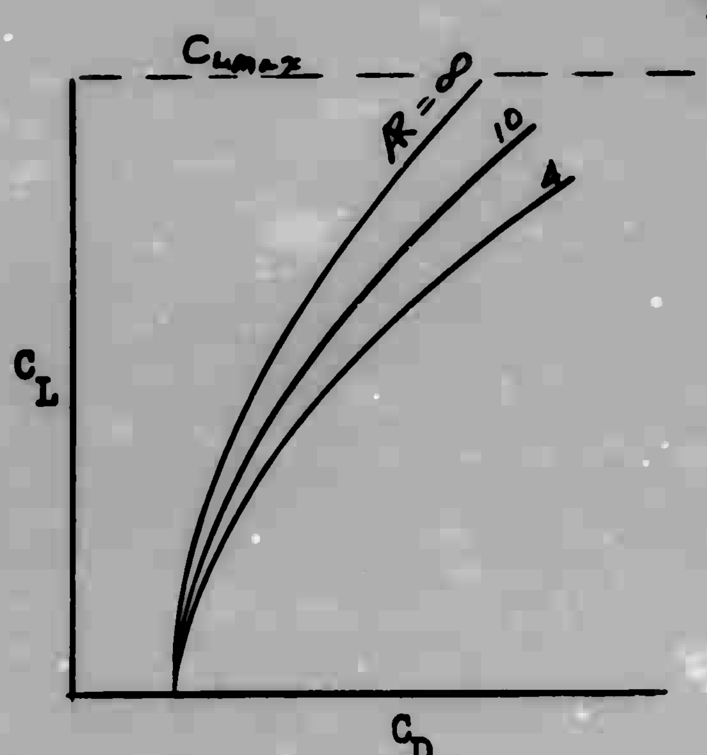


Figure 2.3.18a
Effects of Aspect
Ratio at Constant e

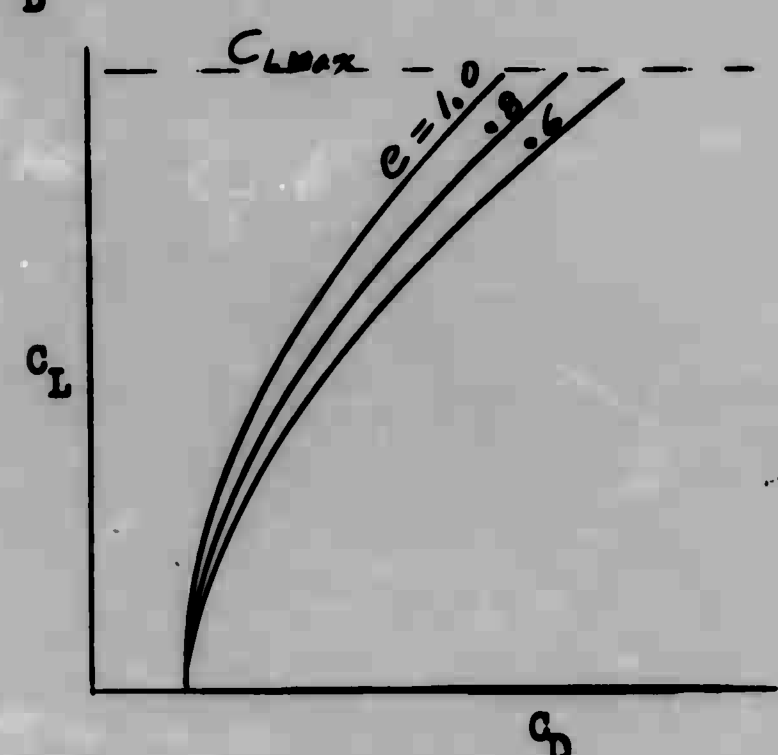


Figure 2.3.18b
Effect of Airplane
Efficiency at Constant AR

Note that $C_{L_{max}}$ is relatively unaffected for changing efficiency factors but is affected by changes in aspect ratio.

The lift-drag ratio, $\frac{L}{D}$, is of importance to flight testing since it directly determines the glide and cruise characteristics and also effects other areas of performance. The lift-drag ratio can be determined directly from the drag polar since

$$\frac{L}{D} = \frac{C_L\, q\, S}{C_D\, q\, S} = \frac{C_L}{C_D}$$

Thus, the slope of the line connecting the origin with any point on the polar is the lift-drag ratio, Figure 2.3.16a. The maximum L/D is the point of tangency of this line from the origin. It will be shown later that the best glide angle and cruise (for reciprocating aircraft) is obtained at L/D max.

<u>Drag as Related to Performance</u>

Important for performance analysis is the variation of the drag coefficient with speed. The induced drag coefficient decreases with increased speed since the lift coefficient required for level flight decreases. This is seen from the following equation

$$L = C_L \tfrac{1}{2} \rho_a V_t^2 S = \text{constant for level flight}$$

Therefore, as V increases C_L decreases inversely as the square of the velocity so that C_{D_i} given by

$$C_{D_i} = \frac{C_L^2}{\pi R e}$$

decreases inversely as the fourth power of the velocity. The induced drag,

however, decreases inversely as the square of the velocity. Figure 2.3.19a and b.

The parasite drag coefficient was seen to be constant. Thus, the parasite drag increases with the square of the velocity. Figure 2.3.19a and b.

The total drag and drag coefficient is shown in Figure 2.3.19b as the sum of the parasite plus induced drag. If the flight speed is greater than the critical Mach number of the aircraft, an additional increment must be added to include the wave drag. This is shown in Figure 2.3.19c and d.

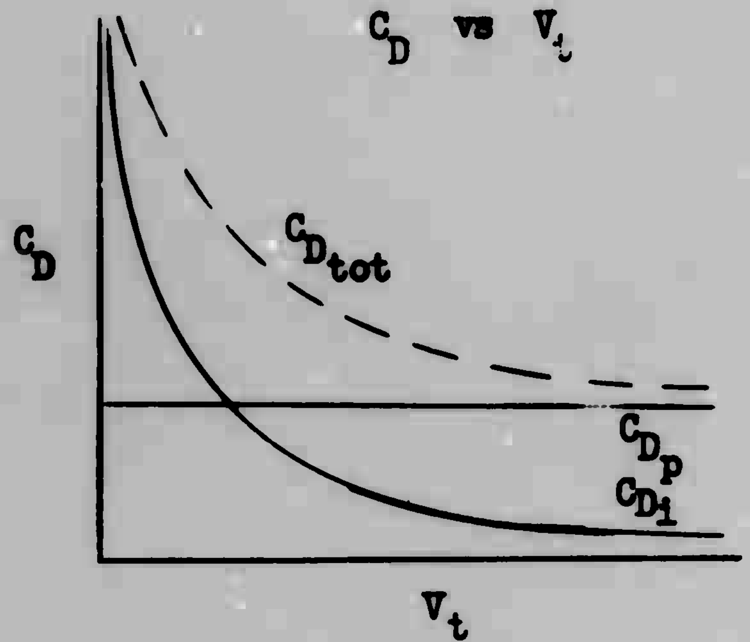


Figure 2.3.19a

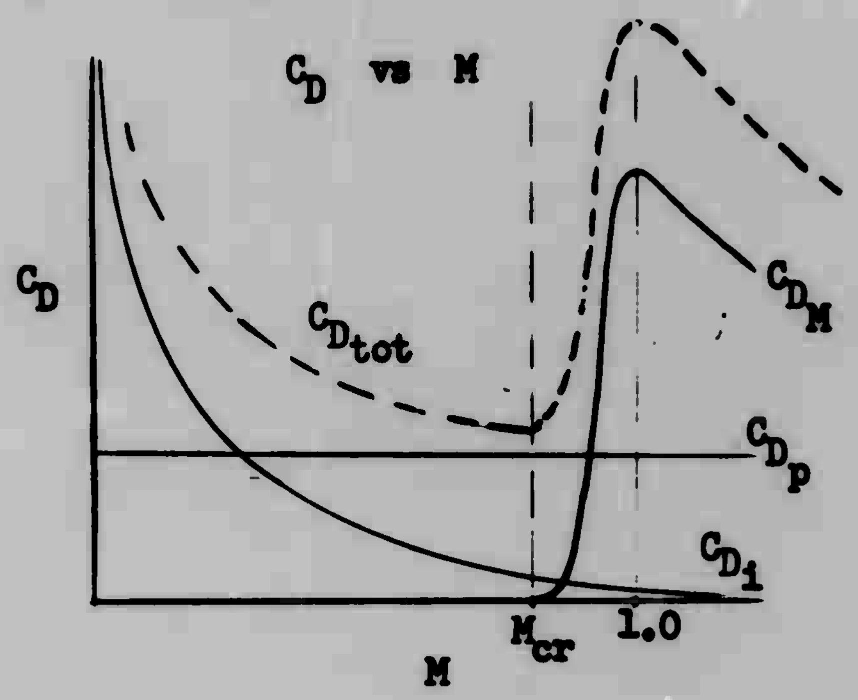


Figure 2.3.19c

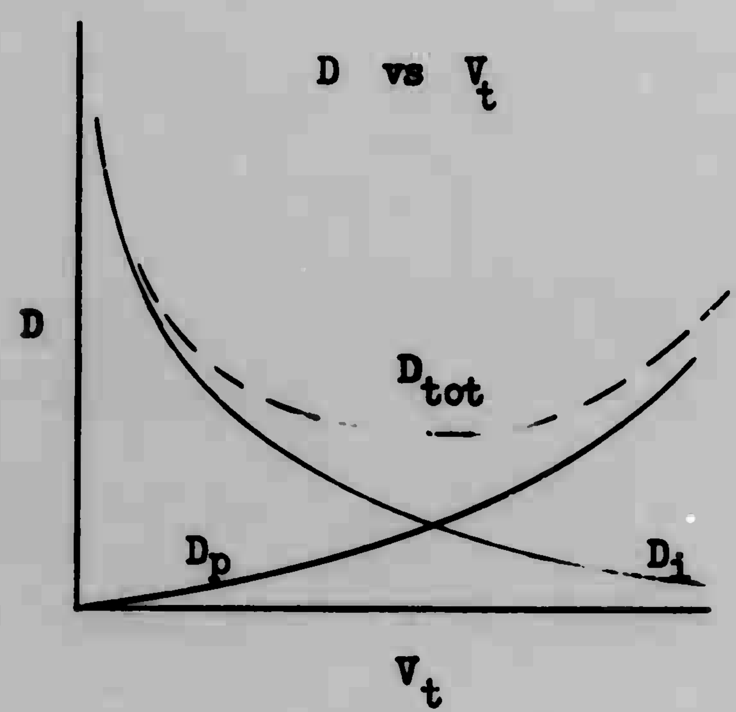


Figure 2.3.19b

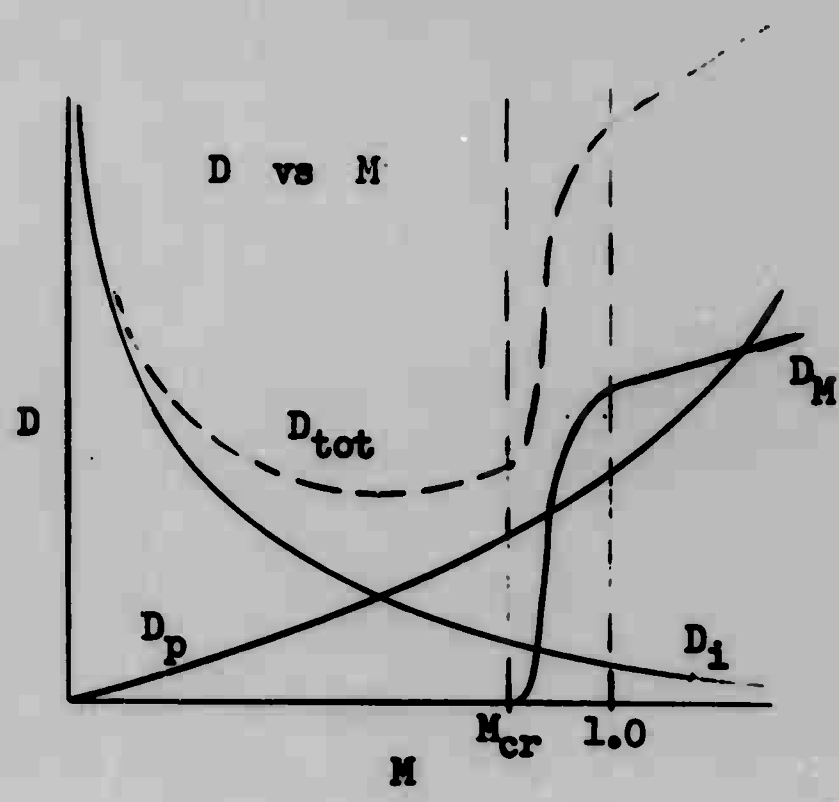


Figure 2.3.19d

Drag Coefficient as Related to Reference Area

Frequently the drag coefficient is related to some area other than the wing area. For instance it might be more realistic to relate drag to the projected frontal area of the aircraft rather than the wing area. Regardless of which area is chosen the total drag must be the same; therefore, the drag coefficients will change depending on the reference area used to obtain the coefficients. Since it is the convention to reference all aerodynamic coefficients to the wing area it may be necessary to convert data based on some other reference to drag coefficients based on wing area. This transformation is shown in the following derivation

$$D = C_{D_1} q S = C_{D_2} q A = \text{constant}$$

where: A is the other reference area

C_{D_1} is the drag coefficient referenced to wing area

C_{D_2} is the drag coefficient referenced to A

cancelling q from the middle two terms gives

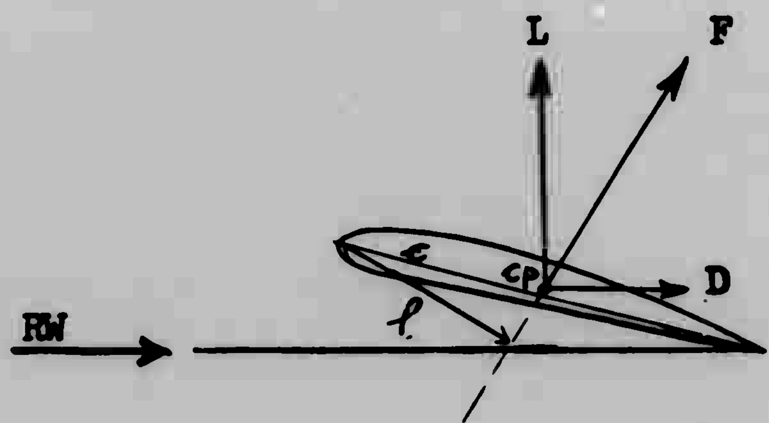
$$C_{D_1} S = C_{D_2} A \qquad \textbf{Equation 2.3.25}$$

or

$$C_{D_1} = C_{D_2} \frac{A}{S}$$

2.3.7 AERODYNAMIC MOMENTS

In addition to lift and drag, an airfoil is acted upon by an aerodynamic pitching moment which may be expressed in coefficient form similar to that of the lift and drag equations. Since a moment is defined as the force times the perpendicular distance to the reference point (Figure 2.3.20), it is necessary to include a unit of length in the coefficient equation, generally the chord length, c.

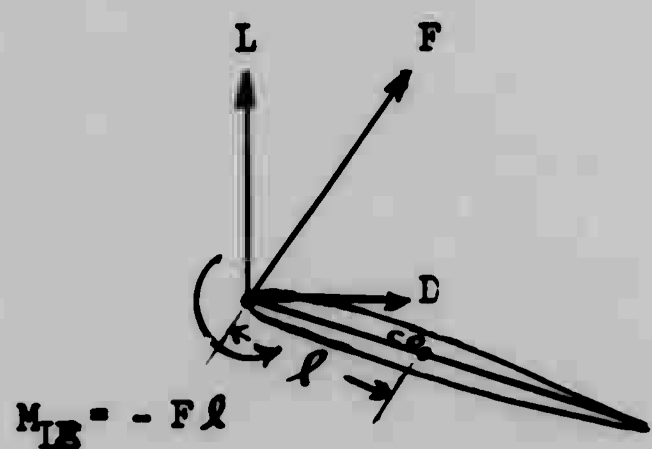


$M_{LE} = F \times l$ and $M_{cp} = 0$

$M = C_m \, q \, c \, S$ Equation 2.3.26

where C_m is the pitching moment coefficient

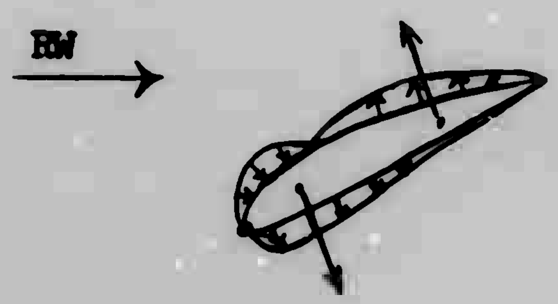
Figure 2.3.20a



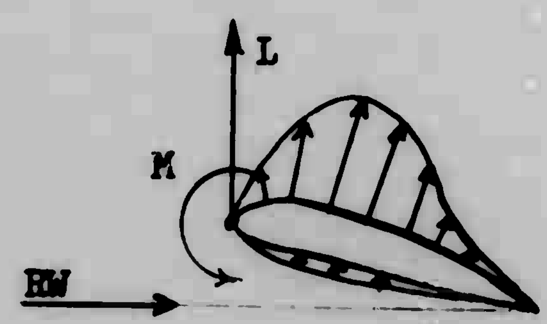
If the aerodynamic force acts at any point other than the center of pressure it must be represented by the aerodynamic force plus a moment as shown in Figure 2.3.20b.

Figure 2.3.20b

The pitching moment is caused by an asymetric pressure distribution over the surface of the airfoil and by the lift vector, Figure 2.3.21. If it tends to pitch the nose up it is positive and if it pitches the nose down it is negative.



Angle of Attack for Zero Lift
Figure 2.3.21a



Positive Angle of Attack
Figure 2.3.21b

Taking moments about the leading edge it is seen that a negative moment is created even when no lift is created.

Since the moment is caused by the pressure distribution over the airfoil, let us re-examine the variables which effect the pressure distribution. In section 2.3.2 it was seen that angle of attack, camber and thickness all affect the pressure distribution. The <u>effects of thickness</u> can be eliminated immediately since increasing the thickness increases the pressure distribution on the upper and lower surfaces proportionately so that there is no net contribution to the moment. The <u>angle of attack</u> affects the pitching moment directly. As the angle of attack increases the lift increases causing the moment to increase in proportion to the moment arm. For thin symmetric airfoils the theoretical location of the center of pressure is at the $\frac{1}{4}$ chord.

For cambered airfoils the c.p. is not constant but moves aft at the lower angles of attack. The effects of camber are apparent in Figure 2.3.21a. The airfoil is at zero lift, that is, the sum of the pressure forces perpendicular to the relative wind are zero; however, the pressure distribution is such that the sum of the moments is not zero. This resultant moment may be represented by a couple which means that the moment is independent of the point about which the moments are summed. From this it is seen that moment coefficient due to camber is independent of the reference location on the airfoil as well as the angle of attack. A symetrical airfoil has no camber and, therefore, no pitching moment at zero lift. The only moments created are those due to angle of attack. Since the above effects are additive the total moment is the sum of the moments due to camber and due to lift. If the moments are summed about a point on the chord line such as the $\frac{1}{4}$ chord for a symetrical airfoil the moment due to lift is zero and the resulting pitching moment is only due to camber and is constant. This then corresponds to the definition of the aerodynamic center given in section 2.3.1.

Some useful insight into the properties of moments as applied to an airfoil is obtained from an expansion of the basic definition of the moment about some arbitrary point on an airfoil, n. From Figure 2.3.22

$$M = L\,c\,(n - cp)\cos\alpha + D\,c\,(n - cp)\sin\alpha$$

assuming small angles of attack and relatively small drag

$$D \ll L$$

$$\cos\alpha \approx 1.0 \qquad \sin\alpha \approx \alpha \approx 0$$

the above equation becomes

$$M_n = L\,(n - cp)c \quad \text{where n and cp are in \% chord}$$

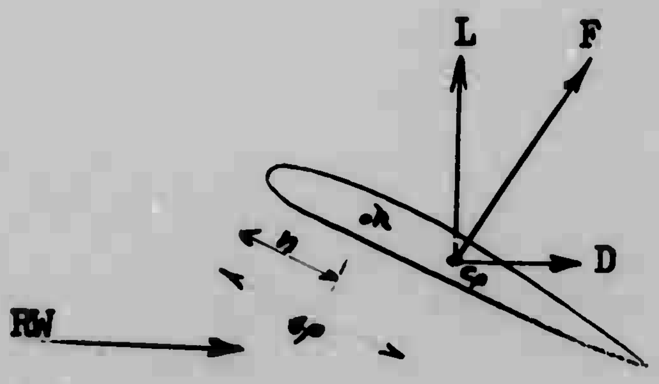


Figure 2.3.22

to get this in coefficient form we divide by q c S obtaining

$$C_{m_n} = C_L\,(n - cp) \qquad \text{Equation 2.3.27a}$$

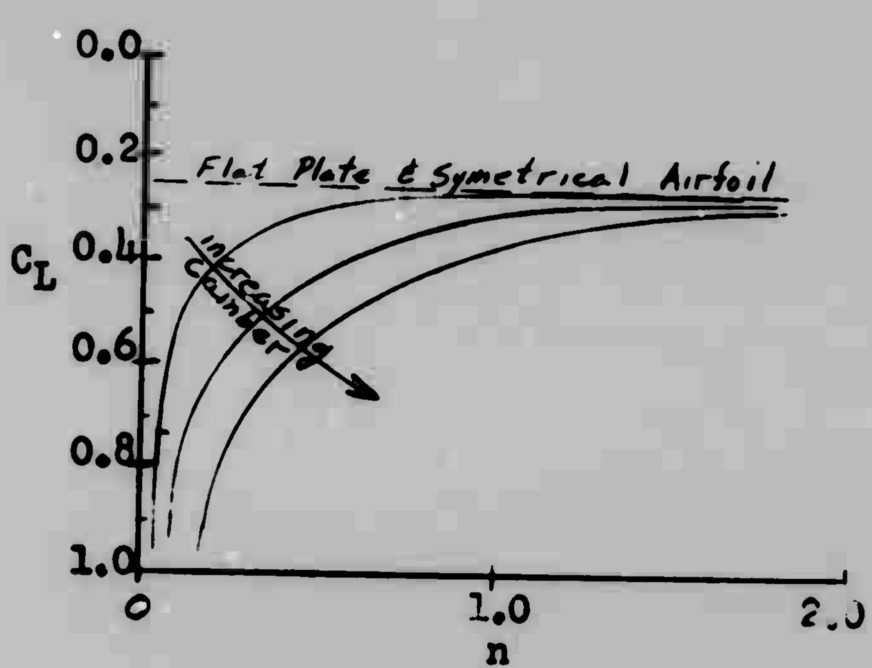
or

$$cp = n - \frac{C_{m_n}}{C_L} \qquad \text{Equation 2.3.27b}$$

Now if, n, is taken as the aerodynamic center, $C_{m_{ac}}$ and ac are constant then the equation is

$$cp = ac - \frac{C_{m_{ac}}}{C_L} \qquad \text{Equation 2.3.27c}$$

From this it is seen that unless $C_{m_{ac}}$ is zero cp becomes infinite at zero lift. Since $C_{m_{ac}}$ is negative for positively cambered airfoils cp is always positive as C_L goes to zero. The cp travel trends are shown in Figure 2.3.22 for varying camber. The cp going to infinity at zero C_L is a result of trying to represent a couple by a single force. To avoid this paradox it is better to admit the moment due to camber as a constant about the ac and consider all lift and drag forces to act at the ac.



Effect of Camber and C_L on cp travel
Figure 2.3.22

An expression for the moment coefficient about any point is obtained when equations 2.3.27b and 2.3.27c are equated

$$n - \frac{C_{m_n}}{C_L} = ac - \frac{C_{m_{ac}}}{C_L}$$

$$\frac{C_{m_n}}{C_L} = \frac{C_{m_{ac}}}{C_L} + (n - ac)$$

or

$$C_{m_n} = C_{m_{ac}} + (n - ac)\, C_L \qquad \text{Equation 2.3.28}$$

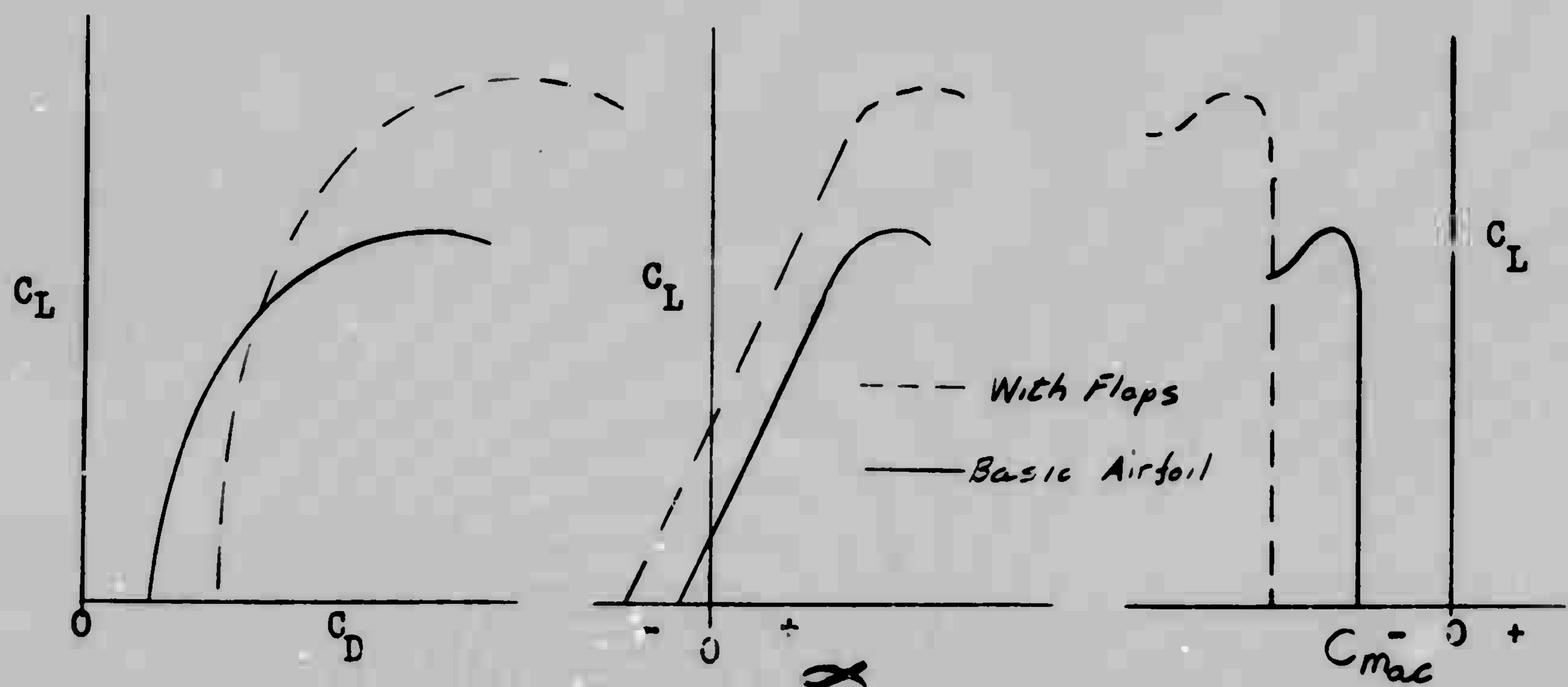
From this it is seen that the total moment coefficient is the sum of the constant $C_{m_{ac}}$ plus the moment coefficient caused by lift. Also note that at zero lift the moment coefficient at any point along the chord equals $C_{m_{ac}}$.

Mean Aerodynamic Chord

In the previous discussion we have talked about the chord as though it were constant. Most wings, however, are tapered, swept, elliptical or are otherwise irregular. When wings of this sort are to be dealt with it is necessary to define some mean chord which would be equivalent to a straight untapered wing. This chord is "mean aerodynamic chord" and is defined as the theoretical wing which has force vectors equivalent to the actual wing throughout the speed range. The means of finding the true mean aerodynamic chord is very complicated and involved; however, it has been found in practice that it corresponds very closely to the mean geometric chord. For almost all calculations use of the mean geometric chord is sufficiently accurate and is the one which is generally used.

Effect of Camber, Reflex and Flaps

We have seen earlier that camber causes an increase in the negative pitching moment and that increasing the camber increases the pitching moment coefficient about the ac or any point on the wing other than the cp. From this it is evident that lowering flaps increases the camber and therefore makes the pitching moment coefficient more negative. A typical example of the effect of lowering flaps is shown in Figure 2.3.23.



Effect of Flaps on Aeordynamic Coefficients

Figure 2.3.23

Flaps increase the maximum lift coefficient but in doing so they negatively increase the moment coefficient. Different types of flaps affect the coefficient to varying degrees. A table showing the affects of different types of flaps are shown in Figure 2.3.24.

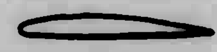
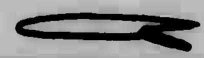
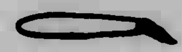
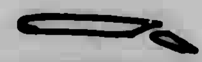
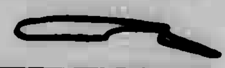
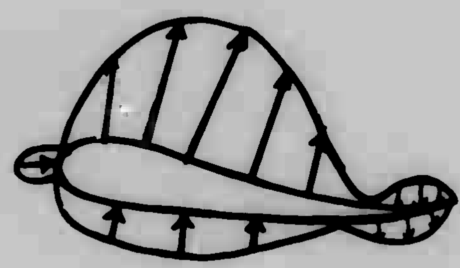
Designation	Deflection	C_{Lmax}	@ C_{Lmax}	C_{Mac}	
Basic Airfoil		1.54	15.5°	-.01	
20% Split Flap	60°	2.53	12.0°	-.18	
20% Plain Flap	60°	2.38	12.5°	-.22	
20% Slotted Flap	50°	2.76	13.5°	-.25	
27% Fowler Flap	30°	2.90	10.5°	-.42	

Figure 2.3.24

The reflexed wing shown in Figure 2.3.25 tends to reduce the effects of camber by creating a down load near the trailing edge of the wing. In fact, if enough reflex is incorporated in a wing either by an upward moving control surface or fixed reflex the negative pitching moment due to camber can be completely overpowered. All tailless aircraft must have some means of adjusting the reflex in order to have stability and control.



Reflex Results in a Down Load on the Trailing Edge
Figure 2.3.25

2.4 FUNDEMENTAL DRAG THEORY

2.4.1 SKIN FRICTION DRAG

2.4.1.1 INTRODUCTION

Although skin friction drag has long been recognized as an important factor in aircraft design, little importance was attached to reducing it until the development of high subsonic through supersonic aircraft. With the advent of high speed aircraft, skin friction drag became of major importance and recent aircraft design reflects this increasing importance. Even greater emphasis in this area of aerodynamics can be expected with the development of more advanced supersonic aircraft since a major portion of their total drag will be skin friction drag.

In a preceding section (1.3), the basic causes, effects, influence of R_e, and some mention of ways to decrease skin friction drag, were discussed. Therefore, this section will be devoted primarily to a discussion of the latest methods used to reduce skin friction drag and why they are effective.

2.4.1.2 REVIEW

Before proceeding with this discussion a quick review of basic skin friction drag theory is probably in order.

Skin friction drag is a portion of profile drag with the other portion being pressure drag. Skin friction drag is caused by the viscosity of the fluid through which the object passes. The viscosity of the fluid manifests itself as a shearing stress ($\tau \sim$ wt/unit area) which in turn acts as a force parallel and opposite to the objects direction.

A good method of depicting and studying skin friction drag is through the study of boundary layers. All skin friction drag originates through

boundary layers. If there is no boundary layer, such as during separation, there is no skin friction drag. There are two types of boundary layers, laminar and turbulent. Turbulent boundary layers create more skin friction drag than laminar boundary layers due to a greater exchange of energy between the surface and boundary layer.

The ratio of inertia to viscous forces ($R_e \approx q/\mu$) is one of the factors that determines what type boundary layer is present. The flow will generally remain laminar up to a certain R_e at which time the flow transitions to turbulent flow. Some of the factors that affect the transition to turbulent flow are:

1. Free stream turbulence.
2. Surface roughness.
3. Mechanical vibrations
4. Sound waves.
5. Pressure gradient in the direction of flow.

The R_e at which transition occurs is called the critical Reynolds number.

Simply stated the, there are two basic methods by which skin friction drag may be decreased. First, delay the transition point in respect to distance from the leading edge by designing smooth airfoil with a more lengthy favorable pressure gradient. Designing an airfoil that decreases free stream turbulence, sound waves and mechanical vibrations, is beyond the scope of this writing and therefore will be ignored in this discussion. Secondly then, design an airfoil that has a small thickness to chord ratio. This reduces the local velocity and hence local R_e so as to increase the distance from the leading edge at which the critical R_e is reached.

2.4.1.3 LAMINARIZATION

Laminarizing an airfoil is accomplished by several design methods. The first design criterion is that it be smooth. No explanation of how this is accomplished is believed necessary. Another important factor in laminarization is where to locate the airfoil's maximum thickness in respect to distance from the leading edge. Experiments have shown that airfoils with a maximum thickness at approximately 30% chord (World War II vintage) will approach a fully turbulent boundary layer at a R_e of 10^6 regardless of T/C ratio. By the same token laminar airfoils with a maximum thickness at 40% to 65% chord will transition to a fully turbulent boundary layer only after reaching an R_e of at least 10^7. The reason for this is that the airfoil with the maximum thickness at a greater distance from the leading edge will have a minimum pressure point further from the leading edge and hence a longer negative pressure gradient. Figure 2.4.1.1 will further explain this phenomenum.

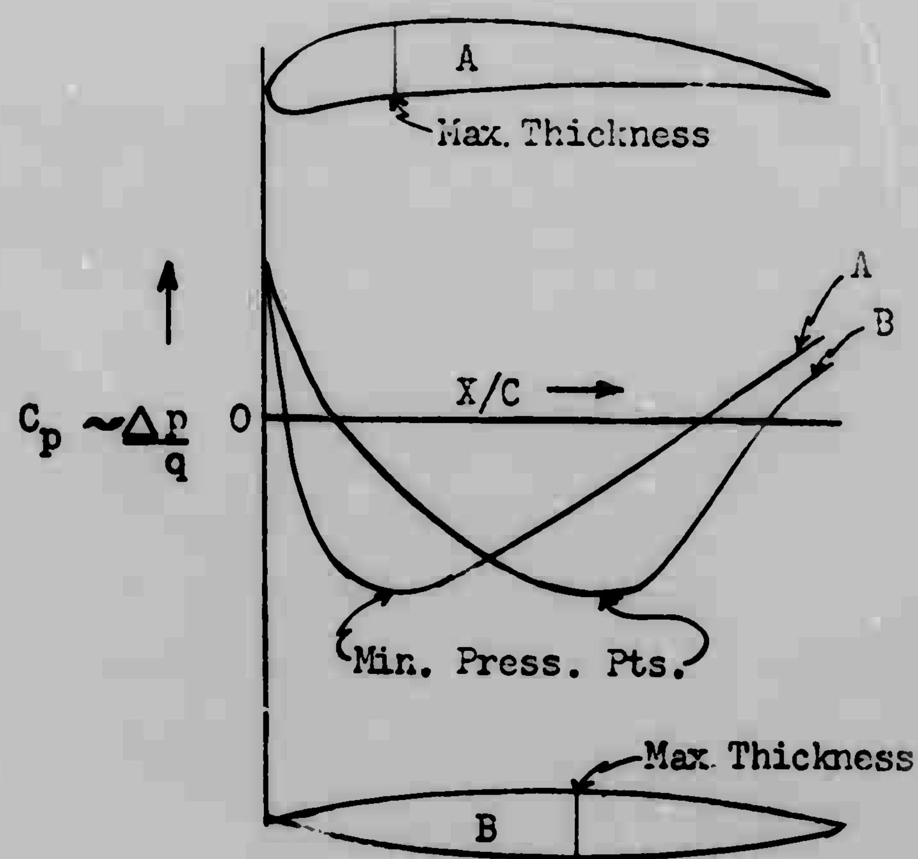


Figure 2.4.1.1 Location of minimum pressure points as a function of points of maximum thickness

However, to realize the optimum advantage from this particular design technique, two other design compromises must be taken into consideration. The first being that the airfoil can not be too thin, with a corresponding small T/C ratio, else the negative pressure gradient developed will be insufficient to help delay transition or provide sufficient lift.

The second design compromise is where to actually locate the maximum thickness point in terms of distance from the leading edge. This is an important item since pressure losses resulting from separation along the trailing edge increase as the maximum thickness point is moved further aft. In other words a point of maximum thickness must be selected that retains the benefits of laminarization in terms of total drag.

The next item to be considered is the actual thickness of the airfoil. As mentioned previously it must be thick enough to provide a useable negative pressure gradient. But, obviously a thick airfoil will tend to increase the local velocity to a greater value in terms of distance from the leading edge in comparison to a thin airfoil. Hence, the local R_e will be much higher at the same distance from the leading edge for the thick airfoil as opposed to the thin airfoil. Consequently the critical R_e will be reached closer to the leading edge on the thick airfoil.

The next design feature necessary for a useable laminar airfoil is a correct leading edge design. Experiments have shown that a parabolic shape is the most suitable in comparison to sharp or circular leading edges. The reasons being that sharp edges force transition on the suction side immediately upon leaving zero angle of attack and rounded edges exhibit transition on both

sides because of minimum pressure points immediately aft of the edge. Figure 2.4.1.2 may better explain this phenomena.

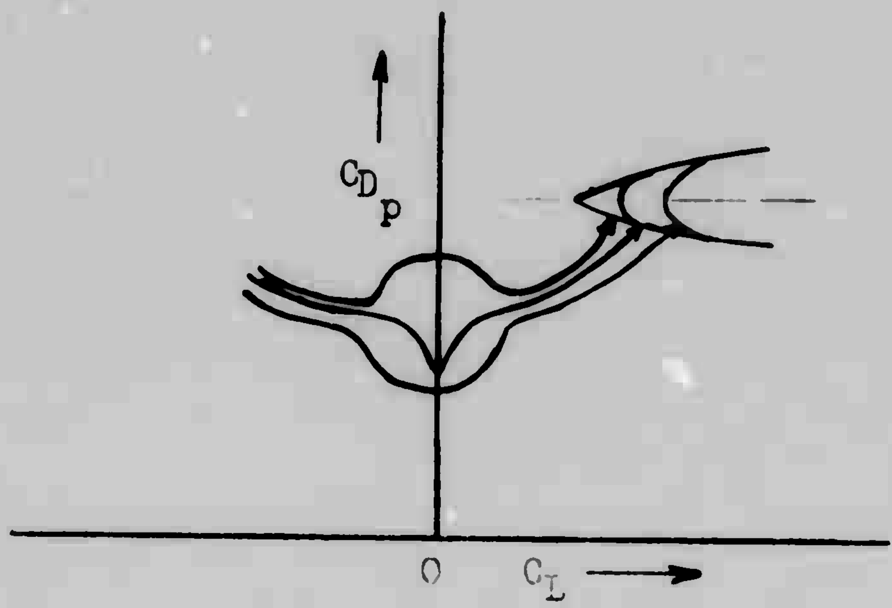


Figure 2.4.1.2 Influence of leading edge shape upon boundry layer and profile drag coefficient

In summary, a laminar airfoil has a maximum thickness at approximately 40-65% chord in order to maintain a favorable pressure gradient over a longer distance from the leading edge and hence delay the point of transition. The maximum thickness point cannot be too far aft or total drag due to separation will negate the decrease in skin friction drag. It is comparatively thin so the local velocity and thus R_e remain low. The airfoil cannot be too thin,else a useable pressure gradient will not be present. And finally it must have a parabolic shaped leading edge or the other design features will not be realized since sharp or rounded leading edges cause transition shortly aft of the leading edge.

2.4.1.4 DRAG BUCKET

A laminar airfoil exhibits a decrease in skin friction drag through a rather restricted range of lift coefficients. In other words it possesses little or no advantage over "normal" airfoils in regard to skin friction drag except in the range of C_Ls where a decrease in skin friction drag is considered most important. Usually the airfoil is designed to give the greatest decrease in skin friction drag during low C_Ls or in other words, high speeds. Figure 2.4.1.3 graphically shows a typical "drag bucket" of a laminar airfoil in comparison to a "normal" airfoil. By employing a suitable value of section camber the "bucket" can be placed around the lift coefficient desired without losing any or much of the laminar effect.

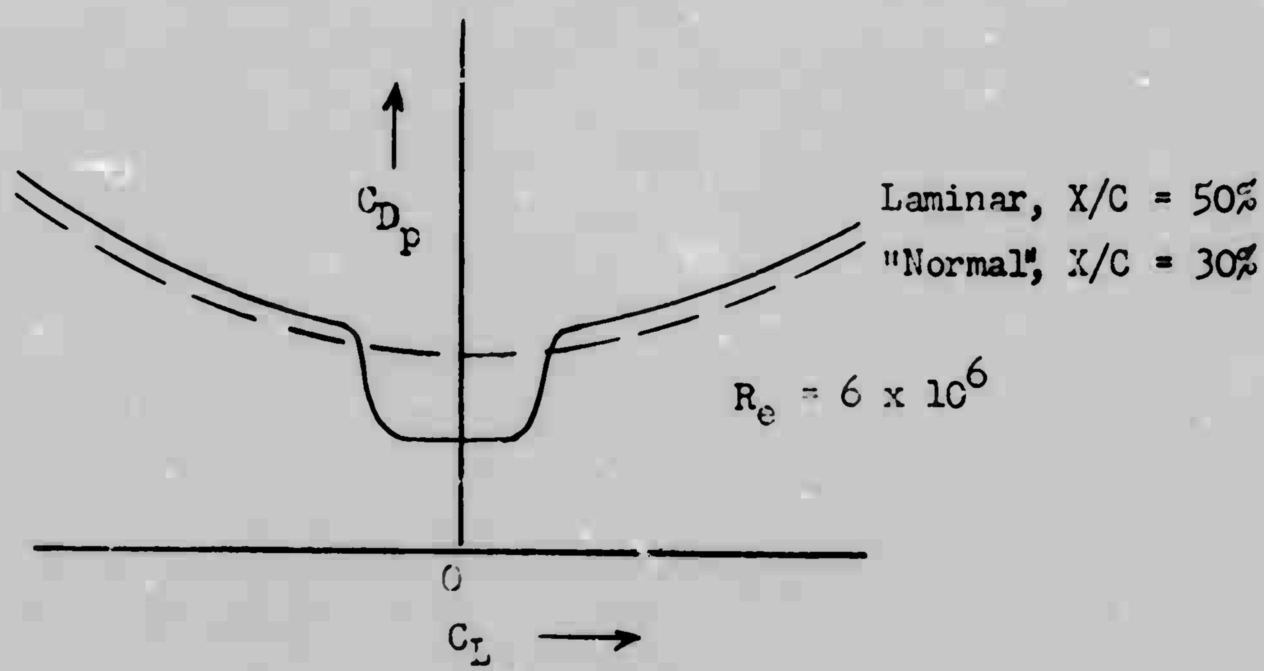


Figure 2.4.1.3 Profile drag coefficient of various foil sections showing the "bucket" shaped drag minimum which is typical of laminar type airfoils.

2.4.1.5 BOUNDARY LAYER SUCTION

An interesting method of reducing skin friction drag that has not as yet been developed to an economically feasible state is a method known as boundary layer suction. Through small slots or a porus surface, a portion of the boundary layer is sucked into a hollow wing. This will decrease the momentum loss in the wake. A simple method of depicting this particular theory is shown in Figure 2.4.1.4.

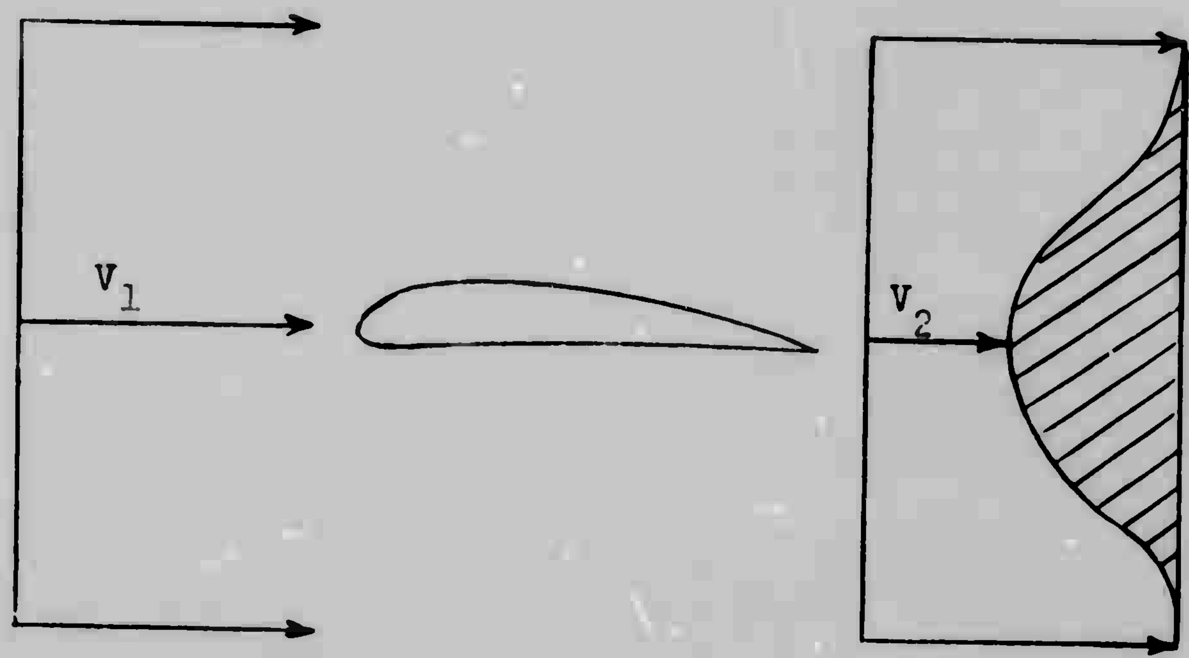


Figure 2.4.1.4 Loss of momentum due to viscosity of air(Shaded Area).

In other words, boundary layer suction decreases the shaded area in Figure 2.4.1.4, by actually eliminating a portion of the momentum losses due to viscosity.

The important limitation to what otherwise appears to be an aerodynamic "breakthrough" is the requirement for a rather complicated suction device that also requires power to operate. This power requirement can be considered as an increase in drag. Presently the drag increase, due to the additional power required for suction, is greater than the benefit gained. However, research in this area is continuing and a feasible system will undoubtedly be perfected in the future.

2.4.2 PRESSURE DRAG

2.4.2.1 INTRODUCTION

We have previously discussed the portion of profile drag caused by a distribution of forces tangential to the body surface. This drag was called skin friction drag. In this section another portion of profile drag called pressure drag will be discussed. In contradistiction, pressure drag is generally considered as the drag resulting from a distribution of forces normal to the body surface. Drag caused by separation is an excellent example of pressure drag. The magnitude of pressure drag, depending on flow conditions etc., is usually many times that of skin friction drag.

2.4.2.2 FLAT PLATE DRAG

A flat plate placed parallel to the flow direction was used to illustrate skin friction drag phenomena. By placing the plate 90 degrees to the flow direction, skin friction drag becomes negligable in comparison to the drag caused by the resulting pressure differential between the front and rear sides of the plate.

By referring to Figure 2.4.2.1 it can be seen that a stagnation point exists on the forward center portion of the plate. Furthermore, flow on either side of the stagnation point is almost parallel to the front side of the plate. From this it can be reasoned that the pressure over the front of the plate, other than the very center, will be less than stagnation point pressure. This pressure distribution on the front side of the plate in combination with lower pressures on the rear, caused by separation, results in a net retarding force or drag. This is a rather extreme example of pressure drag.

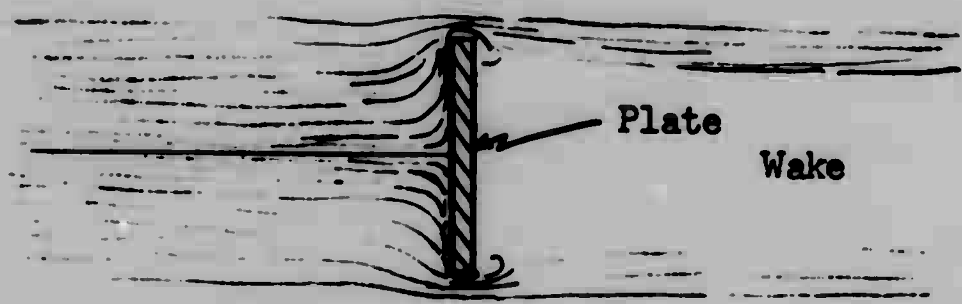


Figure 2.4.2.1

Wind tunnel experiments have shown that a drag coefficient of approximately 1.28 is a good average figure for a flat plate in the flight range of the Reynolds number. The total drag is computed by the equation:

$$D = 1.28\ q\ S_p \qquad \text{Equation 2.4.2.1}$$

where S_p = plate area

For comparison purposes, the parasite drag of an airplane is sometimes expressed in terms of an equivalent flat plate area. That is, the area of a flat plate which yields the same drag as the aircraft. The equivalent flat plate area is then found by using equation 2.4.2.1 and defining A_e as equivalent flat plate area. Then,

$$D = C_D\ q\ S = 1.28\ q\ S_p$$

or $$S_p = \frac{C_D\ S}{1.28} = A_e \qquad \text{Equation 2.4.2.2}$$

This comparison is obtained merely by substituting the C_D and S of each particular aircraft into equation 2.4.2.2. Thus it is seen that the equivalent flat plate area, A_e, is a direct parasite drag comparison between two different types of aircraft.

Obviously if the flat plate drag coefficient had been 1.0 instead of 1.28 the expression for the equivalent flat plate area, equation 1.4.2.2 , would have been simplified considerably. It is convenient to define a fictitious equivalent area called the equivalent parasite area, f, which assumes that the flat plate drag coefficient is 1.0. That is,

$$f = \frac{C_D\ S}{1.00}$$

or simply $$f = C_D\ S \qquad \text{Equation 1.4.2.2a}$$

Like equivalent flat plate area, equivalent parasite area allows easy comparison of the parasite drags of various components of the aircraft or a comparison between different aircraft. All of these quantities are related as follows:

$$D = C_D\, S\, q = 1.28\, A_e\, q = 1.0\, f\, q$$

or

$$C_D\, S = 1.28\, A_e = f \qquad \text{Equation 2.4.2.3}$$

A drag comparison for bodies of unit cross section may be made by comparing either the equivalent parasite area or the equivalent flat plate areas as in Figure 2.4.2.2 below.

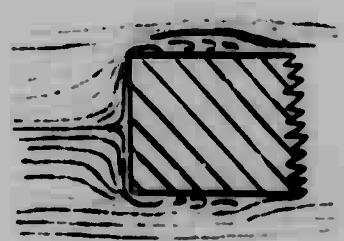
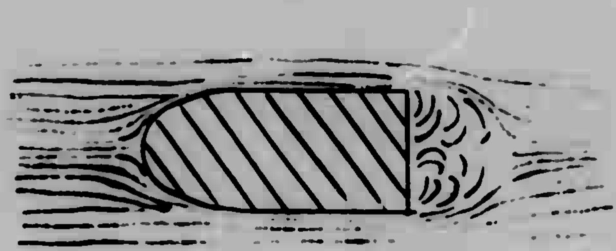
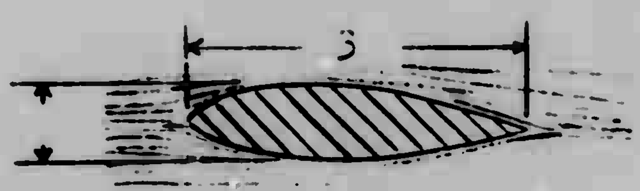
OBJECT	$C_D/1.28$	f
	.80 - .94	1.02 - 1.20
	0.23	0.295
	0.035	0.045

FIGURE 2.3.2.2 Relative Drag of Various Objects

From Figure 2.4.2.2 it can easily be seen that a streamlined shape having a 3/1 finess ratio gives the best value of $C_D/1.28$. Therefore, $C_D/1.28 = .035$ or 3.5% of the drag coefficient of an equivalent flat plate.

2.4.2.3 REYNOLDS NUMBER EFFECT ON PRESSURE DRAG

Experiments using a sphere as the test object have shown a definite relationship between drag coefficients and the particular Reynolds number used during the experiment.

Consider the flow about a sphere. At the stagnation point the pressure is higher than ambient. At approximately 42° the pressure is equal to ambient and at 90° the pressure is at its lowest. This effect is in accord with Bernoullis equation with the decrease in pressure and increase in velocity due to a favorable pressure gradient, Figure 2.4.2.3.

If the flow did not separate, the pressure on both the front and rear sides of the sphere would be equal and there would be no pressure drag. However, due to an adverse pressure gradient after the 90° point the flow will separate. Where the flow separates between 90° and 180° depends on whether the boundary layer is laminar or turbulent. Obviously an early separation will create a large wake and corresponding large pressure drag and conversely, Figure 2.4.2.4. As previously discussed, a laminar boundary layer tends to separate early in comparison to a turbulent boundary layer. Thus, the drag of a sphere will change markedly as the R_e is increased and the boundary layer transitions to a turbulent boundary layer sooner or closer to the stagnation point. This effect is shown in Figure 2.4.2.5.

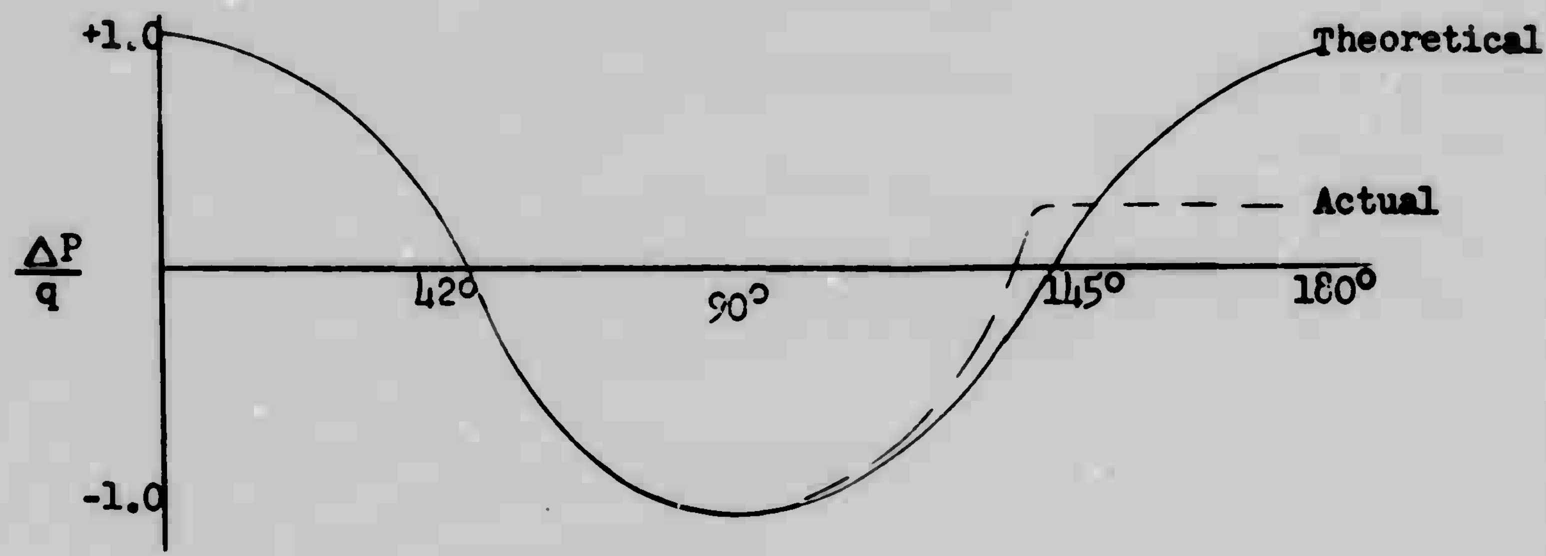


Figure 2.4.2.3 Pressure Distribution About a Sphere

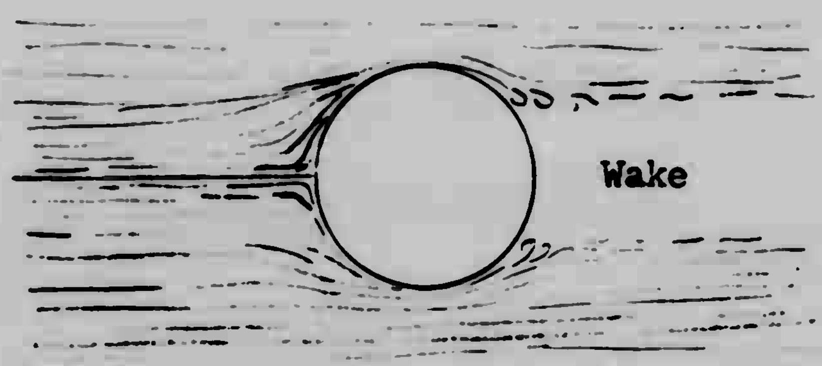


Figure 2.4.2.4 Flow Past a Sphere With a Laminar Boundary Layer (Low R_e)

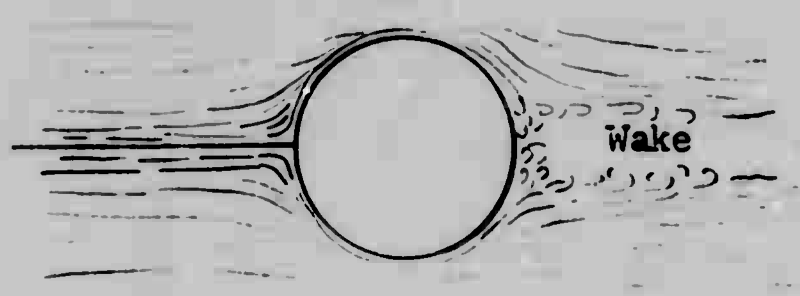


Figure 2.4.2.5 Flow Past a Sphere With a Turbulent Boundary Layer (High R_e)

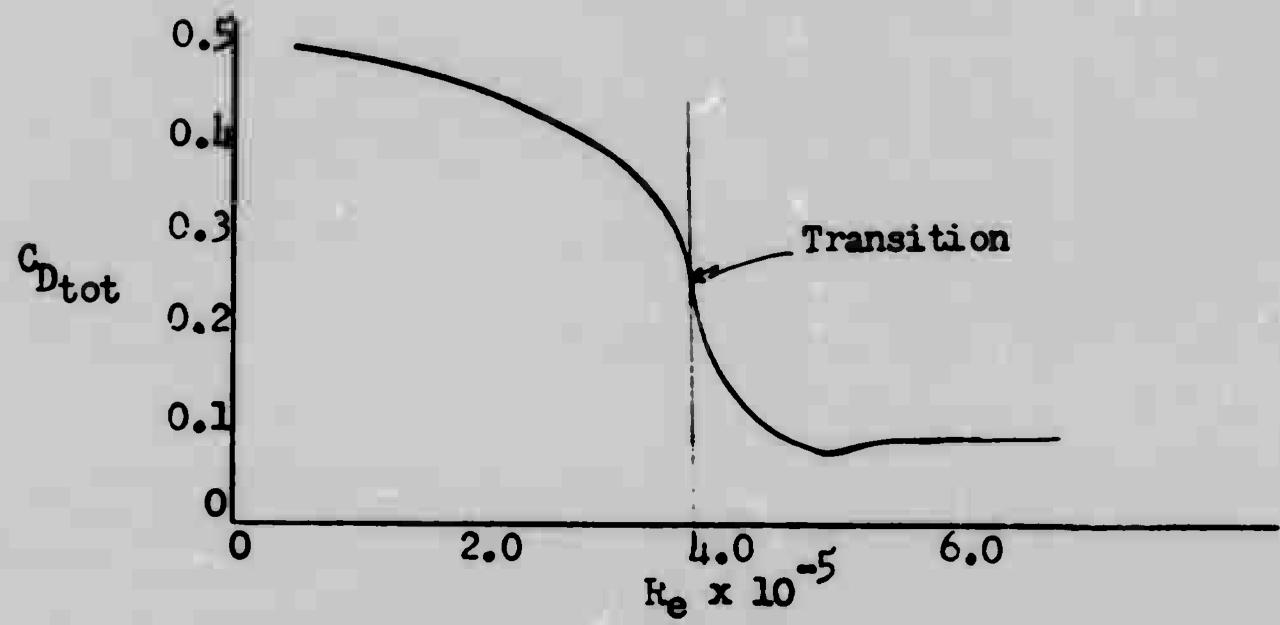


Figure 2.4.2.6 Variation of Sphere C_D With R_e

2.4.2.4 BOUNDARY LAYER CONTROL AND PRESSURE DRAG

From the discussion up to this point it is obvious that a turbulent boundary layer is more desirable than a laminar boundary layer from a standpoint of pressure drag. This is the exact opposite of the case as previously discussed in skin friction drag.

Figure 2.4.2.7 shows the effect of an adverse pressure gradient on flow over an airfoil.

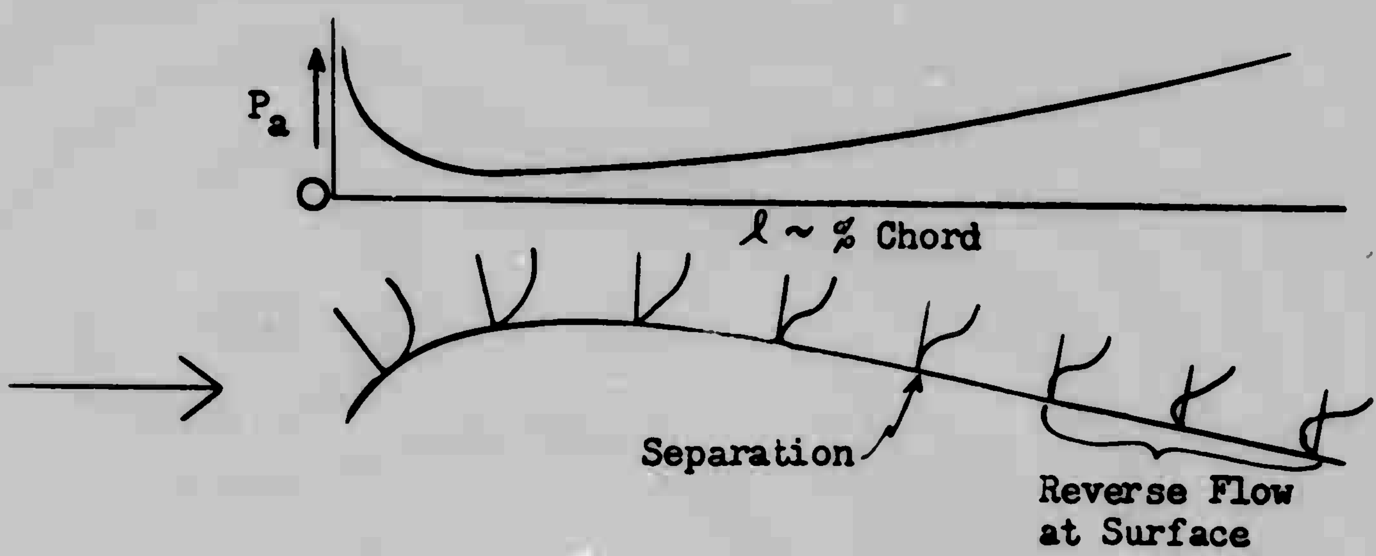


Figure 2.4.2.7 Effect of Adverse Pressure Gradient on B.L. Velocity Profile Including Separation

By using some artificial means of overcoming this effect, and thereby delaying separation, pressure drag can be decreased. This can be achieved in a number of ways.

Sucking air from the upper trailing edge of the airfoil draws off the low velocity boundary layer which inturn delays separation and reduces pressure drag. Blowing high energy air into the boundary layer at the upper trailing edge of the airfoil likewise delays separation and reduces pressure drag.

Blowing high energy air over the leading edge will tend to transition the boundary layer to a turbulent one sooner and will also help to overcome the adverse pressure gradient. Both effects will tend to delay separation and decrease pressure drag.

By strategic placing of vortex generators, laminar boundary layers can be transitioned to turbulent boundary layers with a corresponding delay in separation and decrease in pressure drag. In addition, the vortex generators take high energy air above the boundary layer and mix it with lower energy air at the surface. Another vortex generator use is to increase aircraft control effectiveness by delaying separation near control surfaces. They are also used to prevent or minimize aircraft buffet in certain regimes of flight. All of the above uses tend to delay separation and reduce pressure drag in some regimes of flight. In other regimes of flight when separation is not normally a problem, vortex generators may increase pressure drag due to a small increase in flat plate area.

In summary, all of the above methods of boundary layer control are concerned with delaying separation. However, the primary purpose of boundary layer control as used in present day aircraft is to decrease stall speeds by delaying separation, not merely to decrease pressure drag. The reason being that the energy required to affect a decrease in pressure drag through boundary layer control is more than the benefit gained or energy gained due to a decrease in pressure drag. However, research in this area is being made continually and possibly a method that is economically feasable will be developed in the near future.

2.4.3. INDUCED DRAG

2.4.3.1 INTRODUCTION

The purpose of this section is to derive the equation for induced drag and to give an understanding of the factors affecting induced drag and the existance of upwash and downwash about a wing. This discussion will assume that the air is an ideal fluid, i.e., a fluid which has zero viscosity. Any body moving thru an ideal fluid would encounter no skin friction drag. Since there is no energy lost, as in the boundary layer of viscous flow, there would be no separation of the flow from the surface of the body and, as a result, no pressure drag.

Another assumption made in this discussion is that the air acts as an incompressible fluid, i.e., the density of the air does not change as a body moves thru the air. This assumption prevents the formation of shock waves and any other effects of compressibility found under real conditions.

With these two assumptions a body moving thru the air would not have any parasite drag or drag due to compressibility. The only drag encountered would be induced drag and this drag is present only if lift is being produced by the body.

2.4.3.2 DETERMINATION OF STREAMLINE PATTERNS BY COMBINATION OF FLOWS:

We will consider various types of flows known as potential flows. In potential flow all streamlines have the same energy. The easiest potential flow to visualize is uniform linear flow where all the streamlines are

parallel and straight horizontal lines as shown on Figure 2.4.3.1. This is the type of flow approaching an airplane at a considerable distance ahead of the airplane.

Uniform Linear Flow

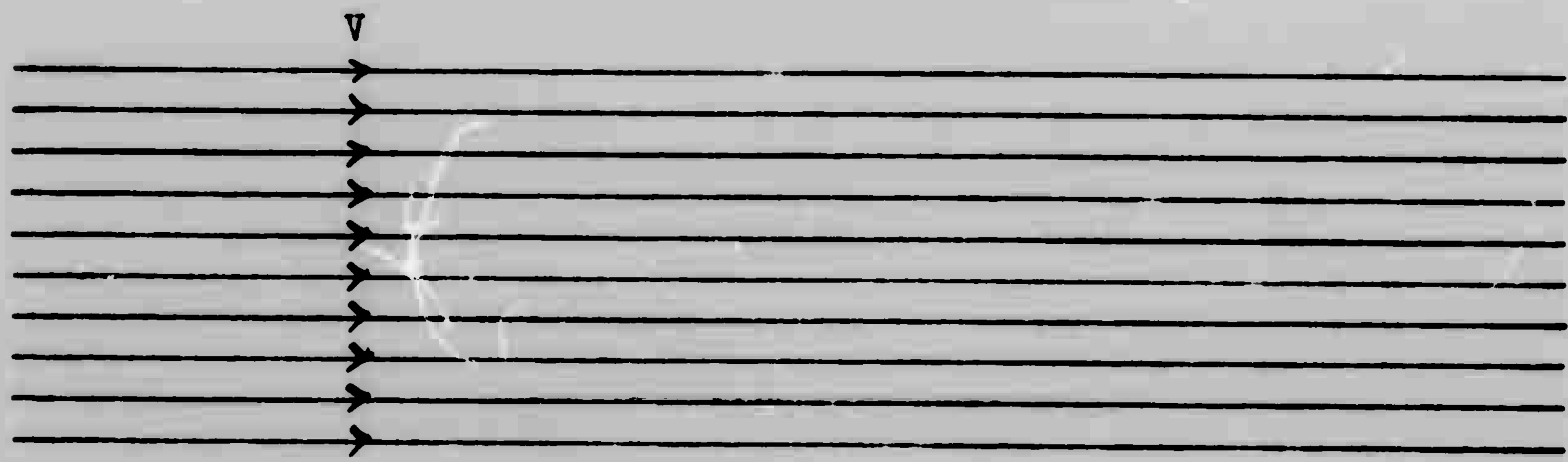


Figure 2.4.3.1

Two other types of potential flows to be considered now are called source and sink. In a source the fluid flows radially away from a singular point in all directions, ref. Fig. 2.4.3.2. The velocity of the flow varies inversely with the distance from the source point. The equation defining the source is:

$$V = \frac{K}{R} \qquad \text{Equation 2.4.3.1}$$

Where: V = velocity of source flow.

R = distance from source point

K = constant (sources of different strengths would have different values of the constant).

Source Flow

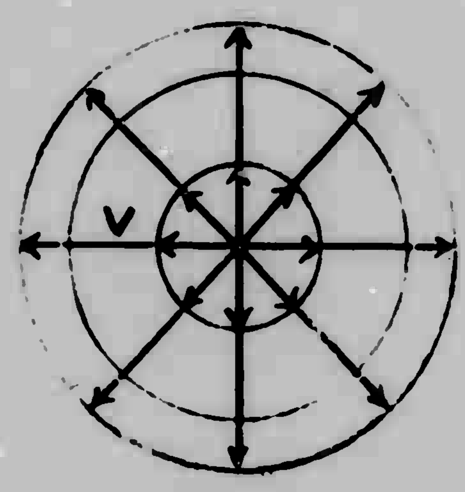


Figure 2.4.3.2

As the radius approaches infinity the velocity approaches zero, and as the radius approaches zero the velocity becomes infinite. This cannot happen under actual conditions but approximations to source flow are possible. Consider flow of water or other non-viscous liquid from an outlet descending vertically to a flat horizontal plate. The flow across the surface of the plate would be similar to souce flow as the velocity of flow must decrease as the distance from the center of the plate increases.

Sink flow is similar to the source flow except that the flow approaches the center, called the sink point, ref. Fig. 2.4.3.3. The equation for sink flow is:

$$V = -\frac{K}{R} \qquad \text{Equation 2.4.3.2}$$

Where: V = velocity of sink flow.

R = distance from sink point

K = constant (depending on strength of sink flow).

Sink Flow

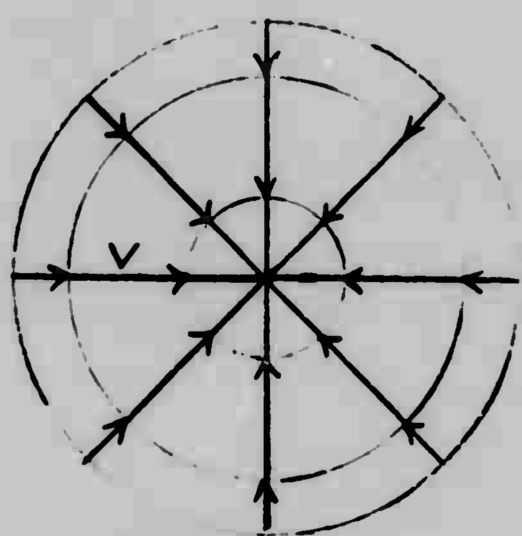


Figure 2.4.3.3

As in source flow, the velocity must be infinite at zero radius and zero at an infinite radius. An example of sink flow is water along a flat horizontal plate into a hole in the center of the plate.

It is possible to combine source, sink and uniform linear flows to determine a streamline pattern of the combined flow. The velocity, (both direction and magnitude) of the flow can be found at any point by adding the velocity vectors of the source, sink and uniform linear flow at the point in question, ref. Fig. 2.4.3.4. If this procedure were done at all points in the flow, streamlines could be constructed such that each streamline would be tangent to all the velocity vectors that it touched.

Addition of Velocity Vectors

$V_1 + V_2 + V_3$ = Resultant velocity

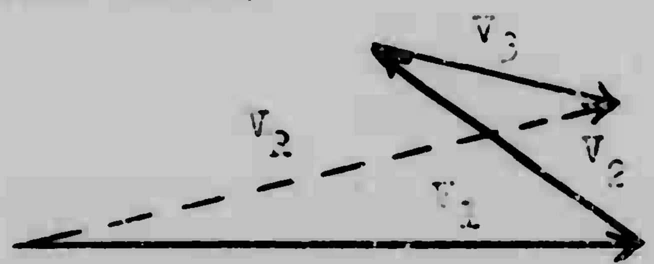


Figure 2.4.3.4

Combination of Source Flow and Uniform Linear Flow

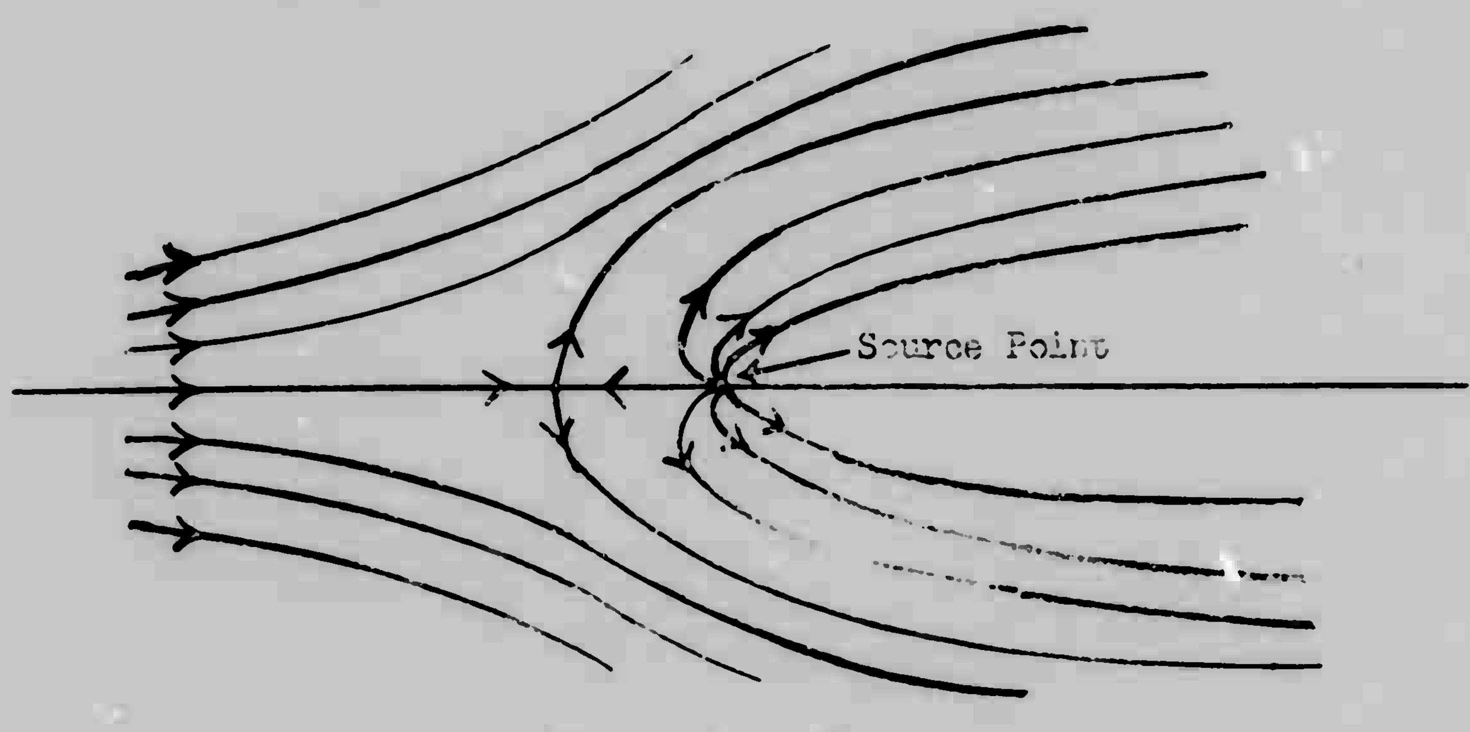


Figure 2.4.3.5

Fig. 2.4.3.5 shows the streamline pattern resulting from a combination of source flow and uniform linear flow. Note that at some distance ahead of the source point the velocity of the source and uniform linear flow are the same magnitude but in opposite directions so that the resulting velocity is zero. This point is called the stagnation point. From this stagnation point a streamline exists showing the direction of flow to the right both above and below the source point. This streamline may be replaced (as may any streamline) by a surface without changing the position of the other streamlines. Remember that this is still considering a non-viscous fluid. If we place a body within the streamlines connecting the fore and aft stagnation points, as indicated by the dotted lines, the external streamlines will not be changed, reference Figure 2.4.3.6.

Source, Sink and Uniform Linear Flow

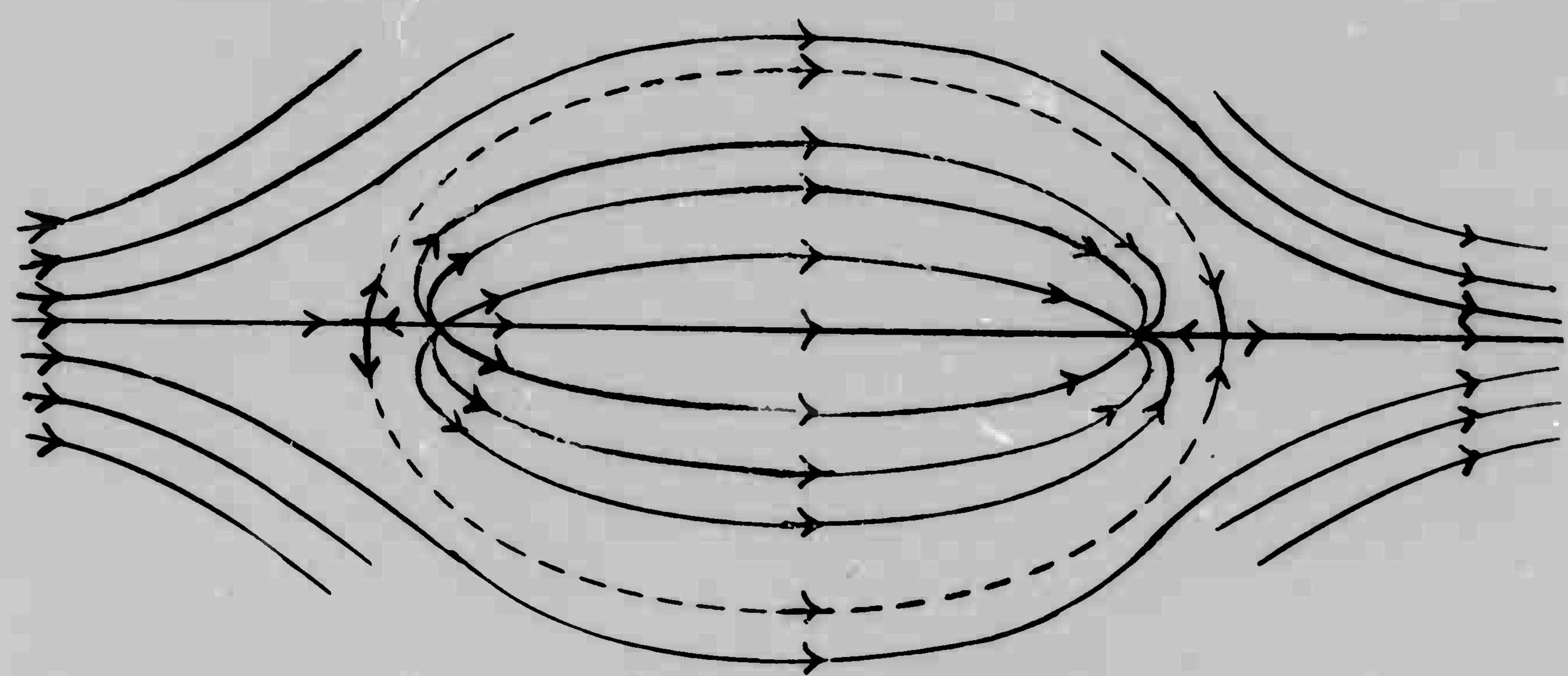


Figure 2.4.3.6

Figure 2.4.3.6 shows the flow pattern determined by the combination of uniform linear flow, source flow and sink flow where the source and sink flow are of equal strength and the centers of the source and sink flow are some finite distance apart. Note that there is a stagnation point behind the sink point as well as ahead of the source point. If the streamline connecting the two stagnation points were replaced by a solid body of the same shape, the remaining streamlines would be identical to the streamlines around a body of this shape. Note that the flow pattern around the rear half of the body is a mirror image of the flow pattern around the front half. Therefore, the pressure on the surface over corresponding locations on the front and rear portions of the body would be identical, so there would be no pressure drag. Similarly, the flow patterns around the upper and lower surfaces are identical, so there is no lift force. It should now be obvious that no drag would result from flow of an ideal fluid around any shape body providing the body is positioned such that it produces no lift force.

If the relative strengths of the source and sink in Figure 2.4.3.6 are the same and if their centers approach each other the corresponding flow field approaches a circle. That is, when the centers coincide, the streamlines connecting the two stagnation points may be replaced with a disk. This combination of source and sink of equal strengths with centers at the same point formed in this way is called a doublet. Figure 2.4.3.7 shows the flow pattern of a doublet and uniform linear flow combination. This represents the flow of an ideal fluid about a cylinder.

Doublet and Linear Flow

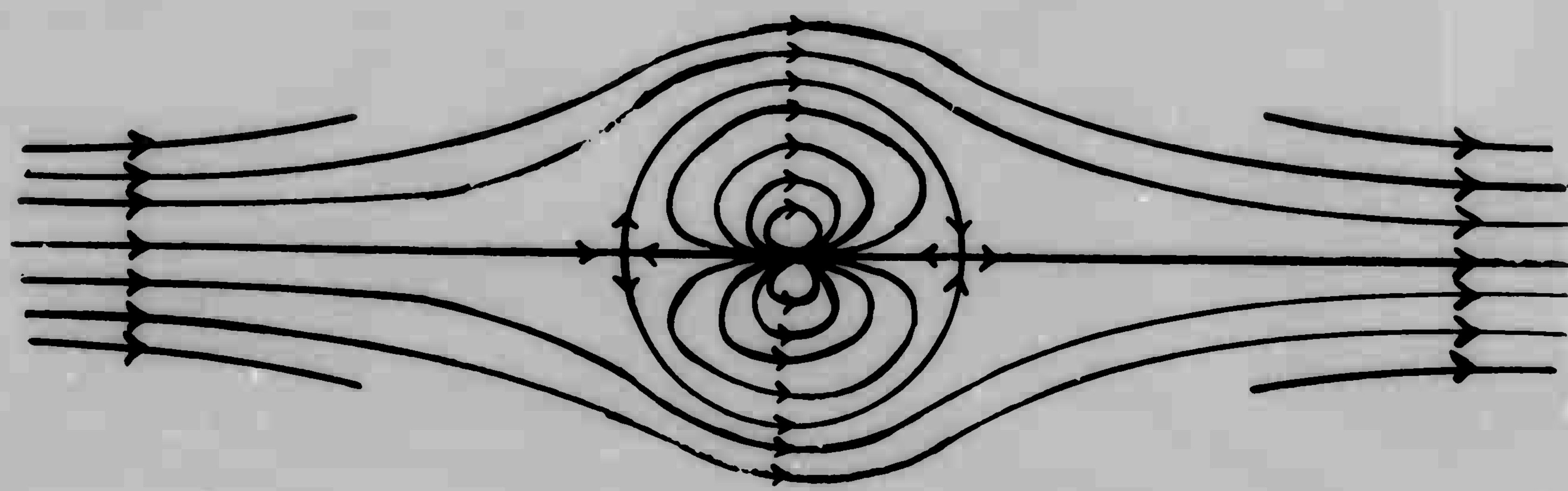


Figure 2.4.3.7

2.4.3.3 CIRCULATION:

Circulation is another type of potential flow which may be added vectorially to the flows previously discussed. In circulation the streamlines are concentric circles about the center of circulation. The velocities of the fluid particles decrease as the radius increases as shown in Fig. 2.4.3.8. The equation for the velocity is:

$$V = \frac{K}{R}$$

Where: V = tangential velocity

R = distance to center of circulation

K = constant (depends on strength of circulation, Γ)

Where: Γ (Gamma) = $2 \pi K$

$$\Gamma = 2 \pi V R \qquad \text{Equation 2.4.3.3}$$

Velocity Distribution for Circulation Flow

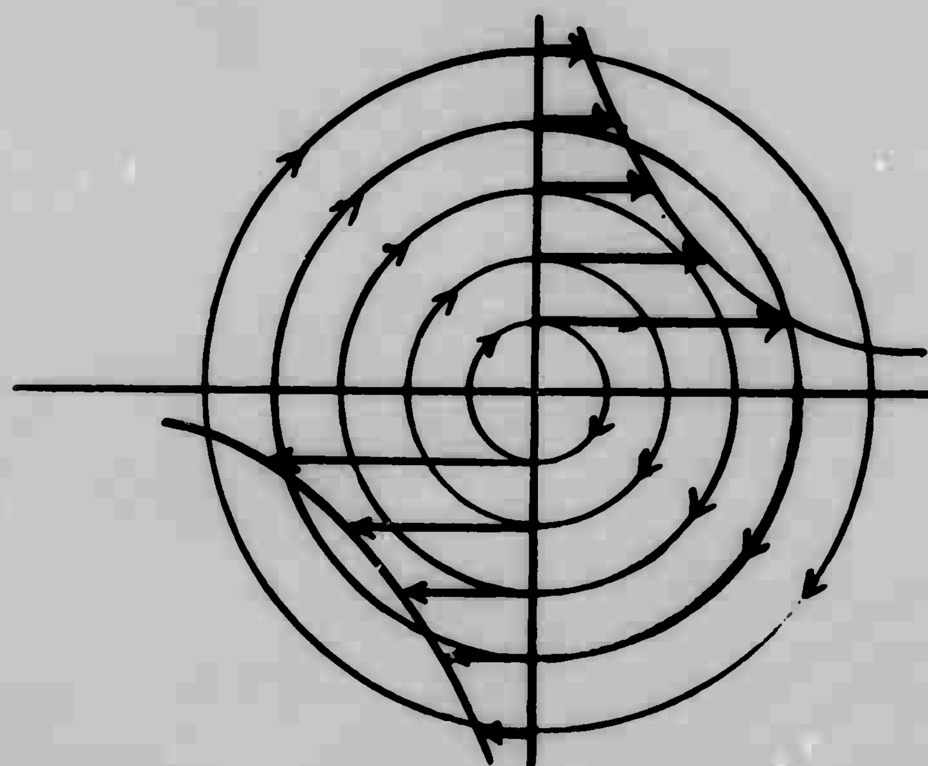
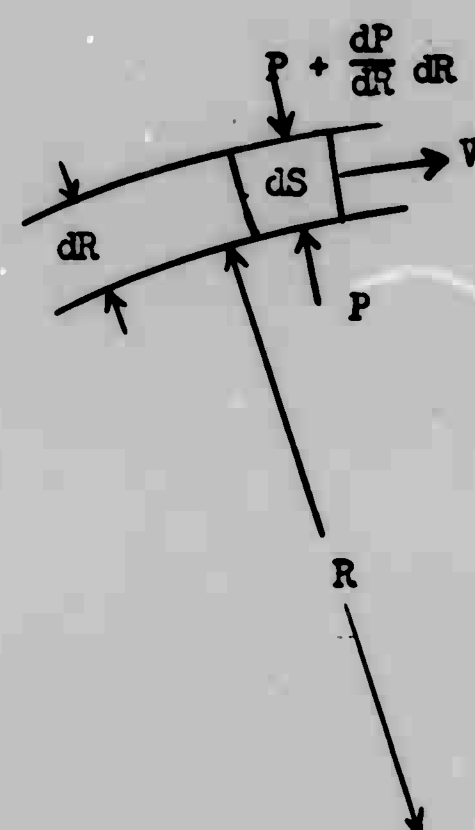


Figure 2.4.3.8

Any potential flow in a curved path is circulation flow. Consider an element of fluid of unit depth moving about some radius.



centrifugal force $= \frac{m\,V^2}{R}$

pressure force $= \left(P + \frac{dP}{dR}\cdot dR\right) dS - PdS$

$$= \frac{dP}{dR}(dR\ dS)$$

$$\frac{m\,V^2}{R} = \frac{dP}{dR}(dR\ dS)$$

mass, $m = \rho(dS\ dR)$

$$\frac{\rho\, dS\ dR\ V^2}{R} = \frac{dP}{dR}(dR\ dS)$$

$$\frac{\rho\, V^2}{R} = \frac{dP}{dR}$$

$$\therefore dP = \frac{dR\,\rho\, V^2}{R}$$

Equation 2.4.3.4

Since the flow is potential flow all the streamlines have the same energy; therefore, Bernoullis equation for incompressible fluid applies through-out the flow field.

$$P + \frac{\rho V^2}{2} = \text{constant}$$

Differentiate

$$dP + \rho V\, dV = 0 \qquad \text{Euler's equation}$$

$$dP = -\rho V\, dV$$

Combine equations:

$$\frac{dR}{R} \rho V^2 = -\rho V\, dV$$

$$\frac{dR}{R} = -\frac{dV}{V}$$

$$\frac{dR}{R} + \frac{dV}{V} = 0$$

Integrate

$$\ln R + \ln V = \text{constant} = K_1$$

$$\ln RV = \text{constant} = K_1$$

$$RV = e^{K_1} = \text{constant} = K_2$$

$$V = \frac{K_2}{R}$$

Any flow in a curved path which follows this equation is called irrotational flow or circulation flow.

The strength of the circulation is called Gamma, Γ. Gamma is 2π times the constant, K_2 in equation

$$\Gamma = 2\pi K_2 = 2\pi V R$$

Circulation flow is different from rotational flow in that the tangential velocity of circulation flow decreases directly with an increased radius, while the velocity of rotational flow increases with an increased radius. Rotational flow is analogous to the velocity of particles of a rotating disk. Circulation is considerably different in that the velocity is infinite at the center of circulation. This is impossible in nature but many examples of circulation are in evidence with the extreme center of the circulation rotating as a solid core. Tornados, water spouts and whirl pools are all examples of circulation flow. Extremely low pressures exist near the center of these circulation systems due to the high velocities attained.

If the flow patterns of circulation and a flow about a cylinder were combined as in Fig. 2.4.3.9 a new set of streamline patterns would result relocating the stagnation points on the cylinder. A rotating cylinder in uniform linear flow would give the same flow pattern.

Circulation and Uniform Flow About a Cylinder

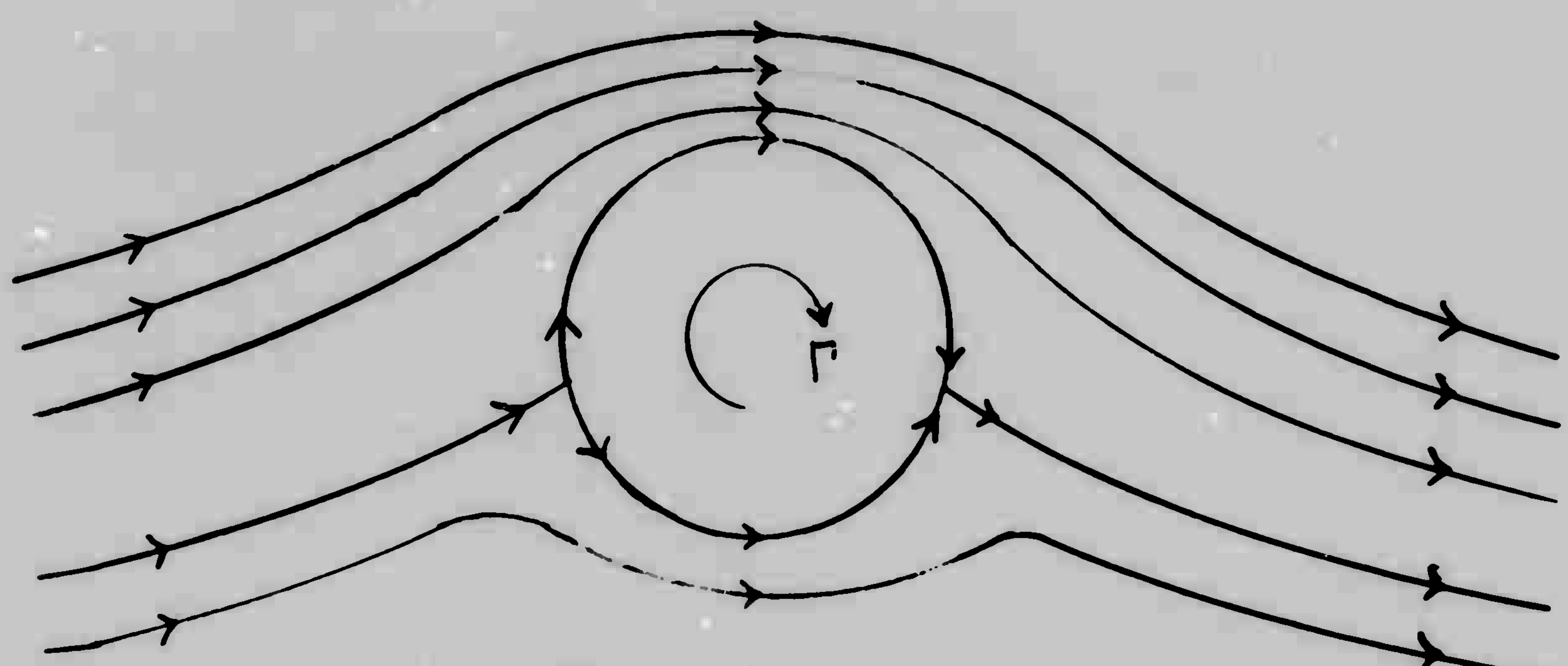


Figure 2.4.3.9

At this point, it should be made clear that the velocities of all of these potential flows may be expressed mathematically so the velocity at any point in the flow may be found, thus the streamline patterns may be determined. Also the pressure on the surface may be found from Bernoullis equation since the velocities are known. Complicated combinations of these potential flows are laborious to calculate by ordinary means but mathematical methods beyond the scope of this course are used for these cases.

Velocity Around Cylinder

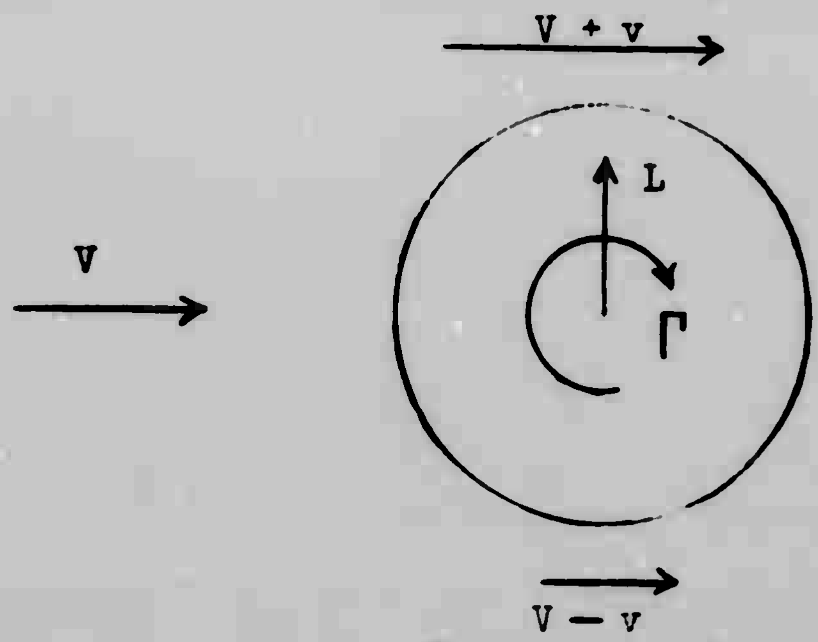


Figure 2.4.3.10

The combination of uniform linear flow and circulation about a body, such as a cylinder, results in unequal pressures about the top and bottom of the body, thus giving a force applied to the body perpendicular to the direction of the uniform linear flow. With the direction of circulation as shown in Fig. 2.4.3.10 the velocity at the surface of the cylinder on the bottom will be less than the free stream velocity, and the velocity at the top will be greater than the free stream velocity. Considering Bernoullis equation, this gives a higher average pressure on the bottom of the cylinder

than at the top so an upward lift force results. This lift force is dependent upon the strength of the circulation. The relationship between lift and the strength of circulation and free stream velocity was derived independently by Kutta and Joukowski and is known as the Kutta-Joukowski relation. A simple derivation of this relation using Bernoullis incompressible equation follows. This derivation is not rigorous so we will make an approximation to arrive at the desired relation. The average velocity along the top of the cylinder is $(V + v)$ and along the bottom of the cylinder is $(V - v)$ for the flow shown in Fig. 2.4.3.10. Now applying Bernoulli's equation for the streamlines above and below the cylinder we have:

$$P_o + \frac{1}{2}\rho V^2 = P_T + \frac{1}{2}\rho(V + v)^2$$

$$P_o + \frac{1}{2}\rho V^2 = P_B + \frac{1}{2}\rho(V - v)^2$$

$$\therefore P_T + \frac{1}{2}\rho(V + v)^2 = P_B + \frac{1}{2}\rho(V - v)^2$$

$$P_T - P_B = \frac{1}{2}\rho\left[(V + v)^2 - (V - v)^2\right]$$

$$P_T - P_B = \frac{1}{2}\rho(4\,V\,v)$$

$$\Delta P = 2\rho V v$$

Equation 2.4.3.5

Now if we consider a one foot length or span of the cylinder, the equation for the lift force is:

$$L = (P_B - P_T)\,D$$

$$L = 2\rho V v D$$

Equation 2.4.3.6

The small velocity (v) used in our average velocities is not the same as the velocity (v') due to circulation at the surface of the cylinder but is related to it.

The equation for the circulation is:

$$\Gamma = 2\pi v' R$$

$$\Gamma = \pi v' D$$

If we assume $v = K v'$

Then $$\Gamma = \frac{\pi}{K} v D$$

$$L = 2 \rho V v D$$

$$L = \frac{2 \rho V \Gamma K}{\pi}$$

$$L = \frac{2K}{\pi} \rho V \Gamma$$ Equation 2.3.3.7

Assume $\frac{2K}{\pi} = 1.0$

Then lift per unit span, $L = \rho V \Gamma$ (Kutta-Joukowski relation).

This equation holds for any shape body with circulation and uniform linear flows. Both flows must be present to produce lift; if either of the flows is non-existant there can be no lift. The airfoil is a device to produce circulation when acted upon by uniform linear flow.

If a circular cylinder is fixed in a uniform stream, it, of course, experiences no lift. Further, if it is given an angular displacement, it's lift does not change, but remains zero. Thus, this degenerate "airfoil" may be said to be in a stall condition, at maximum lift, in fact. Now, if the circular cylinder is given a constant angular velocity of rotation about its axis, then a circulation develops and an aerodynamic force transverse to the

flow direction is exerted. If the stream velocity is from left to right and the rotation is clockwise, then the force is upward (lift). If the rotation is counter clockwise, the force is downward.

This phenomenon is explained by consideration of the boundary layer. In the case of clockwise rotation, the upper surface of the cylinder is moving with, and the bottom surface against, the flow. Consequently, when circulation is present, the boundary layer separates later on the top and sooner on the bottom than is the case when the cylinder is not rotating. On the top, later separation means that the velocity outside the boundary layer is lower at separation. Now, the separation point signifies the beginning of a wake. Therefore, the clockwise vorticity shed into the wake, being proportional to the local outer volocity, is less on the top, and the counter clockwise vorticity shed at the bottom separation is greater, than in the case of no rotation.

Therefore, owing to clockwise rotation, a net increase of counter clockwise vorticity is shed. By the law of conservation of circulation, the circulation therefore cannot be zero, and a clockwise circulation must develop about the airfoil to compensate for the shed vorticity. According to classical hydrodynamics, this circulation results in lift.

There are many examples of this lift force due to circulation besides the wing of an airplane. This effect caused by combining circulation and uniform linear flow is called the magnus effect. It is evident in a rotating ball such as a curve ball pitched in baseball. Ships have been made by replacing the

sail with a rotating vertical cylinder to provide the force normally obtained from the sail. An airplane was constructed (although not flown) where the wing was replaced by a rotating horizontal cylinder.

It is interesting to understand how the circulation starts about an airfoil once the airfoil starts moving thru the air. If there were no circulation the stagnation point on the rear of the airfoil would be on the upper surface just forward of the trailing edge. This requires the flow below the under surface of the airfoil to turn the sharp corner at the trailing edge to reach the stagnation point. Since the viscosity of the air will not allow this (due to energy lost in the boundary layer) separation will occur at the trailing edge which starts a circulation motion counter clockwise called a starting vortex. See Figure 2.4.3.11. The starting vortex leaves the trailing edge of the airfoil and remains with the air at the point where it was formed. The starting vortex causes the circulation around the airfoil to form and this circulation remains with the airfoil as it moves forward. So, in order to have circulation exist, we must have the viscious property of air. If air had no viscosity, wings could not produce lift.

Starting Vortex on Airfoil

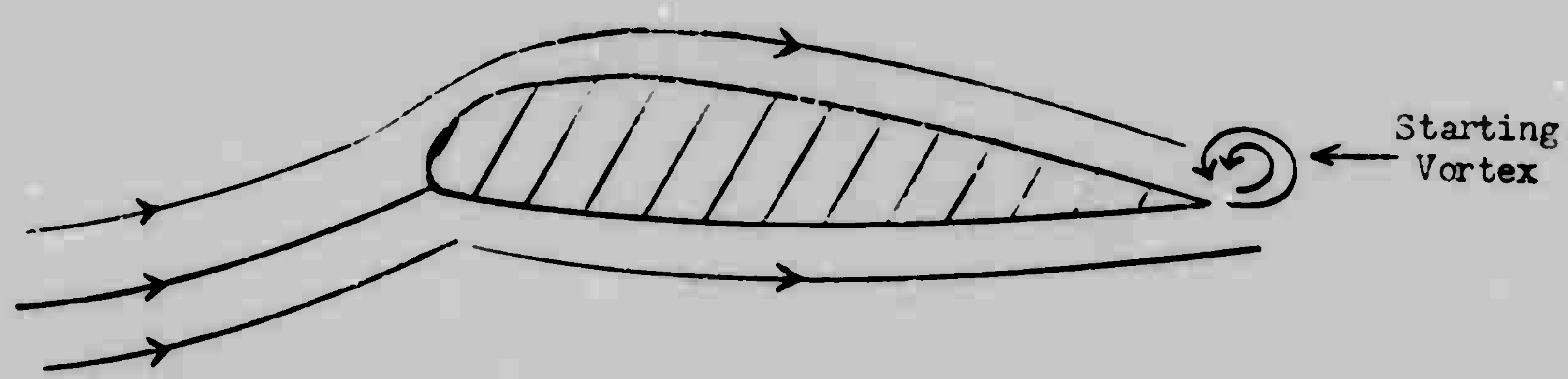


Figure 2.4.3.11

2.4.3.4 FLOW ABOUT TWO-DIMENSIONAL WINGS:

This section considers wings of infinite span or a wing panel mounted from wall to wall in a wind tunnel. Thus the flow pattern about any airfoil section of the wing will be identical to the flow pattern about any similar airfoil section of the wing. All the streamlines move in planes perpendicular to the span, i.e., there is no spanwise flow at any point on the wing.

Flow Pattern About an Airfoil Section

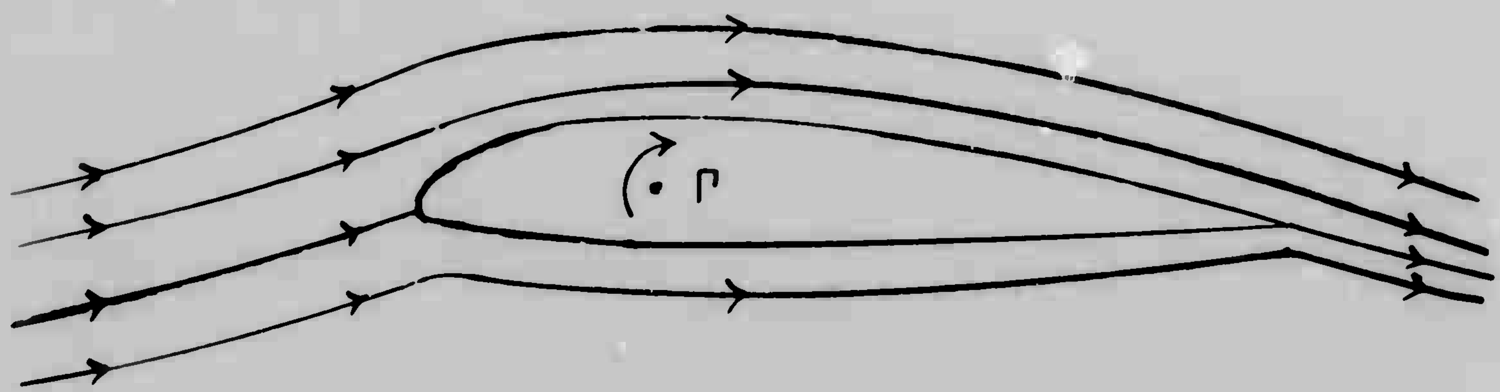


Figure 2.4.3.12

Fig. 2.4.3.12 shows an airfoil of a two-dimensional wing with the flow pattern as obtained by combining circulation and uniform linear flow. Note that, due to the circulation, the resultant velocity ahead of the center of circulation has an upward component, which is called upwash. To the rear of the center of circulation the resultant velocity has a downward component which is called the downwash. As the horizontal distance from the center of circulation increases, the vertical component of the resultant velocity decreases until it is essentially zero for the flow at great distances forward and aft of the airfoil. Therefore, there is no net change in the flow pattern from a great distance ahead of the airfoil to a great distance behind the airfoil. Therefore, a two-dimensional wing has no net downwash velocity

remaining in the air far behind the wing. This condition gives no induced drag for the airfoil as will be shown later.

The upwash ahead of a wing is considered in determining the relative wind approaching a tractor type propeller. This information is necessary in determining asymmetric loads on the propeller for structural reasons.

2.4.3.5 FLOW ABOUT WINGS WITH TIPS:

Before discussing the flow pattern about wings with tips we must cover some vortex laws. In order to have pure circulation flow without the presence of a body, we must have an infinite velocity at the center of circulation. This cannot happen, so near the center of circulation the fluid particles rotate as a solid body called the nucleus. Beyond the nucleus the flow is pure circulation. The velocity profile of this combination of flow is shown in Fig. 2.4.3.13. This combination of flow is called a vortex or eddy. If the circulation flow is about a body such as a cylinder or airfoil, the nucleus of the vortex is within the body so it would be non-existant.

Velocity Profile of a Vortex

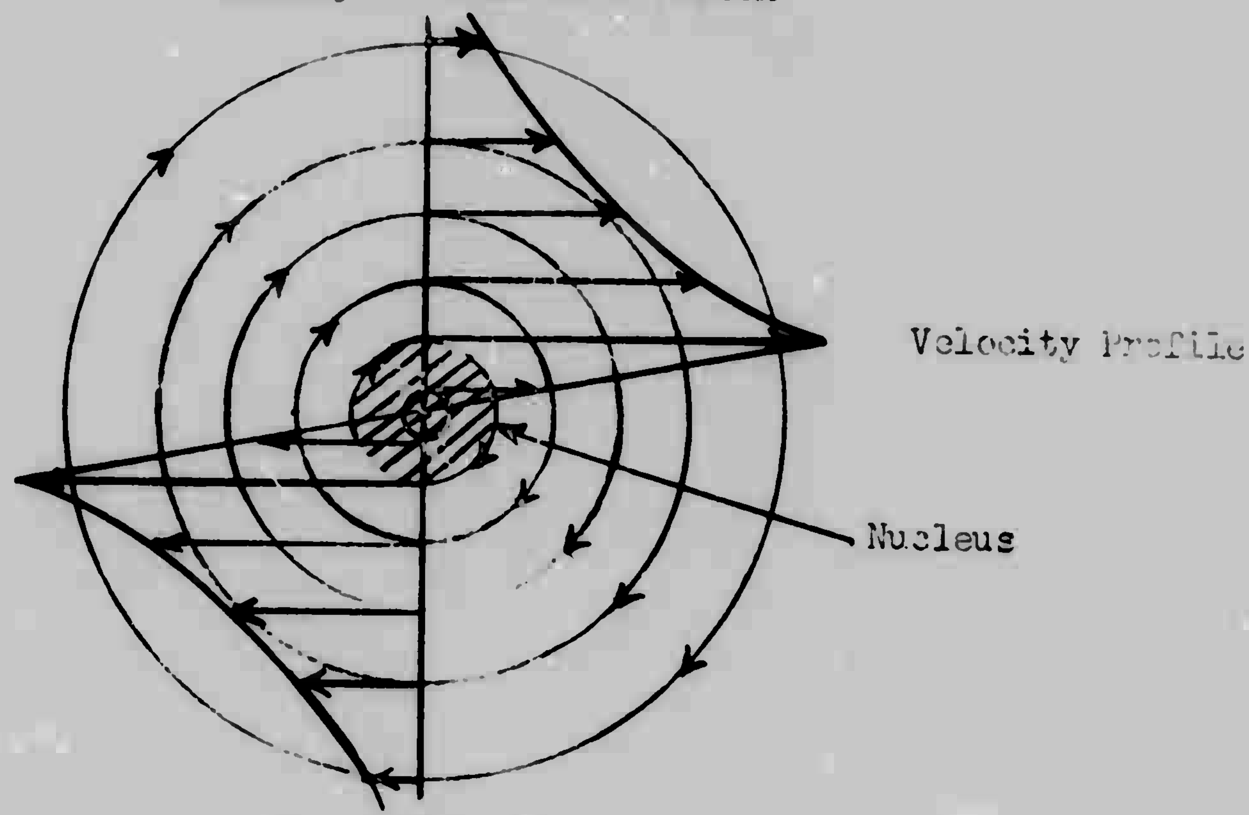


Figure 2.4.3.13

Since a wing causes circulation, the wing may be replaced by a vortex extending from tip to tip. The line defining the center of this vortex replacing the wing is called the lifting line or bound vortex. In general, the line defining the center of a vortex is called the vortex filament.

Two of the vortex laws are of significance at this point.

1. A vortex filament cannot have an end but must either continue to infinity or form a closed path.

2. The strength of a vortex filament is constant along its length.

A smoke ring is an example of a vortex filament which forms a closed path. In a wing with tips, the bound vortex cannot stop abruptly at the tips but must form a closed path or extend to infinity. It could not continue in the same direction beyond the wing tips since it would still be perpendicular to the uniform linear flow and still produce lift which obviously doesn't occur beyond the wing tips. The only logical way for the vortex filament to continue to infinity is to bend perpendicularly back at the tips towards and beyond the trailing edge of the wing as shown in Figure 2.4.3.14. These vortices extending back from the wing tips are called trailing vortices. These trailing vortices produce a downwash in the flow behind the wing in addition to the downwash caused by the bound vortex. An example of the downwash caused by the trailing vortices off the tips of a rectangular wing is shown in Figure 2.4.3.15. Note that these trailing vortices extend back to infinity so that there will still be a net downwash behind the wing with tips even at distances from the trailing edge where the downwash caused by the bound vortex is negligible. This is not true for wings without tips as discussed in the previous section.

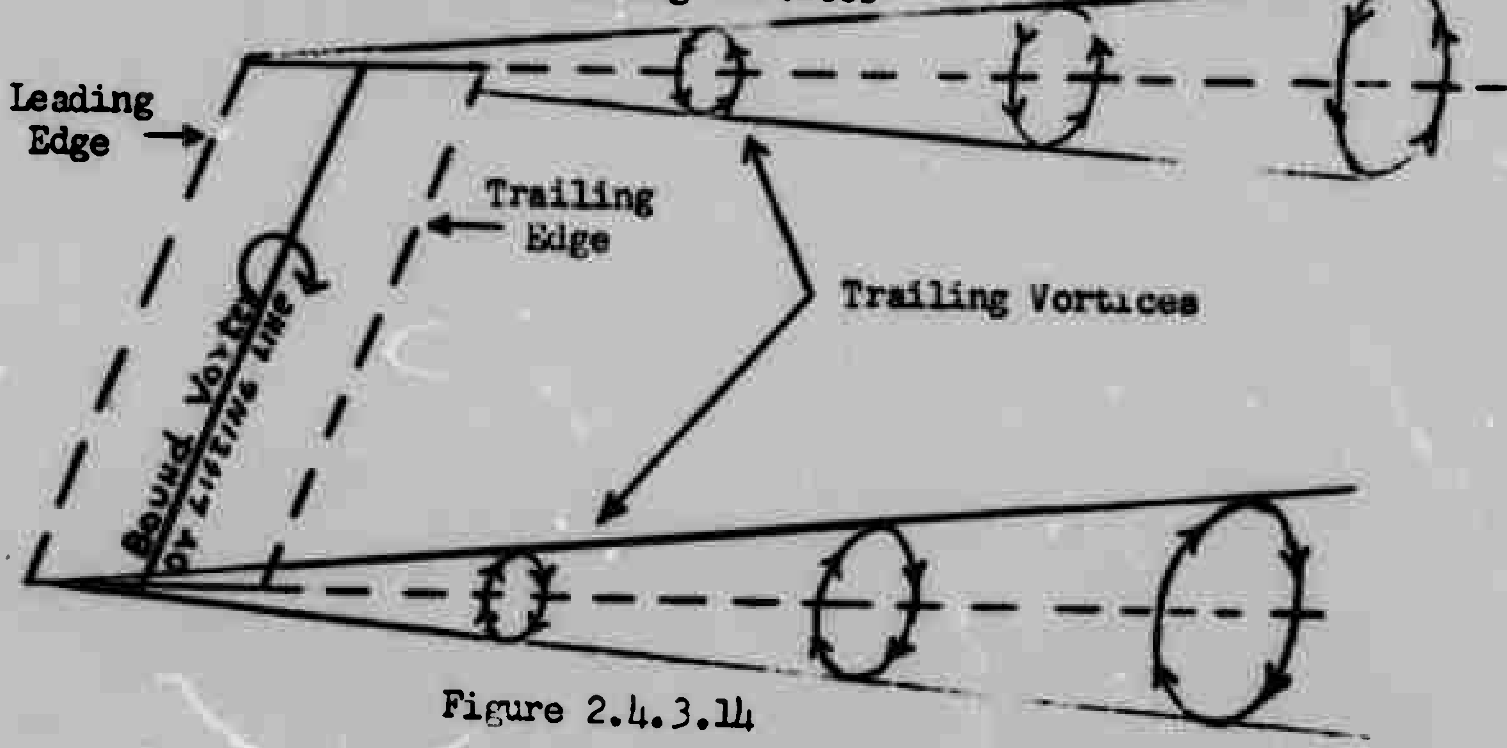


Figure 2.4.3.14

Spanwise Downwash of Rectangular Wing

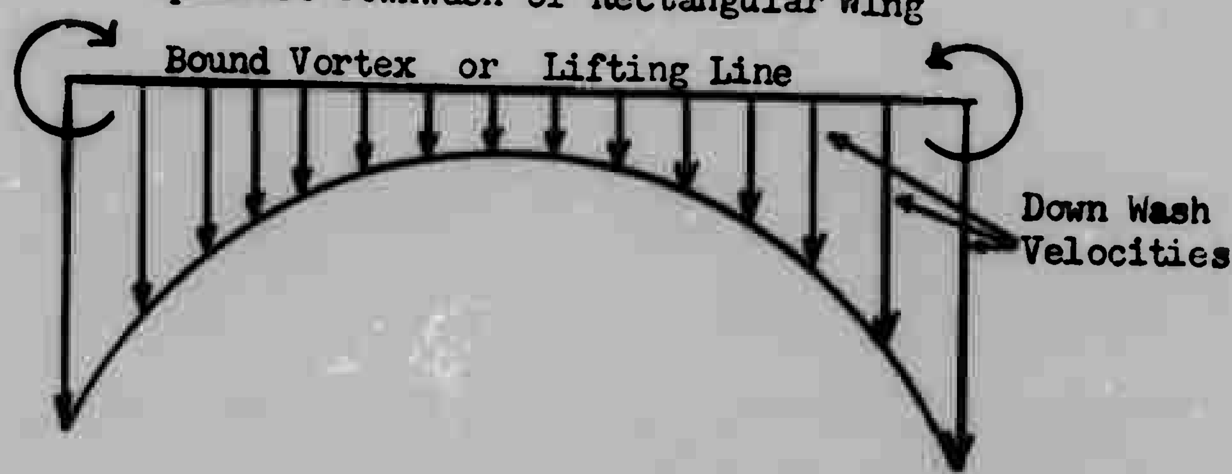


Figure 2.4.3.15

In air with viscosity, the trailing vortices eventually damp out and do not extend to infinity but they remain for a considerable distance behind the wing as is evident from the quote from NASA TN No. 3377: "Experience to date indicates that the exhaust blast from jet airplanes dissipates quickly at

about 300 ft. with minor air disturbances to following aircraft; but the vortex wake turbulence may persist for several minutes in a very powerful form." For an airplane flying 8 miles per minute, this statement indicates that the trailing vortices still have considerable strength 16 miles or more behind the airplane.

Up to now we have considered rectangular wings so only one bound vortex of constant strength is needed to replace the wing. If the wing were tapered or of any planform other than a rectangular planform, each unit span section of the wing would not produce the same lift. Therefore, the circulation must be greater for some sections of the wing than other sections. Since one vortex filament must have a constant strength along its entire length, we must consider many vortex filaments arranged similar to that shown in Fig. 2.4.3.16 to obtain the desired lift distribution along the span. For a tapered wing, or one where the lift contribution of each small section of the wing gradually decreases as the tips are approached, the wing may be considered to be replaced with an infinite number of weak vortex filaments. The trailing vortices of all these filaments would extend off the trailing edge continuously from tip to tip and form a sheet of vortices called the trailing vortex sheet. Each of the trailing vortices would contribute to the overall downwash behind the wing thus tending to give a more uniform downwash velocity from tip to tip as shown in Fig. 2.4.3.17.

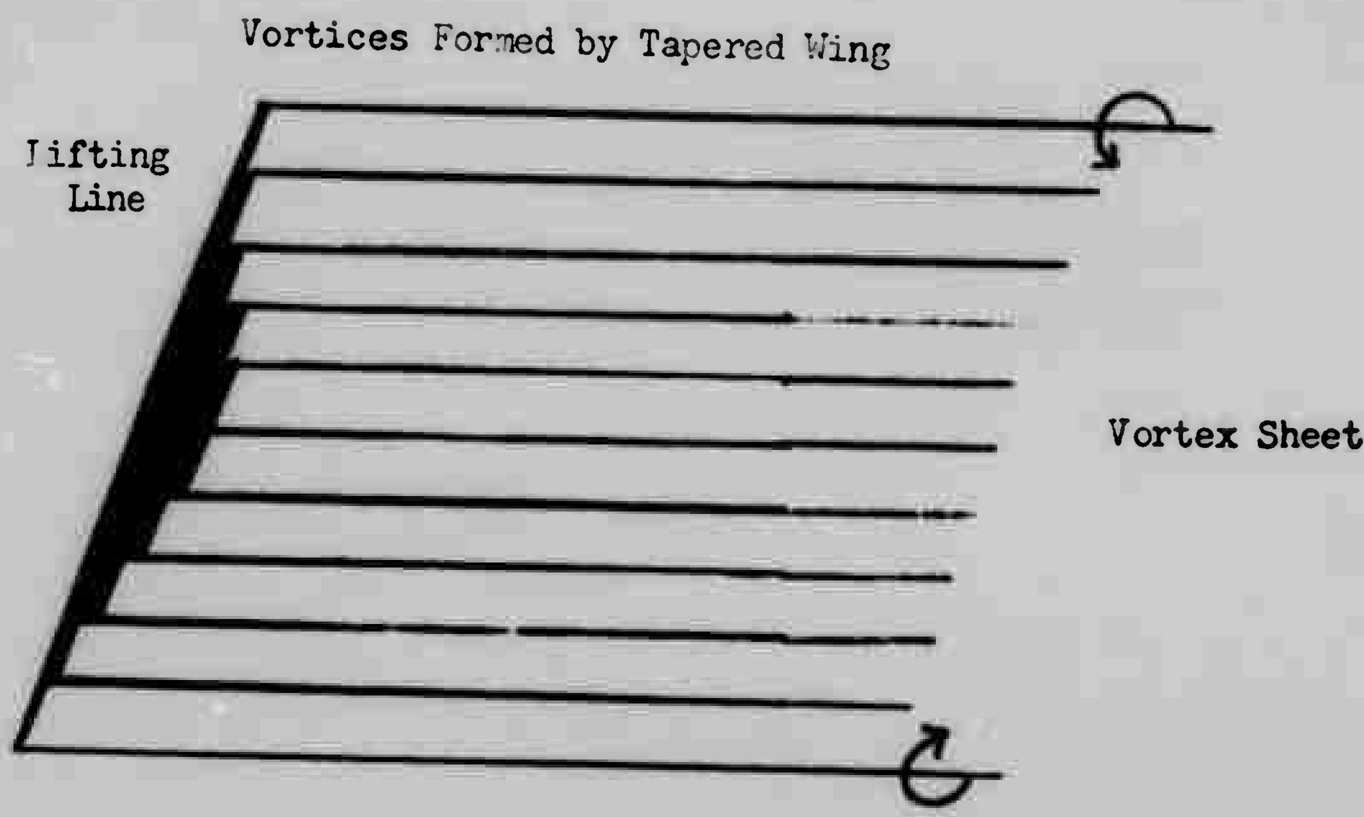


Figure 2.4.3.16

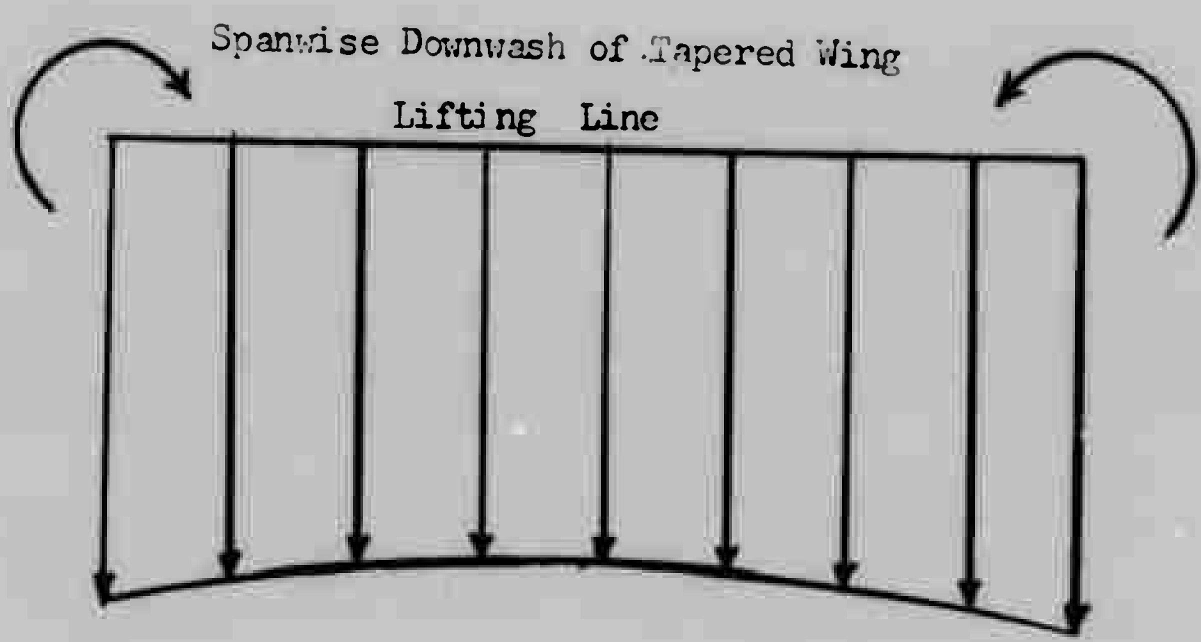


Figure 2.4.3.17

2.4.3.6 INDUCED DRAG ABOUT WINGS WITH UNIFORM DOWNWASH DISTRIBUTION:

If the planform, were properly determined, the wing would have a uniform downwash velocity distribution caused by the trailing vortices. It has been proved by Prandtl that a wing with an elliptical spanwise lift distribution will result in a uniform downwash distribution along the span. Fig. 2.4.3.18 shows an elliptical lift distribution. An elliptical planform wing, similar to the Spitfire wing, will give an elliptical lift distribution provided the wing were constructed from the same airfoil section and had no twist.

Elliptical Spanwise Lift Distribution

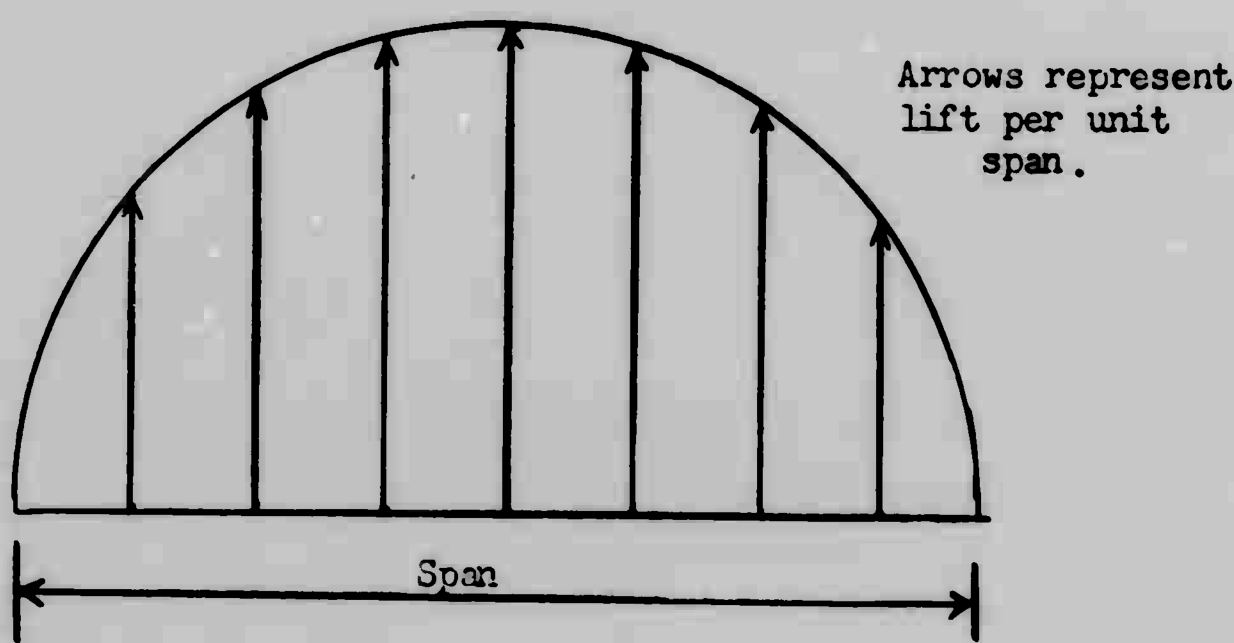


Figure 2.4.3.18

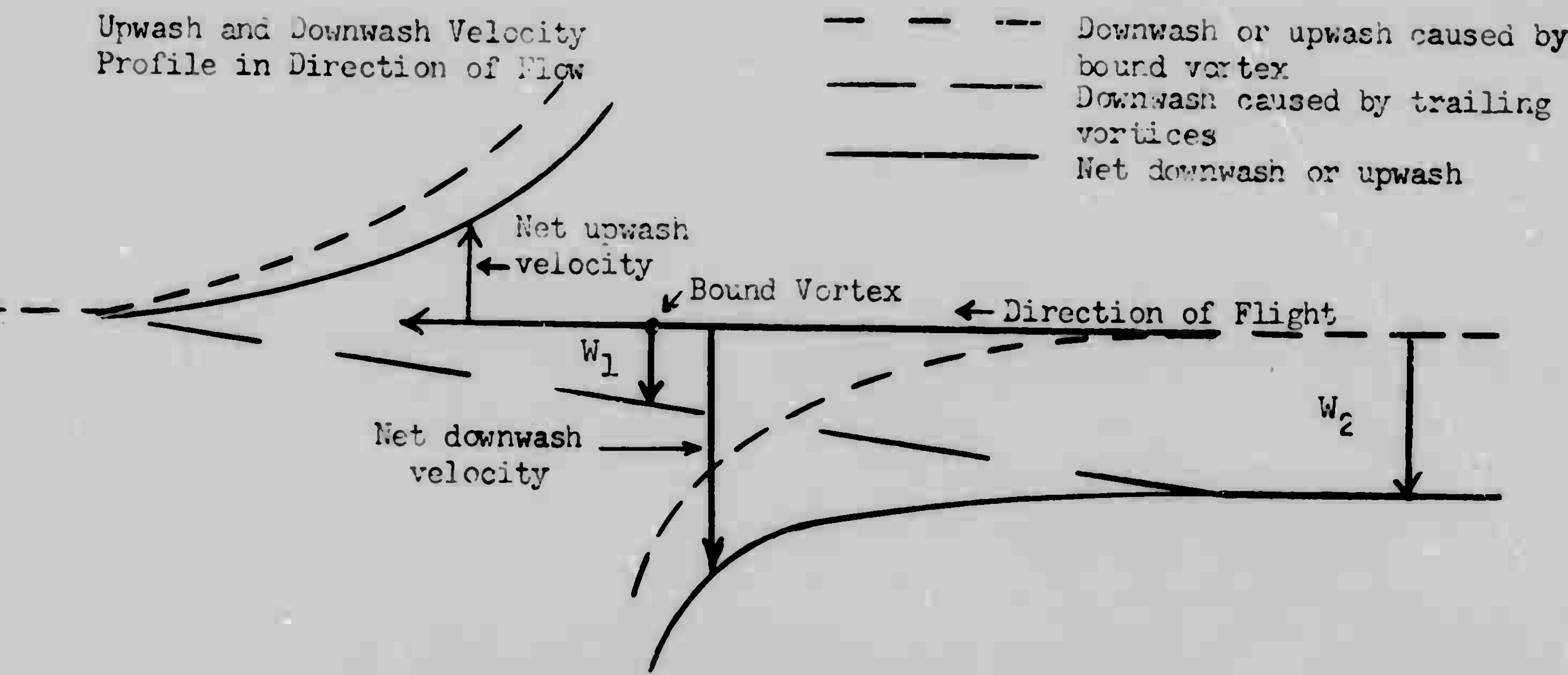


Figure 2.4.3.19

Fig. 2.4.3.19 shows the velocity profile of the upwash caused by the bound vortex, the downwash caused by the trailing vortex and the net upwash and downwash caused by both effects. Note that at a large distance behind the wing, the only downwash is that caused by the trailing vortices.

It should be noted that the trailing vortices extend rearward from the bound vortex but there is a downwash component from the trailing vortices forward of the bound vortex. This situation is analogous to the velocity increase of the air passing thru a propeller where half of the velocity increase occurs before the propeller and the other half occurs behind the propeller. This fact is easily proven by the momentum theory for a propeller. The same condition is true for a wing with tips and the proof is similar to that used for the propeller. Half of the downwash velocity caused by the trailing vortices occurs before the bound vortex and other half occurs behind the wing (W_2). Remember the velocities W_1 and W_2 would be zero if the wing had an infinite span so that no trailing vortices existed.

The local velocity of the bound vortex (which is the center of lift) is the vector sum of the free stream velocity (V_o) and the downwash velocity at the bound vortex (W_1). The angle between the free stream velocity (V_o) and the resultant velocity is the induced angle (α_i) as shown in Fig. 2.4.3.20.

Induced Angle of Attack
and Induced Drag

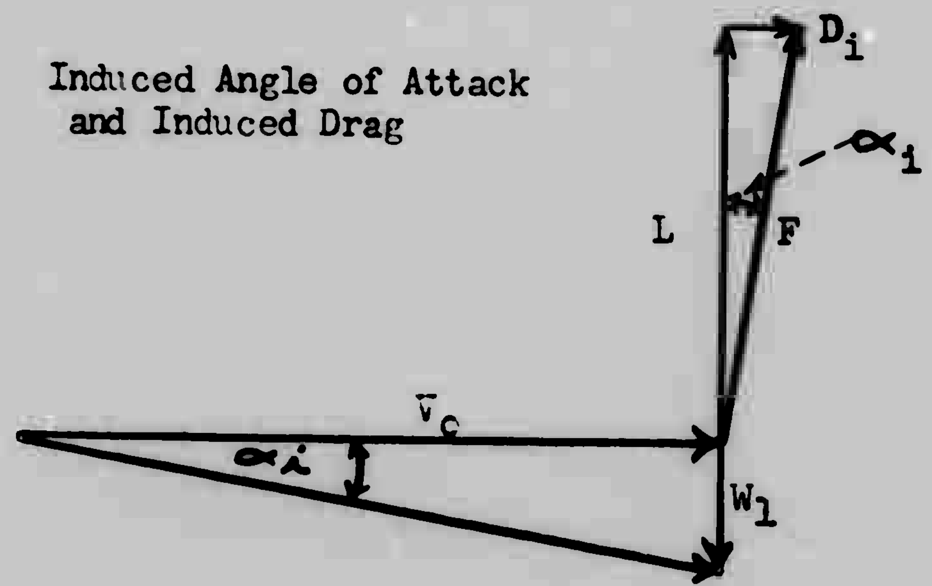


Figure 2.4.3.20

Since the force produced by circulation and uniform linear flow combination is at right angles to the local linear flow at the center of circulation, the force produced by the airfoil with tips is at right angles to the local linear flow at the vortex filament or center of pressure. This force is not perpendicular to the free stream velocity (V_o) but bends back giving a rearward component in the direction of V_o as shown in Fig. 2.4.3.20. The component of the force parallel to V_o is called the induced drag, D_i.

Note that we are still considering flow with zero viscosity so there is no parasite drag. We come up with a drag force known as induced drag which

is caused by the fact that the local linear velocity at the center of circulation is not the same as the free stream velocity. The downwash velocity at the center of circulation causes this difference. The downwash velocity is a result of the trailing vortices which are necessary because the wing has tips. Therefore, the induced drag is the penalty we pay for having a wing with tips. With reference to Fig. 2.4.3.20, the following relationships are apparent:

$$\tan \alpha_i = \frac{W_1}{V_o}$$

$$\sin \alpha_i = \frac{D_i}{F}$$

$$\cos \alpha_i = \frac{L}{F}$$

For small angles of α_i:

$$\tan \alpha_i = \sin \alpha_i = \alpha_i \text{ (in radians)}$$

$$\cos \alpha_i = 1.0$$

Therefore:

$$\alpha_i = \frac{D_i}{F}$$

$$D_i = F \alpha_i$$

$$1.0 = \frac{L}{F}$$

$$L = F$$

Now we will find an expression for lift by considering the momentum change in the air which flows by the wing.

From Newton's second law:

$$F = \frac{d(m\,V)}{dt} = \text{mass flow x } \Delta V$$

Consider a streamtube with a perpendicular area (A) which contains the air affected by the downwash (W_2).

Mass flow of the air in this streamtube = $\rho A V$

Change in velocity of this air (ΔV) = $W_2 - 0 = W_2$

$\therefore$ Force = $\rho A V W_2$ = lift

Since $W_2 = 2 W_1$

Lift = $2 \rho A V W_1$

Now consider the lift of a wing with a elliptical spanwise lift distribution (which gives a uniform spanwise downwash velocity). The German scientist Prandtl has proven that this type of lift distribution effects the air the same as the flow in a streamtube of circular cross section with diameter equal to the span having a velocity change of W_2.

Therefore $A = \frac{\pi b^2}{4}$

Where b is the span.

So lift $= 2 \rho \frac{\pi b^2}{4} \cdot V W_1 = \frac{\rho}{2} \pi b^2 V W_1$

$$\therefore W_i = \frac{2L}{\pi \rho b^2 V}$$

From before $\alpha_i = \frac{W_1}{V}$

$$\alpha i = \frac{2L}{\pi \rho b^2 V^2} = \frac{L}{\pi q b^2}$$ Equation 2.4.3.8

Since

$$L = C_L\, q\, S$$

$$\alpha_i = \frac{C_L\, S}{\pi b^2}$$

Since Aspect Ratio (AR) $= \dfrac{b^2}{S}$

$$\alpha_i = \frac{C_L}{\pi\, AR}$$

From before

$$\alpha_i = \frac{D_i}{F} = \frac{D_i}{L}$$

$$\therefore\ D_i = L\alpha_i = C_L\, q\, S.\ \frac{C_L}{\pi\, AR}$$

$$D_i = \frac{C_L^2}{\pi\, AR}\, q\, S$$

$$C_{D_i} = \frac{D_i}{q\, S}$$

$$\therefore\ C_{D_i} = \frac{C_L^2}{\pi\, AR} \qquad \text{Equation 2.4.3.9}$$

Remember that this equation was derived for a wing with a uniform downwash velocity at all points along the span. This condition will occur for a wing with an elliptical planform with no twist and using the same airfoil section throughout the span. Other planforms such as a tapered wing with proper taper will approach this condition. The equation as derived is the ideal induced drag coefficient or the lowest value of induced drag coefficient one can hope to attain.

2.4.3.7 OSWALD'S EFFICIENCY FACTOR:

Since most wings do not have an elliptical lift distribution, their induced drag will be greater than that found by the equation:

$$D_i = \frac{C_L^2}{\pi AR} \cdot q\,S$$

To account for this difference, an efficiency factor is used called the Oswald's efficiency factor (e) or airplane efficiency factor. This factor which is usually slightly less than unity is included in the denominator of the induced drag coefficient equation.

Therefore: $$C_{D_i} = \frac{C_L^2}{\pi\,AR\,e} \qquad \text{Equation 2.4.3.10}$$

For an ideal elliptical wing the Oswald's efficiency factor is 1.0. The quantity $AR\,e$ is called the effective aspect ratio.

Wings with straight leading and trailing edges are much easier to construct than an elliptical planform wing so tapered wings are primarily used on airplanes. Fortunately the tapered wing with a proper taper ratio has an efficiency factor close to 1.0. There are two common ways of increasing the efficiency factor of a given taper wing. One way is to change the airfoil section along the span to provide a spanwise lift distribution which more closely approximates an elliptical lift distribution. This may be done by gradually changing a characteristic of a given airfoil along the span, such as the thickness ratio or camber. The second way is to twist the wing gradually along the span so that the tip airfoil incidence will be different from the root airfoil incidence.

To review, there are five possible ways to obtain a high Oswald's efficiency factor.

1. Design an elliptical planform wing.
2. Twist the wing.

3. Change airfoil sections along the span.

4. Design a tapered wing with points 2 & 3 above.

5. Design a wing with proper fences or tip tanks.

It is often convenient to calculate the induced drag from the equation derived below:

$$D_i = C_{D_i} \; q \; S$$

$$D_i = \frac{C_L^2}{\pi \, AR \, e} \; q \; S$$

$$D_i = \frac{C_L^2 \; S^2 \; q}{\pi \, b^2 \; e} \times \frac{q}{q}$$

$$D_i = \frac{L^2}{\pi \, q \, e \, b^2}$$

$$D_i = \frac{(L/b)^2}{\pi \, q \, e} = \frac{(n\,W/b)^2}{\pi \, q \, e} \qquad \text{Equation 2.4.3.11}$$

The Oswald's efficiency factor of a wing may be found from flight test once the drag polar (the variation of total drag coefficient with lift coefficient) is known. This method assumes that the parasite drag coefficient remains constant and there is no drag due to compressibility. The following relationship for total drag coefficient applies for lift coefficients which are reasonably below $C_{L\,max}$, such as from $C_L = 0$ to $C_L = 1.0$.

$$C_D = C_{D_p} + \frac{C_L^2}{\pi \, AR \, e}$$

A plot of C_D vs C_L^2 will be a straight line if our assumptions hold.

The equation for a straight line is:

$$y = y_o + mx$$

where y_o is the y-axis intercept and m is the slope of the line.

In the plot of C_D vs C_L^2, C_{D_P} is the y-axis intercept and $\frac{1}{\pi AR e}$ is the slope of the line. By measuring the value of the slope from the plot and knowing the aspect ratio, the Oswald's efficiency factor may be found.

$$\therefore e = \frac{1}{\pi AR m} \qquad \text{Equation 2.4.3.12}$$

The Oswald's efficiency factor of an airplane with properly located tip tanks or other end plate effect will be higher than for the basic airplane. These devices may allow the Oswald's efficiency factor to be greater than 1.0, and may therefore increase the effective aspect ratio of the wing. Therefore, induced drag and, the Oswald's efficiency factor have a noticeable effect on the performance of airplanes flying at low speeds or high altitudes.. Sailplanes, which are generally flown at speeds just above the stall speed to obtain the minimum rate of descent, have high aspect ratio wings to reduce the induced drag. On some airplanes a high aspect ratio wing has disadvantages which are more important than the advantage of reduction in induced drag. A high aspect ratio wing presents a structural problem, and thus requires more weight for a given wing area. Also a high aspect ratio wing will have a short chord so a wing of given thickness ratio has a lower physical thickness. This may not allow sufficient space to retract the landing gear into the wing or to provide adequate fuel cells in the wing. Airplanes designed to operate at

high speeds do not have much induced drag at these speeds,so a small reduction in induced drag will not appreciably effect the performance. In fact,the lower the aspect ratio the lower will be the drag due to compressibility on airplanes intended to operate at transonic and supersonic speeds.

The induced drag, never the less, still is important in take-off and landing performance, cruise and climb performance and maneuverability, especially at high altitude.

SECTION III

PERFORMANCE

3.1. PERFORMANCE EQUATIONS

3.1.1 INTRODUCTION

The following section will show the origin of the basic performance equations which apply to steady flight. The derivations will use the following assumptions:

1. All the forces acting on the airplane act through the center of gravity.

2. The thrust force is parallel to the direction of flight.

3. The mass or weight of the airplane is constant.

4. The aerodynamic forces are in the plane of symmetry of the airplane; that is, the airplane is in coordinated flight.

We will derive the performance equations for straight and level flight, gliding and climbing flight and level turning flight. These equations are basis for estimating aircraft performance and for reduction of flight test data to standard conditions.

A review of Newton's three laws of motion is appropriate at this time. They are:

1. A body at rest remains at rest, and a body in motion continues to move at a constant speed along a straight line, unless the body is acted upon in either case by an unbalanced force.

2. The time rate of change of momentum is proportional to the impressed force. This law is stated in equation form as F (force) $= \frac{d}{dt}(m V)$ where m V is momentum. With the mass held constant this equation becomes the familiar expression of Newton's second law which is $F = m \frac{dV}{dt} = m A$.

3. For every force or action there is an equal and opposite reaction.

We will make use of the first two of Newton's laws in the following derivations.

3.1.2 STRAIGHT AND LEVEL FLIGHT

According to Newton's first law, if an airplane is flying straight and level at a constant speed all the forces must balance out so the net force is zero. The forces acting on an airplane are lift, drag, thrust and weight. Lift force is the component of the aerodynamic forces perpendicular to the relative wind. The drag force is the component of the aerodynamic forces parallel to the relative wind. The thrust force is produced by the airplane propulsive unit (propeller, turbo-jet, rocket) and is assumed parallel to the relative wind. The weight force acts towards the center of the earth.

Level Flight

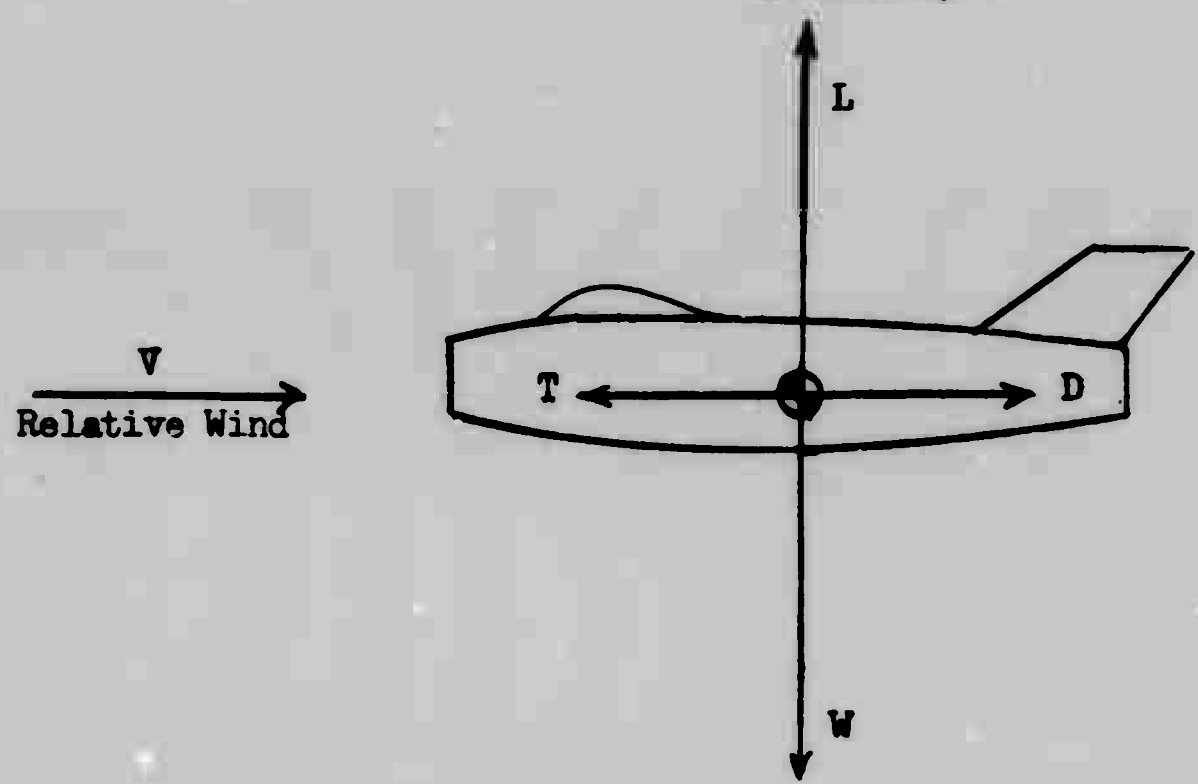


Figure 3.1.1

In this case the four forces lie along only two directions, the horizontal and vertical. The lift opposes the weight and the thrust opposes the drag. Refer to Fig 3.1.1. Therefore, we easily obtain the following equations:

$$L - W = 0$$

$$\therefore L = W$$

$$T - D = 0$$

$$\therefore T = D$$

These equations apply only to straight and level, unaccelerated flight.

If the thrust is increased, there will be an unbalanced force in the direction of flight so the airplane will accelerate. The amount of acceleration is determined by Newton's second law, $F = m\,a$. The unbalanced force is (T-D) so we have:

$$T - D = m\,a = \frac{W}{g}\,a$$

The acceleration is therefore: $a = \frac{(T-D)\,g}{W}$ Equation 3.1.1

The units of acceleration of the airplane (a) are the same as the units of the acceleration of gravity (g). The term (T-D) is called excess thrust and is often denoted as T_{ex}.

This equation applies as long as the airplane is flying level. The thrust and drag may change as the speed changes so the acceleration will be different at different speeds. The speed for maximum acceleration is the speed where the net force, (T-D), or excess thrust is the maximum. At some speed the drag will equal the maximum thrust output of the engine so the airplane will no longer accelerate. This is obviously the maximum level flight speed of the airplane.

Note that, if the flight is straight and level, the lift must be equal to the weight at all speeds. In order to fly straight and level the pilot adjusts the airplane attitude such that it is producing just enough lift to oppose the weight force.

3.1.3 GLIDING FLIGHT

Consider an airplane in a glide holding a constant true airspeed with power off. Since the thrust is zero there are only three forces acting on the airplane; lift, drag and weight. In a steady straight glide these forces must cancel so there is no net force acting on the airplane. This condition is shown in Fig 3.1.2. The angle is the angle of climb measured between the horizon and the flight path. The angle will have a positive sign when the airplane is climbing and a negative sign when the airplane is descending. The weight force can be considered as having two components, one opposing the lift and the other opposing the drag.

Gliding Flight

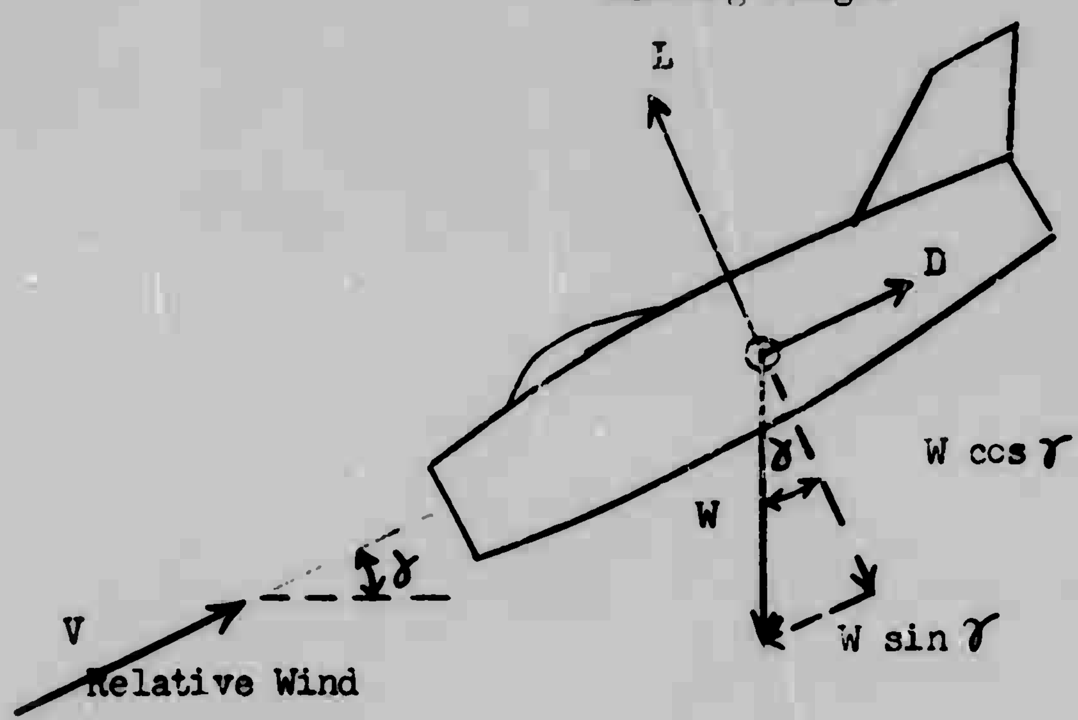


Figure 3.1.2

From Fig. 3.1.2 it is evident that the weight components are W cos γ and W sin γ opposing the lift and drag respectively. This gives us the following equations:

$$L = W \cos \gamma$$

$$D = W \sin \gamma$$

Dividing the above equations we have:

$$\frac{D}{L} = \frac{W \sin \gamma}{W \cos \gamma} = \tan \gamma \qquad \text{Equation 3.1.2}$$

It is then evident that the angle of glide (γ) is determined by the ratio of drag to lift (D/L). The minimum glide angle occurs at a glide speed where D/L is a minimum or L/D is a maximum. In order to glide a maximum distance from a given altitude, the minimum glide angle is desired. The minimum glide angle (or L/D max) is obtained when the airplane glides at a optimum angle of attack. This angle of attack is determined by the drag characteristics of the airplane so an aerodynamically "clean" airplane will glide farther than a "dirty" airplane. Note that a heavy airplane will glide as far as a light airplane providing both are flying at the optimum angle of attack for gliding flight. In order to do this the heavy airplane must indicate a higher speed than the light airplane.so its rate of descent will be higher.

At low angles of glide the lift will nearly equal the weight since the cosine of a small angle is approximately equal to 1.0.

3.1.4 CLIMBING FLIGHT

If an airplane is in a steady climb and maintaining a constant true airspeed all the forces acting on the airplane must cancel. The forces act as shown in Fig 3.1.3a; γ is the angle of climb.

Climbing Flight

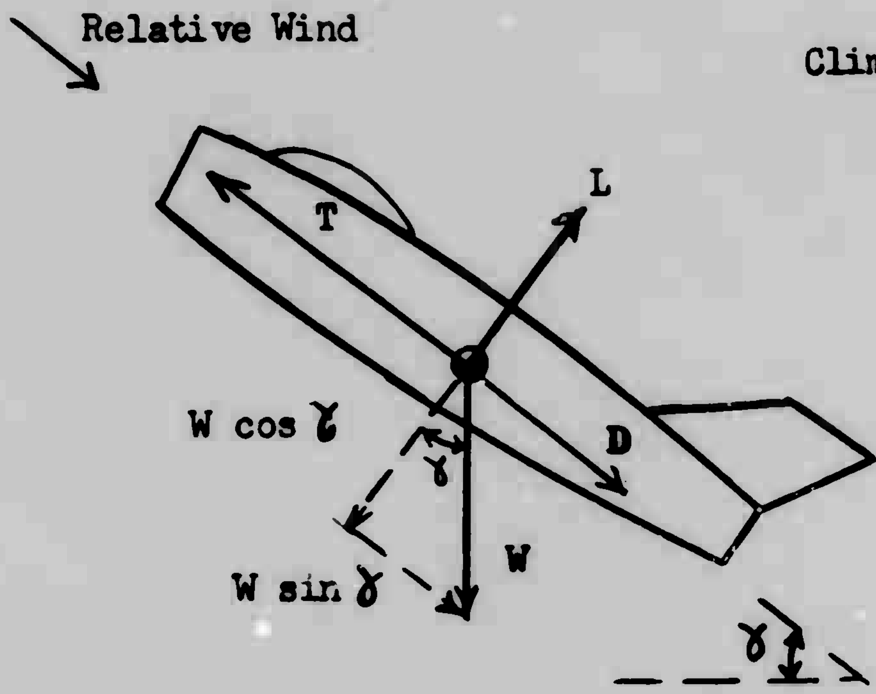


Figure 3.1.3a

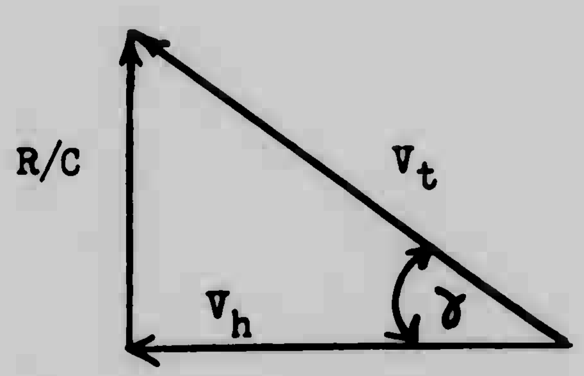


Figure 3.1.3b

If we consider the forces paralled to the direction of flight, we will obtain:

$$T - D - W \sin \gamma = 0$$

$$(T-D) = W \sin \gamma$$

$$\sin \gamma = \frac{(T-D)}{W} \qquad \text{Equation 3.1.3}$$

Therefore, the speed for maximum angle of climb is the speed where the excess thrust (T-D) is a maximum. If the thrust were zero, $\sin \gamma = -\frac{D}{W}$ so the speed for minimum drag would give the minimum angle of descent. This is identical to the results obtained in Section 3.1.3. It can be proven that the speed for minimum drag is the same as the speed for maximum L/D ratio.

From Fig 3.1.3b

$\sin \gamma = \frac{R/C}{V_t}$ where R/C is the rate of climb. Therefore equation 3.1.3 becomes:

$$R/C = \frac{(T-D)\, V_t}{W} \qquad \text{Equation 3.1.4}$$

The units of the rate of climb are the same as the units of the true airspeed when determined from equation 3.1.4; that is, if V_t is in ft/sec, the rate of climb will be in ft/sec. The rate of climb equations with rate of climb expressed in ft/min are:

$$R/C\ (\text{ft/min}) = \frac{60\ V_t\ (T-D)}{W} \qquad (V_t \text{ in ft/sec})$$

$$= \frac{88\ V_t\ (T-D)}{W} \qquad (V_t \text{ in mph})$$

$$= \frac{101.3\ V_t\ (T-D)}{W} \qquad (V_t \text{ in knots})$$

Equation 3.1.3 may be expanded to:

$$R/C = \frac{1}{W}\ (T\ V_t - D\ V_t)$$

It will be shown in a later section that thrust horsepower available $(THP_a) = \frac{T\ V_t}{550}$ and thrust horsepower required $(THP_r) = \frac{D\ V_t}{550}$ when V_t is in ft/sec.

Therefore $R/C = \frac{1}{W}\ (THP_a \times 550 - THP_r \times 550)$

$$R/C\ (\text{ft/sec}) = \frac{(THP_a - THP_r)\ 550}{W}$$

or $$R/C\ (\text{ft/min}) = \frac{(THP_a - THP_r)\ 33{,}000}{W} \qquad \text{Equation 3.1.5}$$

The difference between the thrust horsepower available and thrust horsepower required is called the excess thrust horsepower and given the symbol of Δ THP or THP_{ex}.

Therefore

$$R/C\ (\text{ft/min}) = \frac{\Delta THP\ 33{,}000}{W} \qquad \text{Equation 3.1.6}$$

Either equation may be used to determine the rate of climb for unaccelerated flight. From equation 3.1.6, it should be noted that the maximum rate of climb will occur at the speed where the maximum excess thrust horsepower (Δ THP) is obtained. This speed is different than the speed for maximum excess thrust (T-D) where maximum level acceleration will occur. At the speed for maximum excess thrust horsepower, the quantity $(T-D) \times V_t$ is a maximum. Therefore, the speed at which the best acceleration occurs is different than the speed for maximum rate of climb. The speed for the maximum angle of climb is the speed for the maximum acceleration.

In case an airplane is climbing or descending and accelerating or decelerating, the above equations for climb and acceleration will not apply. There will be a net or unbalanced force in this case so the value of $T-D-W \sin \gamma$ will not equal zero.

From Newton's second law we have:

$$T-D-W \sin \gamma = m a = \frac{W}{g} a$$

$$T-D = W \sin \gamma + \frac{W}{g} a_c$$

$$T-D = W \left(\frac{R/C_a}{V} + \frac{a_c}{g} \right) \qquad \text{Equation 3.1.7}$$

where R/C_a is rate of climb when accelerating or decelerating, a_c is acceleration with climb or descent.

If an airplane has some excess thrust (T-D) it may be climbed without acceleration or accelerated in level flight or any combination of climb and acceleration. With a combined climb and acceleration, the excess thrust used for the climb is represented by the first term of equation 3.1.7. The remaining excess thrust will provide an acceleration as determined by the second term of equation 3.1.7.

For the combined climb and acceleration condition, we can determine the instantaneous excess thrust if we know the instantaneous rate of climb, acceleration, weight and true velocity. With this excess thrust we can calculate the rate of climb the airplane would have at the same speed if the acceleration were zero. An equation which can be used under this condition is derived below.

Solving for R/C_a from equation 3.1.7 we have:

$$R/C_a = \left(\frac{T-D}{W} - \frac{a_c}{g}\right) V_t$$

$$R/C_a = \frac{T-D}{W} \cdot V_t - \frac{a_c}{g} \cdot V_t$$

$$R/C_a = R/C - \frac{a_c}{g} V_t$$

$$\therefore R/C = R/C_a + \frac{a_c}{g} V_t \qquad \text{Equation 3.1.8}$$

Equation 3.1.8 gives the rate of climb the airplane would have with no acceleration if the combined rate of climb and acceleration are known. Similarly we can find the level flight acceleration the airplane would have when the combined rate of climb and acceleration are known.

Solving for a_c we have:

$$a_c = \left(\frac{T-D}{W} - \frac{R/C_a}{V_t}\right) g$$

$$a_c = a - \frac{R/C_a}{V_t} \cdot g$$

$$a = a_c + \frac{R/C_a}{V_t} \cdot g \qquad \text{Equation 3.1.9}$$

Equation 3.1.8 is used to determine the level flight acceleration potential of an airplane if the instantaneous rate of climb and acceleration are known. Equations 3.1.8 and 3.1.7 are helpful to understand the corrections which are used in the reduction of the data from the sawtooth climb and level acceleration test. The acceleration correction to the sawtooth climb data may be derived directly from equation 3.1.8.

3.1.5 CONCLUSIONS

At the beginning of this paper four simplifiying assumptions were listed. We will discuss the effect of these assumptions on the results obtained.

The first assumption states that all the forces acting on the airplane act through the center of gravity. Whenever an airplane is in steady flight, the attitude of the airplane remains constant. In order for this to occur, the sum of the moments about the center of gravity must be zero. The wing lift, tail force, thrust and drag may not act through the center gravity but they are of such magnitudes and directions that no unbalanced moment about the center of gravity exists. With a condition of equilibrium the resultant force in each

direction may be considered acting at the center of gravity with the same magnitude with no effect on the resulting equations.

The second assumption states that the thrust force is parallel to the direction of flight. The thrust is seldom exactly parallel to the direction of flight so it does have an effect on the equations developed in this paper. If the angle between the thrust force and direction of flight is small (on the order of 6 to 8° or less) and the thrust to weight ratio or thrust to lift ratio is small (on the order of 1/3 or 1/4), this thrust effect will be relatively unimportant. Most airplanes, except rocket powered airplanes and other very high performance airplanes, are generally flown within the limitations listed above. The performance equations considering this thrust effect may be easily derived.

The third assumption states that the mass or weight of the airplane is constant. Since fuel is being continuously consumed the weight of an airplane changes in flight. This effect is insignificant for most airplanes since the change of weight over a small time interval is very small compared to the total weight of the airplane. Rocket powered airplanes and airplanes using after-burner consume fuel at a very high rate so this assumption cannot always be used to accurately determine the performance of such airplanes. In order to account for the rapid change of weight during flight, and for other reasons, the energy method of performance has been developed. This method will be discussed in detail in Section 3.6.

The fourth assumption is that the airplane is in coordinated flight. We are generally not interested in the performance of an airplane in yawed flight or with large aerodynamic side loads. Therefore, this assumption is appropriate and the equations are valid. One exception of major importance is the determination of performance of a twin or multi-engined aircraft with an engine inoperative. Most of the derived equations will apply for this case but the additional drag due to rudder deflection and the inoperative engine must be considered. Also, the wings may not be level for straight flight. The reasons for this will be discussed in the stability section of the course.

3.2 LEVEL FLIGHT PERFORMANCE

3.2.1 DRAG AND POWER CHARACTERISTICS

3.2.1.1 INTRODUCTION

It is essential that a thorough understanding of the aerodynamic drag characteristics be obtained in order to understand the factors affecting performance of aircraft and the reduction of test flight data. This paper will explain various forms of drag and power required curves and will indicate the methods of obtaining performance data for standard conditions from flight test data obtained during test conditions. Of foremost consideration is the correction of flight test data for non-standard temperatures to obtain standard temperature performance data. With proper methods it is also possible to correct the data for non-standard weight and altitude.

At the USAF Experimental Flight Test Pilot School we are primarily concerned with the corrections necessary to determine standard temperature performance from non-standard temperature flight test data.

The first portion of the section will deal with subsonic drag curves and reduction of data obtained from subsonic airplanes. The latter portion of the section will discuss the theory of data reduction for transonic airplanes.

3.2.1.2 DRAG CURVES

The drag and thrust characteristics of an airplane at all altitudes and speeds will determine the performance of an airplane. This is evident by reviewing performance equations. The equations which determine the performance have the terms thrust and drag included. The level acceleration equation is $a = \frac{(T-D)}{W} g$ and the rate of climb equation is $R/C = \frac{(T-D)\,V_t}{W}$. It has been

explained that for straight and level flight the thrust must equal the drag. To better understand performance of an airplane we must know how the thrust and drag changes with airspeed and altitude.

At subsonic speeds the total drag is the sum of the parasite drag and the induced drag. Subsonic speeds are the speeds of the airplane where the flow around the airplane remains subsonic at all times. The parasite drag increases as the square of the velocity. It may be expressed in equation form as

$$D_p = C_{D_p} \tfrac{1}{2} \rho V_t^{2} S \qquad \text{Equation 3.2.1}$$

where D_p - parasite drag

C_{D_p} - parasite drag coefficient

ρ - density of the air

V_t - true airspeed

S - wing area

The parasite drag coefficient (C_{D_p}) is approximately constant for an airplane of given configuration for all speeds and altitudes. We will consider it is a constant in this discussion.

The induced drag may be expressed as

$$D_i = C_{D_i} \tfrac{1}{2} \rho V_t^{2} S$$

where D_i - induced drag

C_{D_i} - induced drag coefficient

It has been proven that the theoretical value of induced drag coefficient will be

$$C_{D_i} = \frac{C_L^{2}}{\pi AR}$$

where C_L - lift coefficient $= \frac{2L}{\rho V_t^2 S}$

The actual induced drag of an airplane may be larger than the induced drag obtained from theory. The actual induced drag of an airplane may be obtained from wind tunnel or flight tests. A correction factor, e, known as the airplane efficiency factor or Oswald's efficiency factor is commonly used to account for the difference between the actual and theoretical induced drags. This factor, e, is defined as:

$$e = \frac{D_{i\ theo.}}{D_{i\ actual}} = \frac{C_{D_{i}\ theo.}}{C_{D_{i}\ actual}}$$

therefore

$$C_{D_{i}\ actual} = \frac{C_{D_{i}\ theo.}}{e} = \frac{C_L^2}{\pi\ AR\ e}$$

The value of e of an airplane of a given configuration is considered to remain constant throughout its subsonic speed range.

For straight and level flight, the lift is equal to the weight so the equation for lift coefficient may be written as $C_L = \frac{2W}{\rho V_t^2 S}$ (for straight and level flight).

Therefore, as the velocity increases the lift coefficient will decrease. Since the lift coefficient decreases with velocity, so will the induced drag coefficient.

We are more concerned with how the induced drag changes with airspeed. We can develop an equation for induced drag by substituting the equations for C_L and AR where appropriate.

Thus $D_i = C_{D_i} \frac{1}{2} \rho V_t^2 S$

$$D_i = \frac{C_L^2}{\pi AR\, e} \frac{1}{2} \rho V_t^2 S$$

$$D_i = \left(\frac{2W}{\rho V_t^2 S}\right)^2 \times \frac{S}{2\pi b^2 e} \cdot \rho V_t^2 S$$

$$D_i = \frac{2(W/b)^2}{\pi \rho V_t^2 e} \qquad \text{Equation 3.2.2}$$

The term $\frac{1}{2} \rho V_t^2$ is called the dynamic pressure (q). Thus

$$D_i = \frac{(W/b)^2}{\pi q e} \quad \text{(straight and level flight)}$$

It can be seen that for straight and level flight, the induced drag will decrease as speed increases.

The various drags will change with speed according to Figure 3.2.1.

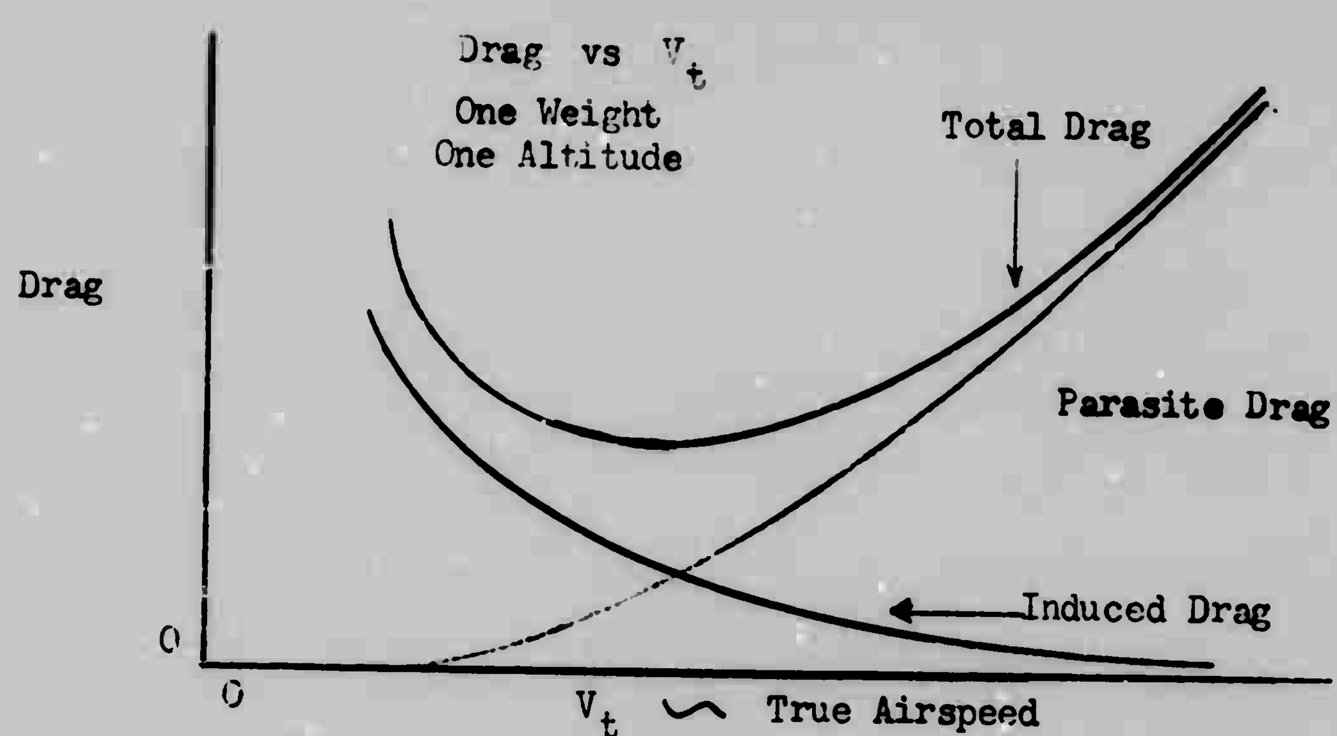


Figure 3.2.1

At the speed for minimum drag, the parasite drag equals the induced drag. This is the speed for best endurance for jets and the best glide speed if minimum glide angle is desired. Note that at lower speed the drag increases as the speed decreases.

If the thrust variation with speed were known we could determine from the plot, the rate of climb and acceleration potential at various speeds besides the maximum level flight speed. Note that at any speed the quantity (T-D) or excess thrust is known so this term may be used in the equations for rate of climb and level accelerations. Figure 3.2.2 presents the thrust variation with speed which would apply for a turbo-jet at full throttle.

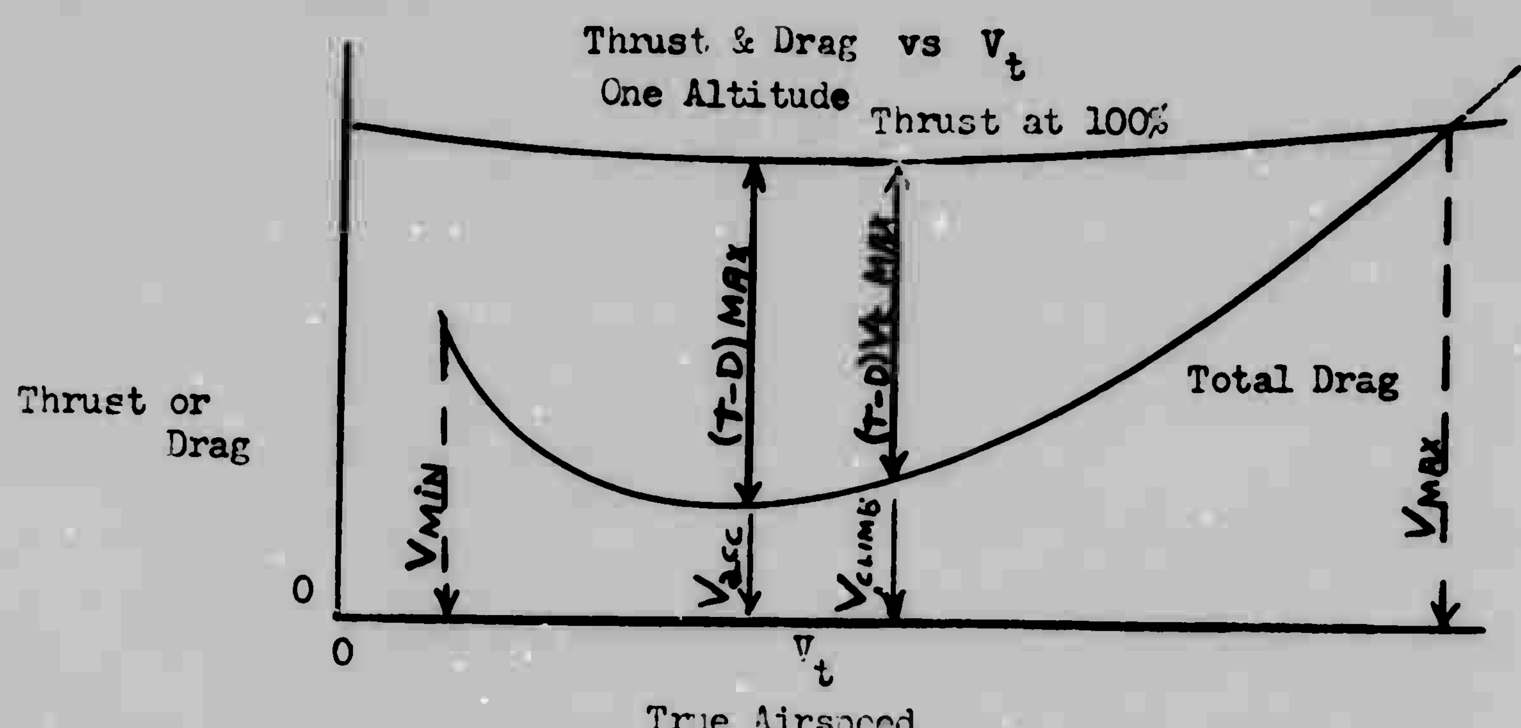


Figure 3.2.2

Note that the maximum level flight speed is the speed where the thrust is equal to the drag. The speed for maximum acceleration is the speed where the excess thrust is a maximum. The speed for maximum rate of climb is the speed where the product of the excess thrust (T-D) and true airspeed is a maximum. Thus, the speed for best rate of climb is a higher speed than the speed for best acceleration.

3.2.1.3 WEIGHT EFFECT:

We have only considered the drag curve for one weight. We will obtain a different drag curve for other weights since the induced drag depends on the weight. The total drag curves for various weights are shown in Figure 3.2.3.

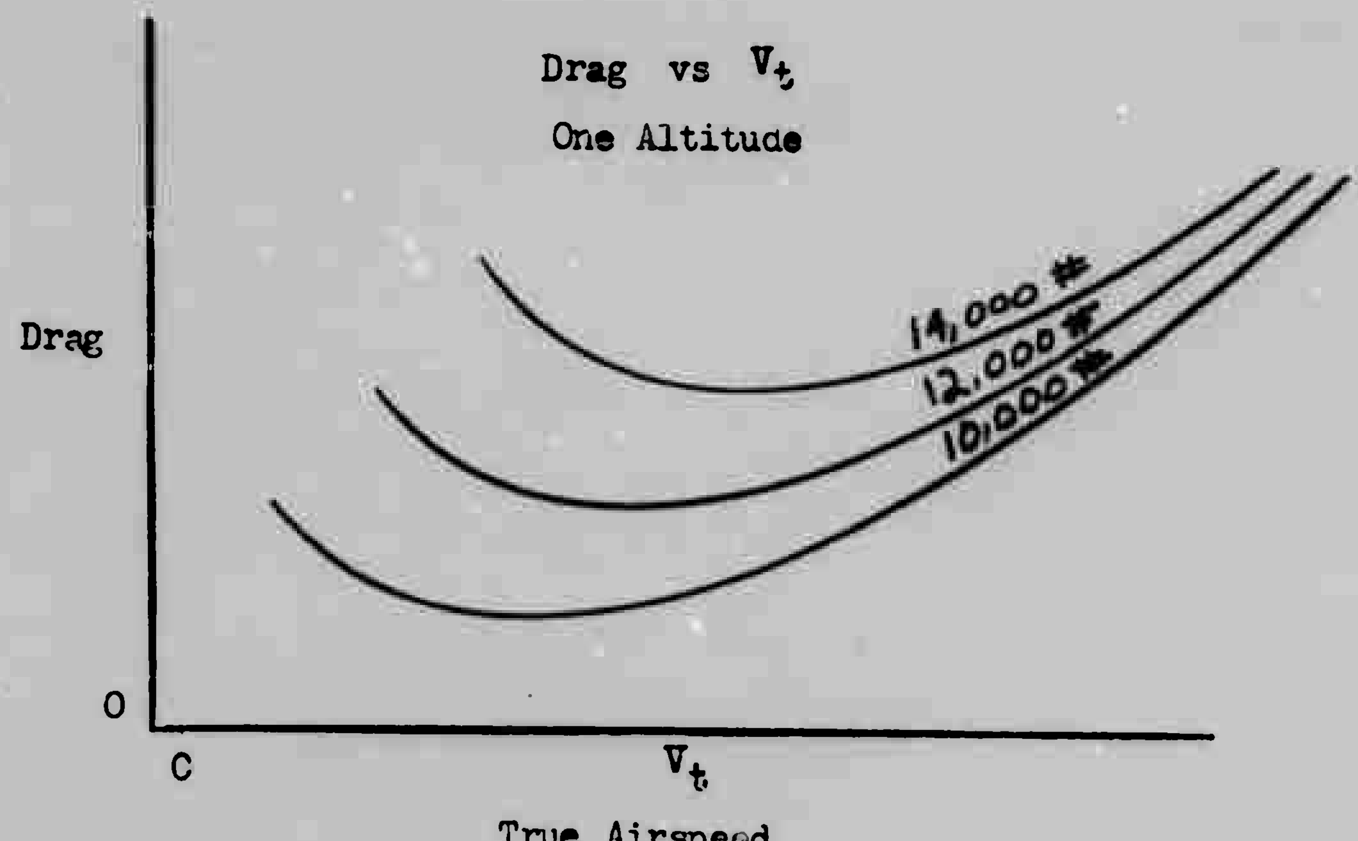
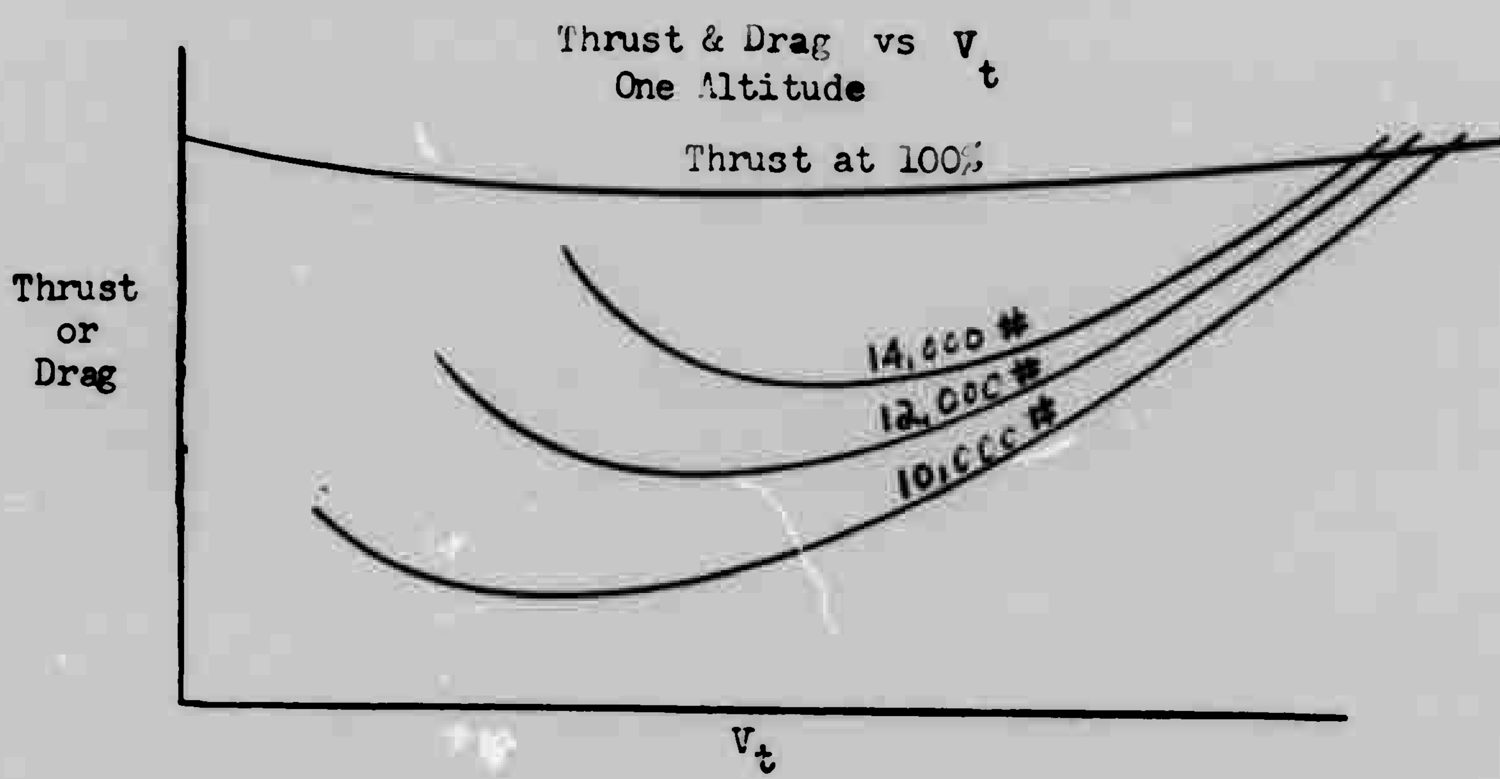


Figure 3.2.3

Note that the weight affects the drag curves more at low speeds than at high speeds, and that the minimum drag speeds increase with a weight increase. Therefore, the speed for best endurance and best glide will change to some extent with changes in weight. If we consider the thrust curve without drag curves as shown in Figure 3.2.4 we will see that our maximum speed will depend on the weight of the airplane.



True Airspeed

Figure 3.2.4

Also, the excess thrust is lower at all speeds for the heavy weight airplane as compared with the light weight. Therefore, the accelerations and rate of climb will be less for the heavy airplane. By looking at the equations for acceleration and rate of climb we see that the weight of the airplane enters in the denominator. This will also reduce the acceleration and rate of climb potential of a heavy airplane as compared to a light air-

plane. Therefore, two factors reduce the acceleration and rate of climb performance of heavy airplanes; namely, decrease in excess thrust and increase in weight. The converse is true when going from a heavy airplane to a light airplane such as when fuel is consumed.

Cruise performance is likewise affected greatly by changes in weight. To review, we will get specific range curves which have the appearance as shown in Figure 3.2.5.

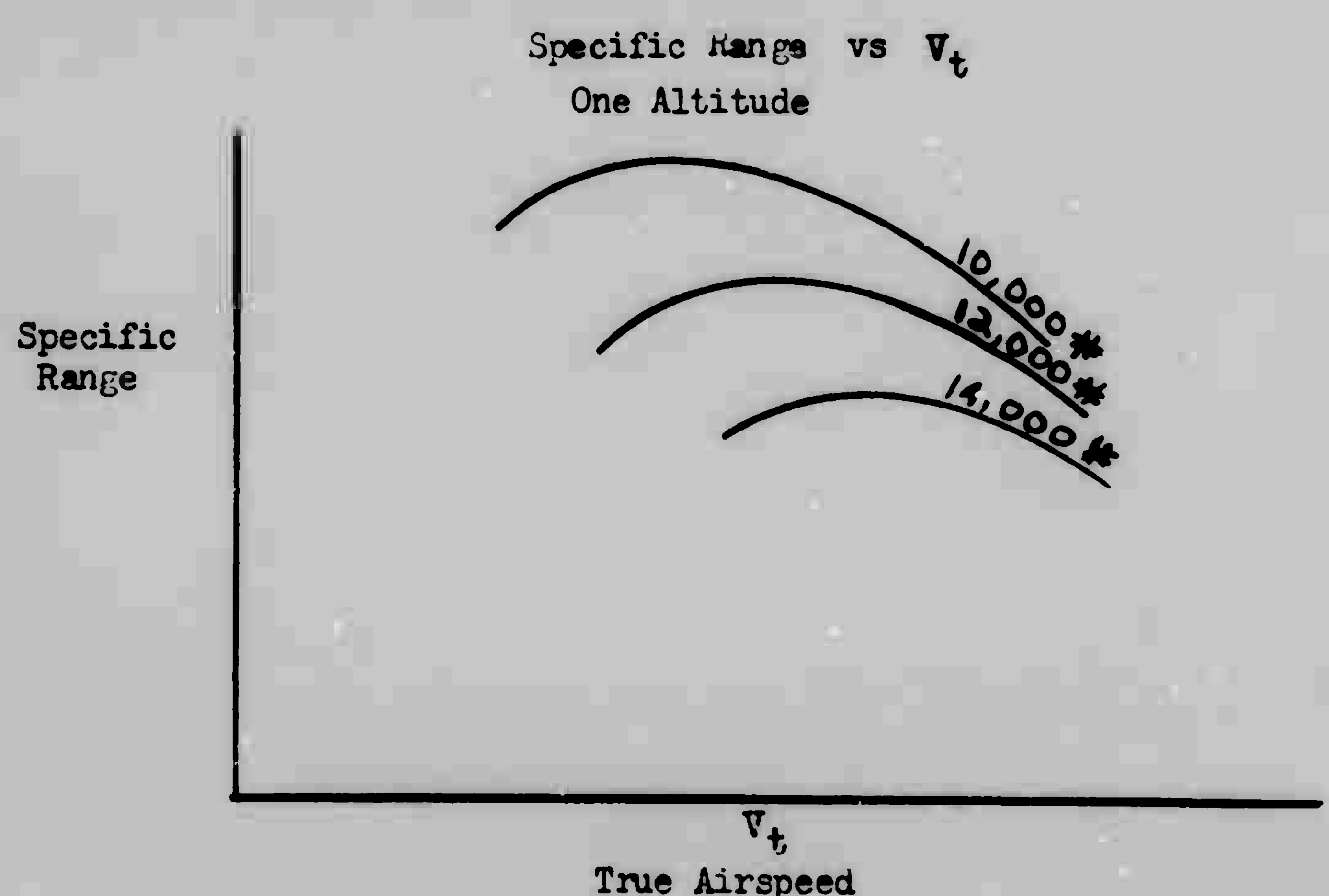


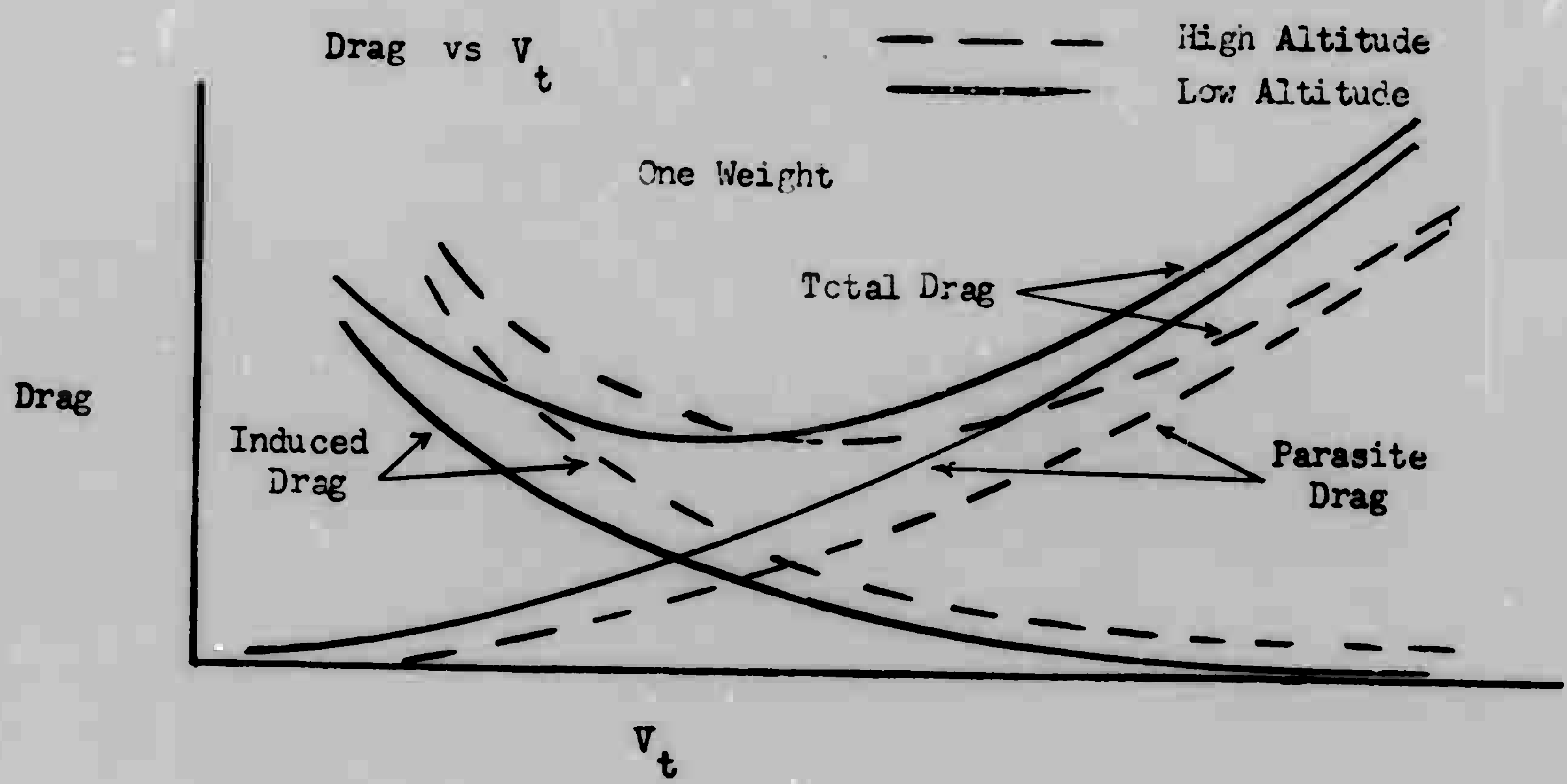
Figure 3.2.5

BLANK PAGE

Note that the best cruise speed decreases as fuel is consumed (or weight decreases). As the airplane loses weight it will tend to increase in speed if the throttle is not adjusted. Therefore, to remain at the same speed the throttle must be retarded, and as the weight lowers the best cruise speed decreases so the throttle must be further reduced to remain at the best cruise speed for the weight of the airplane. This reduction in throttle setting gives a corresponding reduction in fuel flow, thus an increase in specific range. Note that this discussion assumes cruising at a constant altitude.

3.2.1.4 ALTITUDE EFFECT

Changing altitude has an effect on both the drag and thrust curves. Since the air is less dense at high altitude than at low altitudes the drags will be different when compared at the same true airspeed. The parasite drag will decrease at high altitudes at all airspeeds (ref. equation 3.2.1 for parasite drag) and the induced drag will increase at all speeds. (ref. equation 3.2.2 for induced drag). A comparison of the drag curves is given in Figure 3.2.6.



True Airspeed
Figure 3.2.6

The minimum drag value will remain the same for both high and low altitudes, but the true airspeed for minimum drag will be higher for the high altitude. This fact tells us that we can loiter a jet for maximum endurance equally well at all altitudes provided the engine efficiency doesn't change with altitude. In any case, it may be detrimental to waste fuel by climbing to a higher altitude for maximum endurance.

The thrust at full throttle will decrease appreciably with increased altitude. The effects of altitude on both thrust and drag are shown in Figure 3.2.7.

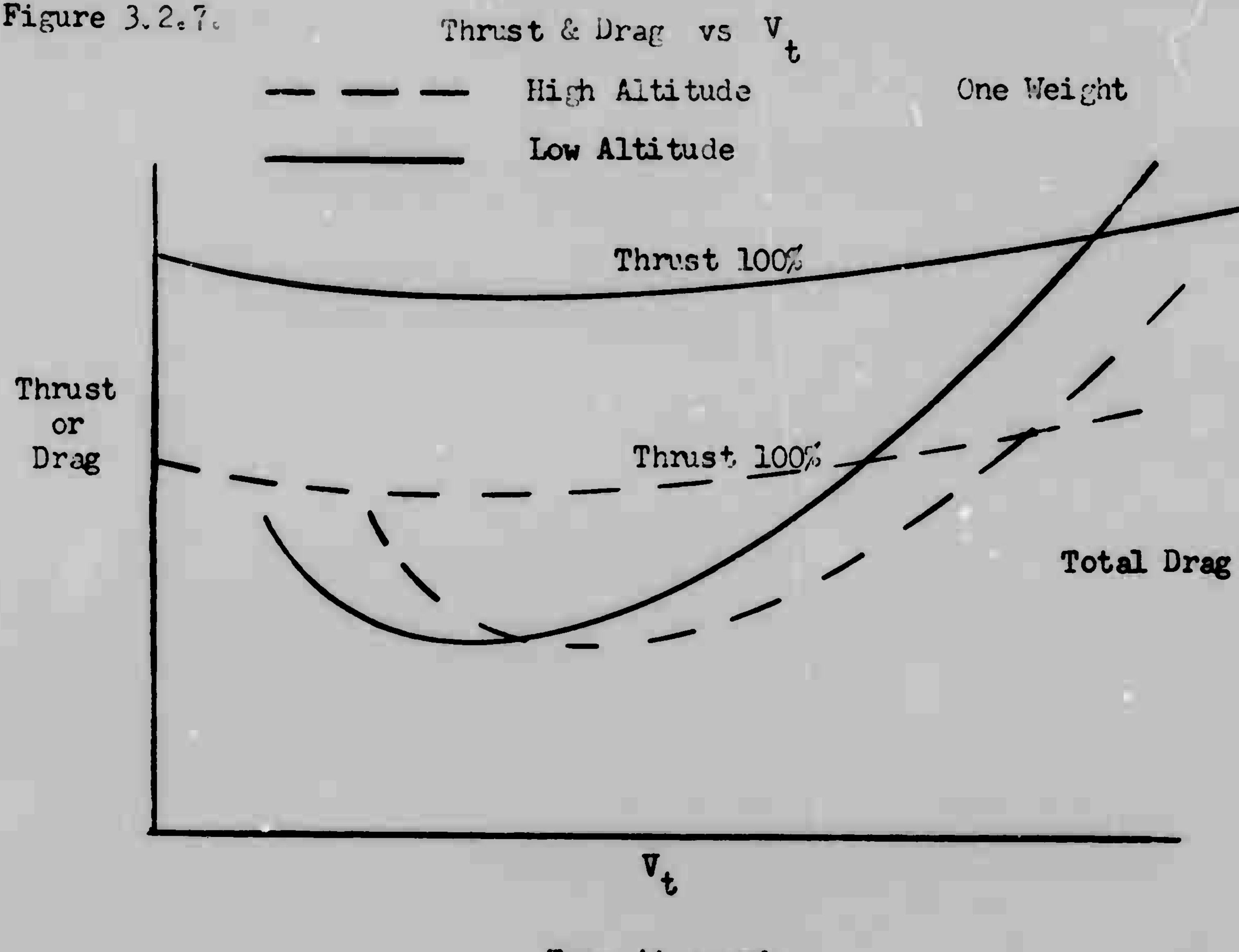


Figure 3.2.7

The maximum level flight speed may or may not increase with altitude. This performance item depends on how much the thrust will change with altitude. Recall that so far we are only discussing flight at subsonic speeds. Therefore, the conclusions regarding the maximum level flight speeds which we obtain from these curves do not apply for transonic airplanes such as the F-86 and F-100. Generally the drag curves discussed will apply for all airplanes at subsonic speeds such as the loiter, climb and cruise speeds.

3.2.1.5 TEMPERATURE EFFECT

The effects of temperature on the drag curves have not been discussed. The previous discussions have been considered with standard temperature conditions. A temperature change at a constant pressure altitude will cause a change in density of the air in the same manner as altitude will cause a density change. This is evident from the equation of state. Therefore, a hot temperature will affect the drag curve in the same manner as a higher altitude. This effect is shown in Figure 3.2.8.

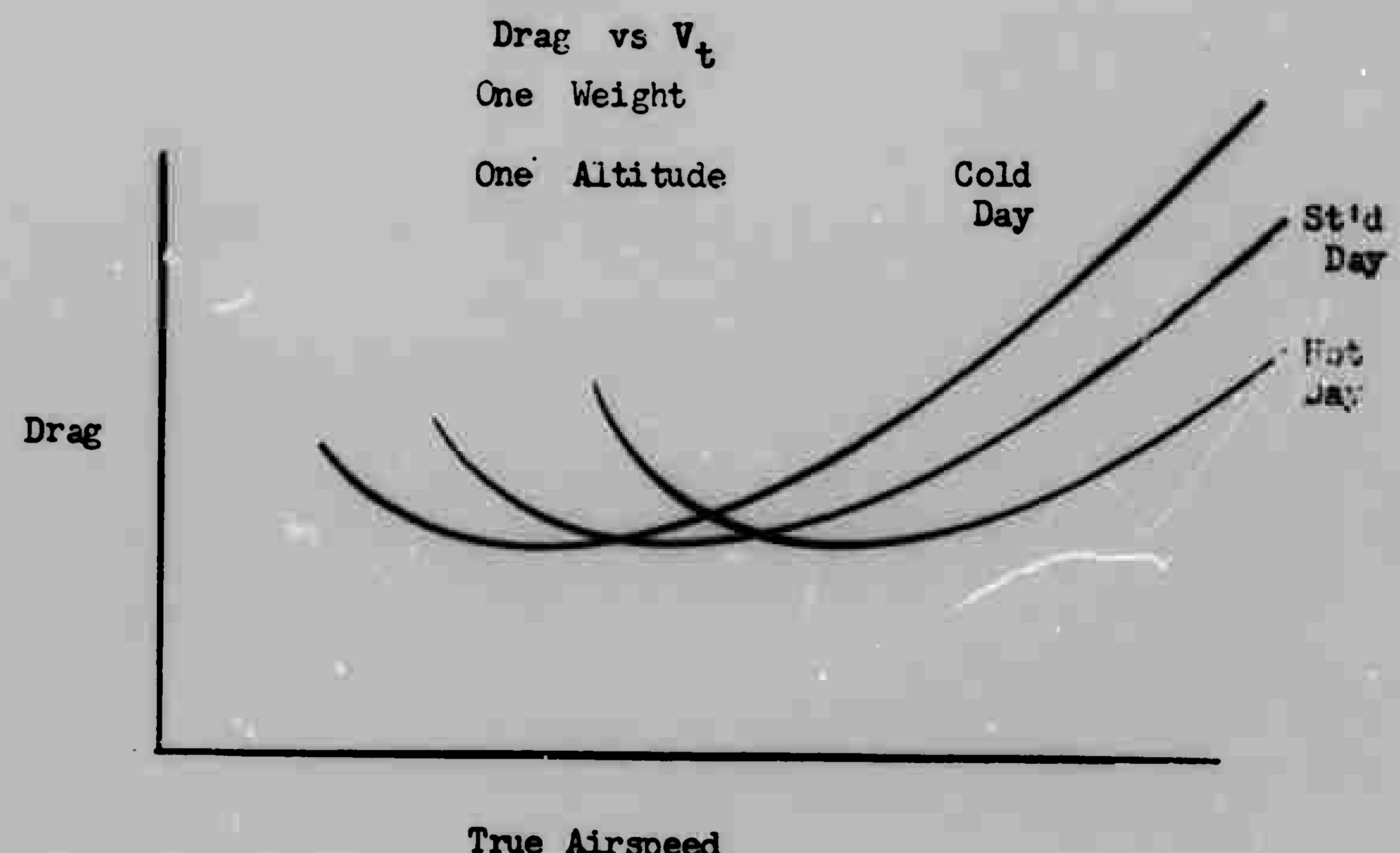


Figure 3.2.8

The minimum drag will remain the same just as it did in changing the altitude. At the same altitude, a change in temperature will change the thrust of the engine at a constant throttle setting. This effect is shown in Figure 3.2.9.

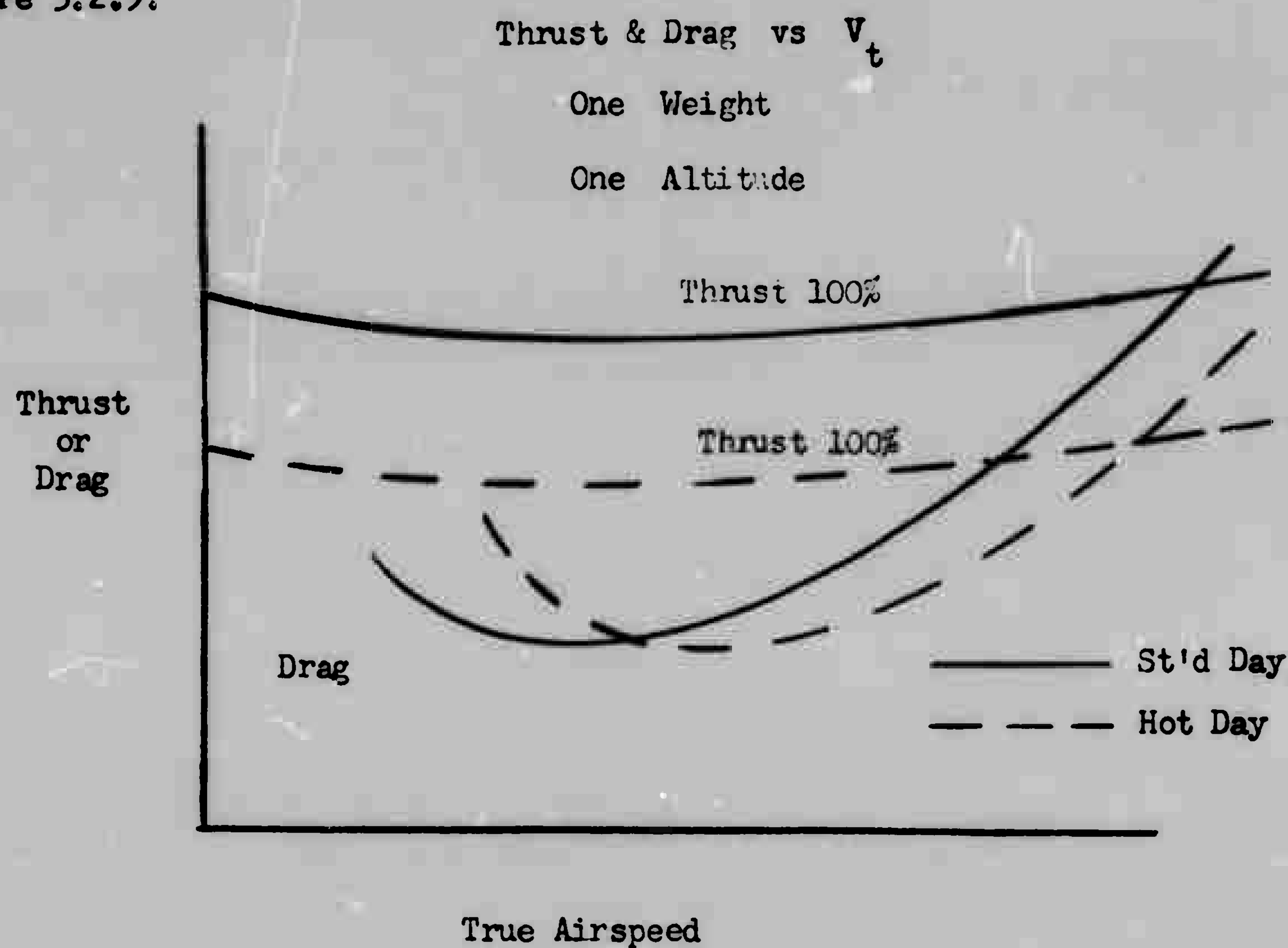


Figure 3.2.9

For a subsonic airplane the maximum speed will decrease as the temperature increases since the temperature has more effect in reducing the thrust output than in decreasing the drag at a given true airspeed.

This effect of temperature on performance is one of the major corrections made during the data reduction of performance flight test results. It is obvious from the above plot that tests results obtained during a non-standard day will not apply directly for standard day conditions. The maximum level speed, rate of climb, acceleration and cruise performance depends on the temperature of the air, therefore, the test data must be corrected to standard day temperature conditions. The method of making these corrections depends on the manner with which the test data is plotted. The drag of an airplane may be determined from flight tests if a means of measuring the thrust accurately is provided, since in stabilized level flight the thrust is equal to the drag. Therefore, we can determine the drag curve by stabilizing the airplane with various power settings to obtain the complete range of airspeeds desired. If we plotted the drag data against true airspeed we would get a different curve from one temperature day to another so we cannot determine the standard day drag curve directly.

3.2.1.6 DRAG vs EQUIVALENT AIRSPEED

By looking at our drag equation we see that we can express it as:

$$D = C_D\left(\tfrac{1}{2}\rho V_t^2\right) S = C_D\, q\, S \quad \text{since } q = \tfrac{1}{2}\rho V_t^2$$

$$\text{but } q = \tfrac{1}{2}\rho V_t^2 \cdot \frac{\rho_0}{\rho_0} = \tfrac{1}{2}\rho_0 \sigma V_t^2$$

$$= \tfrac{1}{2}\rho_0 V_e^2 = \frac{V_e^2}{841} \quad (V_e \text{ in ft/sec})$$

$$\text{since } V_e = \sqrt{\sigma}\, V_t$$

$$\text{therefore } D = \frac{C_D V_e^2 S}{841} \quad (V_e \text{ in ft/sec})$$

We can apply the same reasoning to the parasite and induced drag equations. Thus

$$D_P = \frac{C_{Dp} V_e^2 S}{841} \qquad D_i = \frac{841 (W/b)^2}{\pi V_e^2 e}$$

Note that the density (ρ) does not appear in either equation so if we plotted drag against equivalent airspeed (V_e) instead of true airspeed (V_t) we will get one drag curve for all conditions of density. Therefore, this drag curve obtained will apply for one weight at all altitudes and temperatures. Figure 3.2.10 shows the drag curve vs V_e for three gross weights:

Drag vs V_e

All Altitudes

All Temperatures

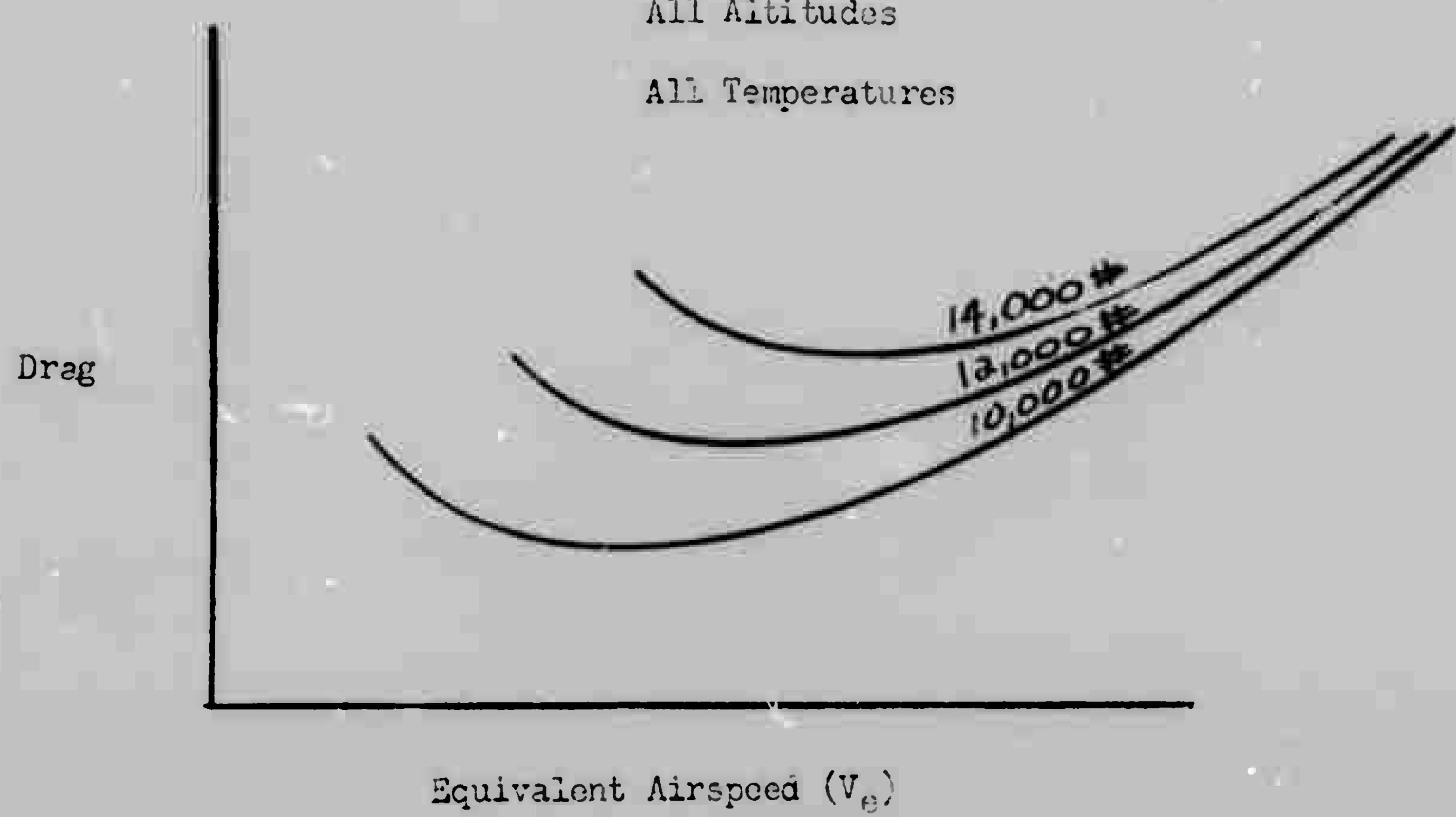


Figure 3.2.10

3.2.1.7 POWER REQUIRED CURVES

So far in this discussion we have considered the forces acting on an airplane, principally the drag and thrust forces. Instead of considering the forces we may think in terms of power. Power is the time rate of doing work. If the force is not acting parallel to the motion of the object, the parallel component of the force is used in defining the work.

$$\text{Power} = \frac{\text{Work}}{\text{unit time}} = \frac{F \times d}{t}$$

Since d/t is velocity, power may be expressed as:

$$\text{Power} = F \times V_t$$

The term horsepoweris a unit of power which is commonly used. one Horsepower (HP) = 33,000 ft-lb/min = 550 ft-lb/sec. Thus, when the velocity is expressed in ft/sec, horsepower is expressed as:

$$HP = \frac{F \times V_t}{550} \qquad V_t \text{ in ft/sec.}$$

If we consider thrust as the force, we have a power term known as thrust horsepower available (THP_a).

$$THP_a = \frac{T \times V_t}{550} \qquad V_t \text{ in ft/sec.}$$

And if we consider drag as the force we have thrust horsepower required (THP_r).

$$THP_r = \frac{D \times V_t}{550}$$

Assuming we know a drag and thrust curve for an airplane, the power required and power available curves can be calculated from the equations above.

We will obtain the same performance data from the power curves as we will obtain from the drag and thrust curves. The maximum speed where the thrust is equal to the drag is the speed where the THP_a equals the THP_r.

The use of the power curves has an advantage in determining the best climb speed and rate of climb. The rate of climb equation is derived considering the forces:

$$R/C = \frac{(T-D)\ V_t}{W}$$

$$R/C = \frac{T \cdot V_t - D \cdot V_t}{W}$$

Since $T \times V_t = THP_a \times 550$

and $D \times V_t = THP_r \times 550$

$$R/C\ (ft/sec) = \frac{(THP_a - THP_r)\ 550}{W}$$

and $$R/C\ (ft/min) = \frac{(THP_a - THP_r)\ 33{,}000}{W}$$ Equation 3.2.3

The term $THP_a - THP_r$ is called the excess thrust horsepower and given the symbols of Δ THP or THP_{ex}. From the equation above it can be seen that the rate of climb will be a maximum at the speed where the Δ THP is a maximum. So the best climb speed can be easily determined from the power curves by noting the speed where there is the largest difference between the THP_a and THP_r curves.

Figure 3.2.11 shows typical power curve for a subsonic airplane. The THP_a curve is typical for a turbo-jet engine. Note that the speed for minimum drag is at a higher speed than the speed for minimum thrust horsepower required.

THP vs V_t

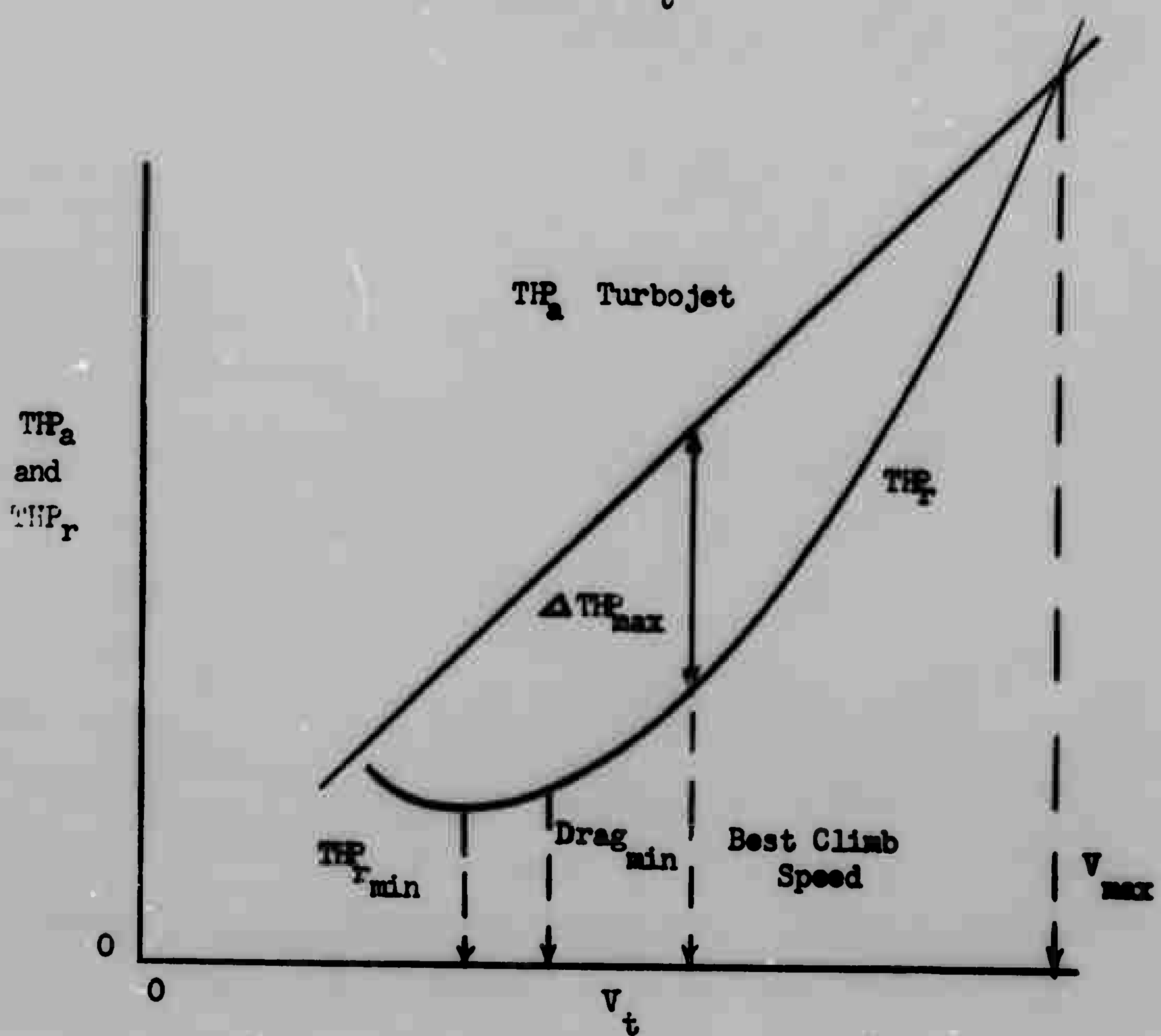


Figure 3.2.11

The speeds less than the speed for minimum THP_r are known as the speeds on the "back side" of the power required curve.

The thrust horsepower available curve for a reciprocating engine has a different shape than for the turbo-jet engine. At a given manifold pressure and RPM the reciprocating engine delivers the same brake horsepower (BHP) at all speeds. The THP_a delivered by the engine propeller combination will vary with speed as determined by the propeller efficiency (η_p) since:

$$THP_a = \eta_p \times BHP$$

Propeller efficiency may be as high as 86% to 88% under optimum conditions, but at other conditions the propeller efficiency will vary. At high speeds the propeller efficiency drops rapidly due to propeller tip losses. A typical plot of THP_a for a reciprocating engine - propeller combination is shown in Figure 3.2.12

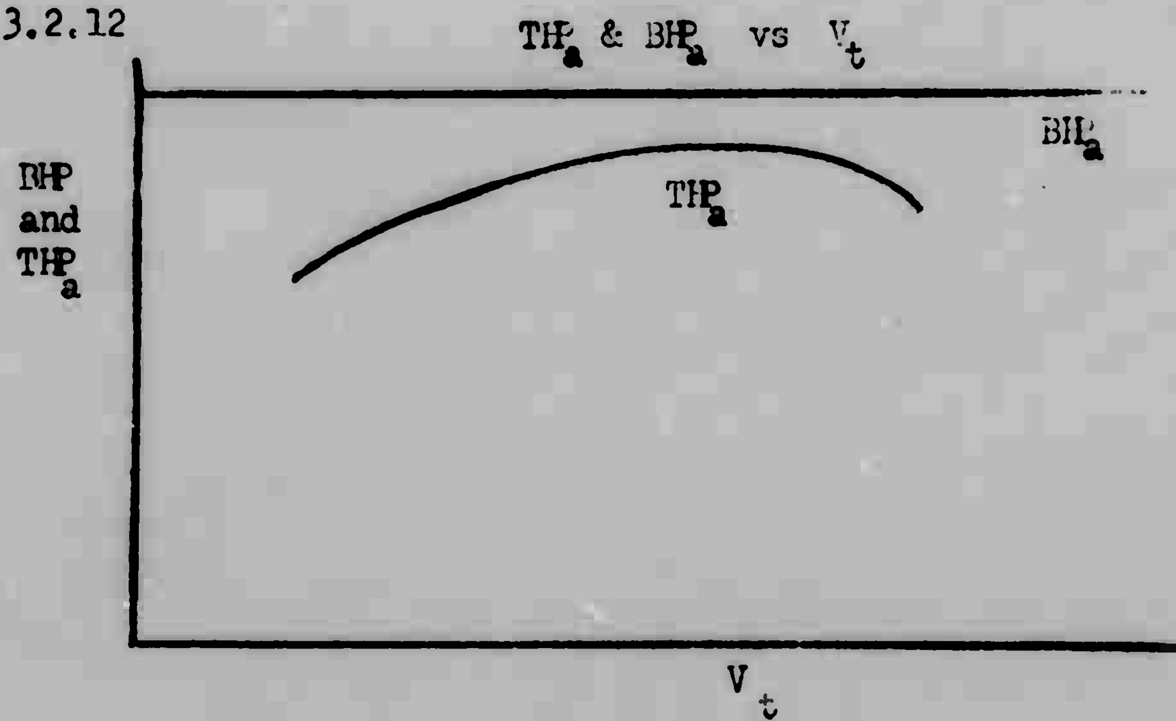


Figure 3.2.12

Since it is easier to determine the BHP of a reciprocating engine than the thrust delivered, the power curves are used in flight testing of propeller-type aircraft. The BHP may be determined from engine charts or by the use of a torque meter on the crank shaft. These methods are applicable to turbo-prop engines as well as reciprocating engines. The output of the turboprop engine is termed equivalent shaft horsepower. It should be noted that in level stabilized flight the THP_a equals the THP_r since the thrust equals the drag.

The thrust horsepower required curves will change with weight, altitude and temperature for the same reasons that the drag curves changed. We will first investigate the effects of altitude and temperature on the power required curves. We know that altitude and temperature have similar effects on the curves since the change is due to a change in the density of the air. The power required curves would not only move to the right as did the drag curves (with decreasing density) but would also move upward. Thus, the value of the minimum power required would increase as the density decreased.

THP_r vs V_t

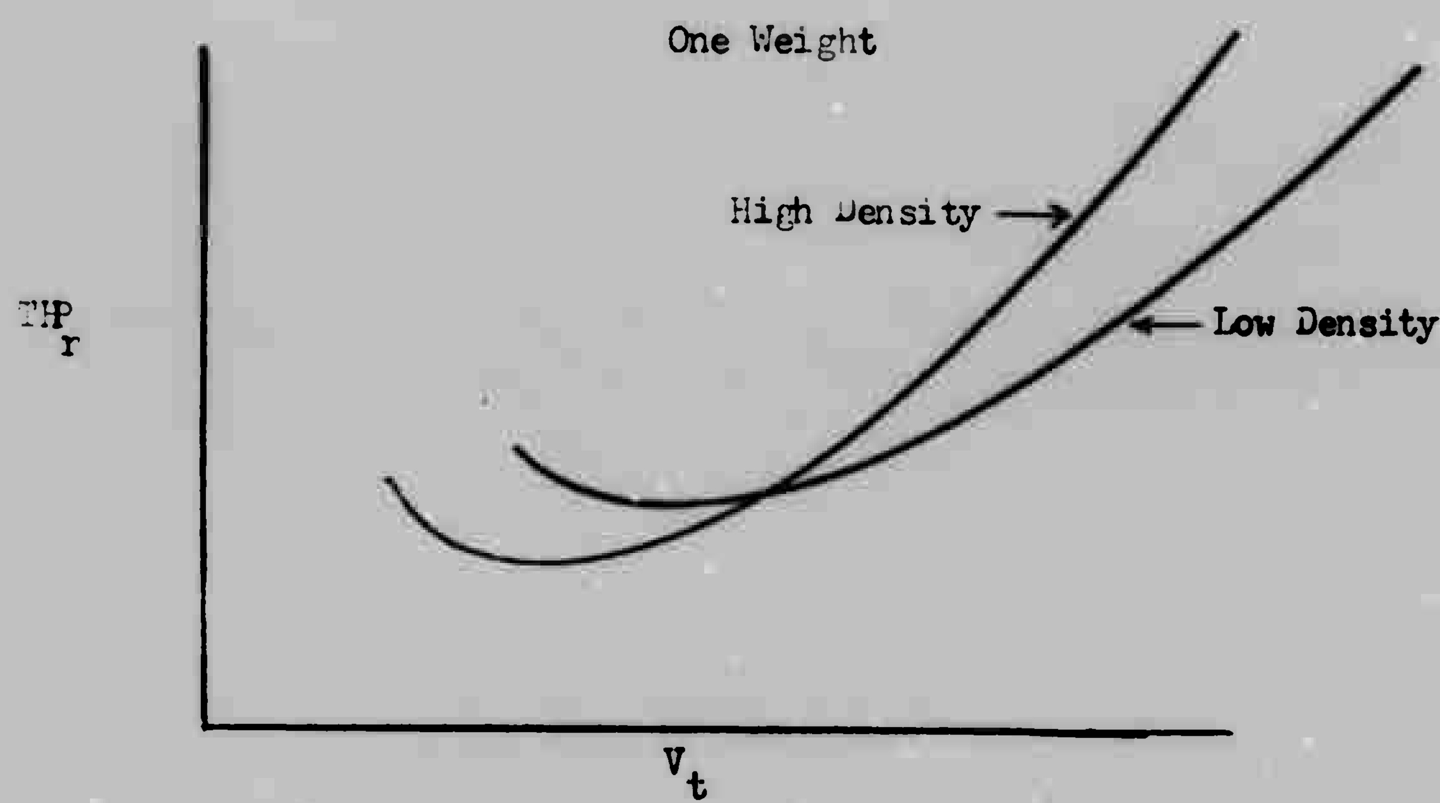


Figure 3.2.13

In the case of the drag curves, we would obtain one curve if we plotted the drag vs V_e. This is not the case for the power required curves. If we plotted the THP_r vs V_e the low density curves would shift to the left. The resulting curves are shown in Figure 3.2.14.

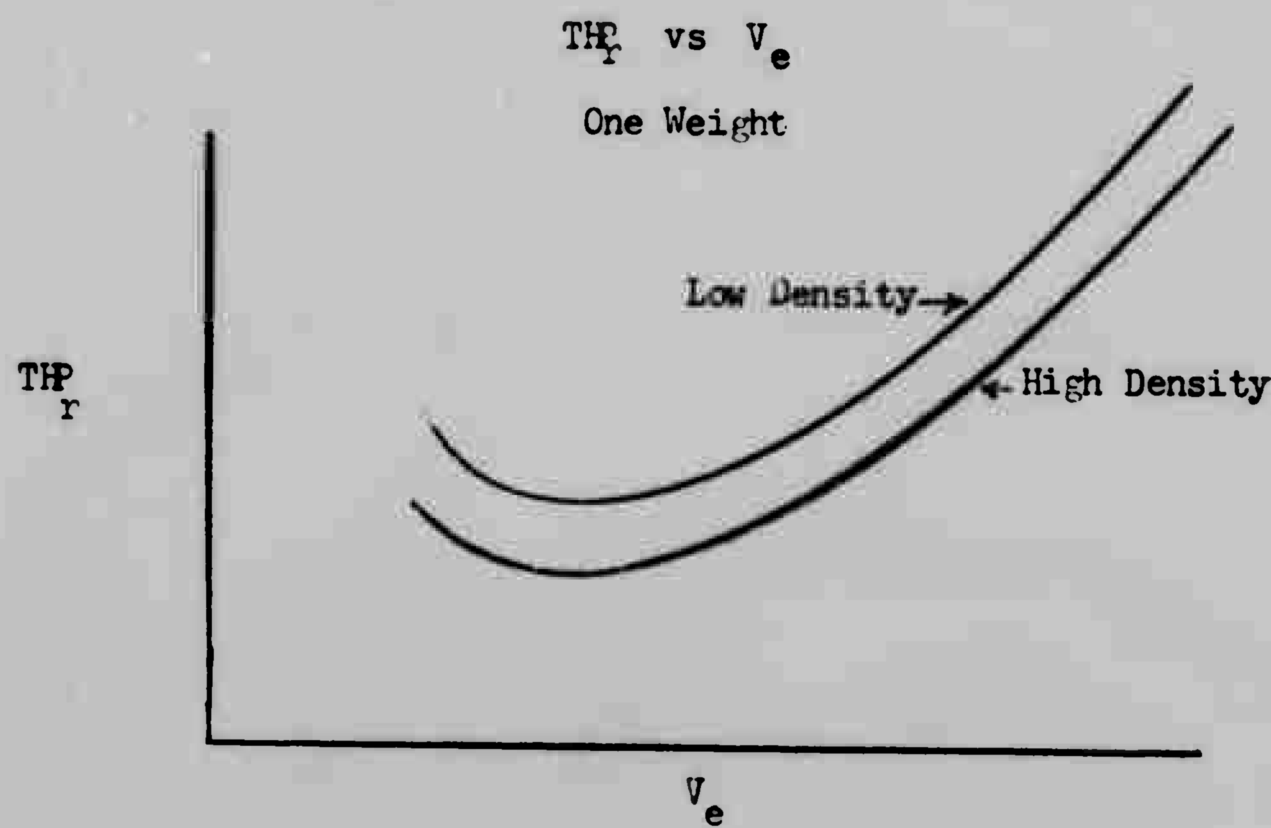


Figure 3.2.14

This fact may be proved by the following derivation:

$$THP_r = \frac{D \times V_t}{550}$$

$$D = \left(C_{D_p}\ q\ S\right) + \frac{L^2}{b^2 \pi q\ e}$$

Dynamic pressure may be expressed as:

$$q = \frac{\sigma V_t^2}{841} \quad \text{with } V_t \text{ in ft/sec.}$$

$$D = C_{D_p} \cdot \frac{\sigma V_t^2 S}{841} + \frac{L^2 \; 841}{b^2 \pi \sigma V_t^2 \; e}$$

$$THP = \frac{C_{D_p} \sigma V_t^3 \; S}{841 \times 550} + \frac{L^2 \; 841}{b^2 \pi \sigma V_t \times 550 \; e}$$

This equation may be expressed interms of equivalent airspeed instead of true airspeed since

$$V_t = \frac{V_e}{\sqrt{\sigma}}$$

Thus, $$THP_r = \frac{C_{D_p} V_e^3 \; S}{\sqrt{\sigma} \; 841 \times 550} + \frac{L^2 \; 841}{b^2 \pi \sqrt{\sigma} V_e \; 550 \; e}$$

From this equation we can see that the density will have an effect on the THP_r curves when plotted against V_e. The lower the density the higher the THP_r.

3.2.2 PROPELLER-AIRFRAME PERFORMANCE

In order to have one curve represent the power required for one weight at all values of density, we can modify the power required further. If we multiply both sides of the equation by $\sqrt{\sigma}$ we will have:

$$\sqrt{\sigma}\, THP_r = \frac{C_{D_p} V_e^3 \; S}{841 \times 550} + \frac{L^2 \; 841}{b^2 \pi V_e \; 550 \; e}$$

$$\sqrt{\sigma}\, THP = K_1 V_e^3 + K_2 / V_e$$

where $$K_1 = \frac{C_{D_p} \; S}{841 \times 550}$$

and $$K_2 = \frac{L^2 \; 841}{b^2 \pi \; 550 \; e}$$

It can be seen that the parameter, $\sqrt{\sigma}\ THP_r$, for an airplane at a constant weight depends only on the equivalent airspeed. Thus, we can plot THP_r as shown in Figure 3.2.15.

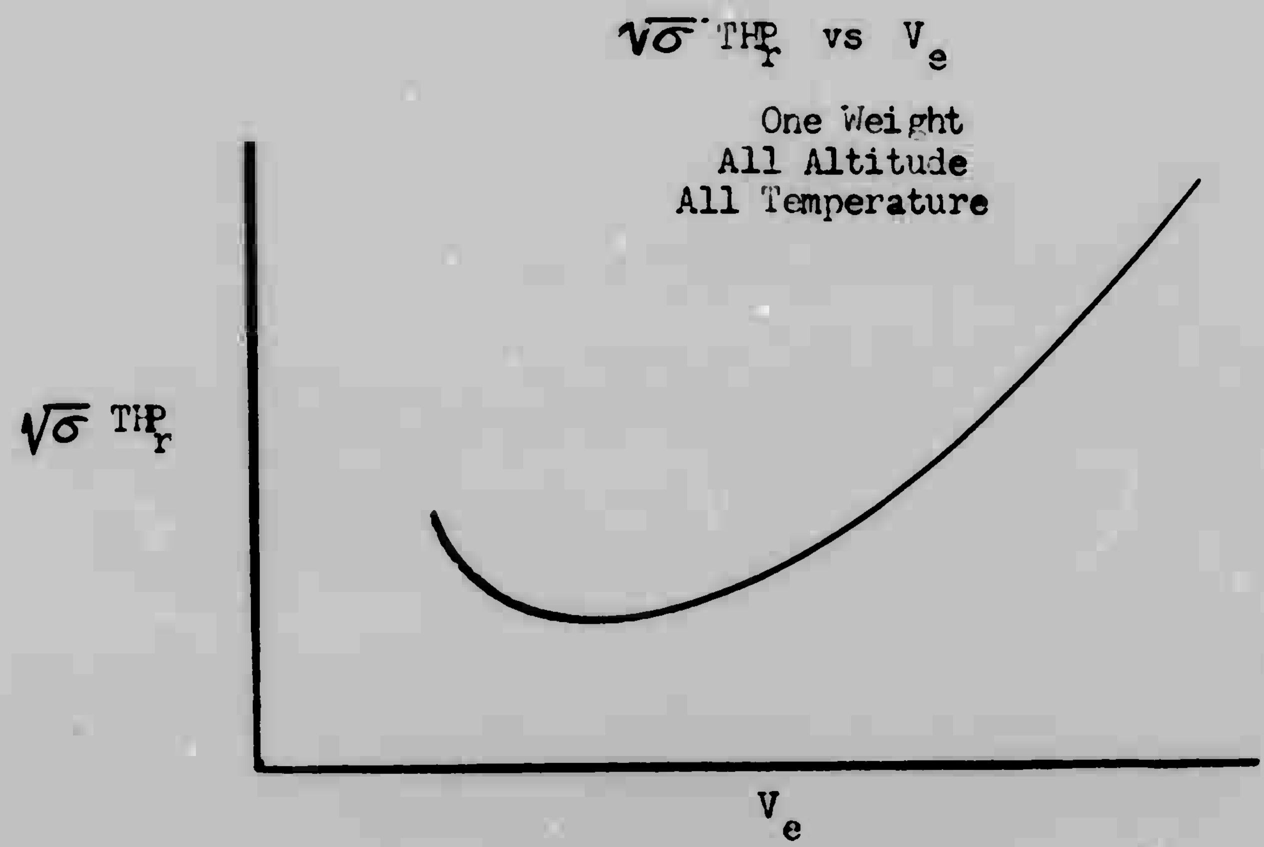


Figure 3.2.15

We will obtain different curves for different weights since the constant K_2 contains the lift produced by the airplane. We can consider lift as a variable and write our power required equation in the following manner:

$$THP_r = K_1 V_e^3 + \frac{K_3 L^2}{V_e}$$

$$\text{where } K_3 = \frac{841}{b^2 \pi\ 550\ e}$$

To introduce the weight of the airplane we will replace lift with nW.

$$\sqrt{\sigma}\ THP_r = K_1 V_e^3 + \frac{K_3 n^2 W^2}{V_e}$$

For load factors of one we get the curves on Figure 3.2.16 for various gross weights.

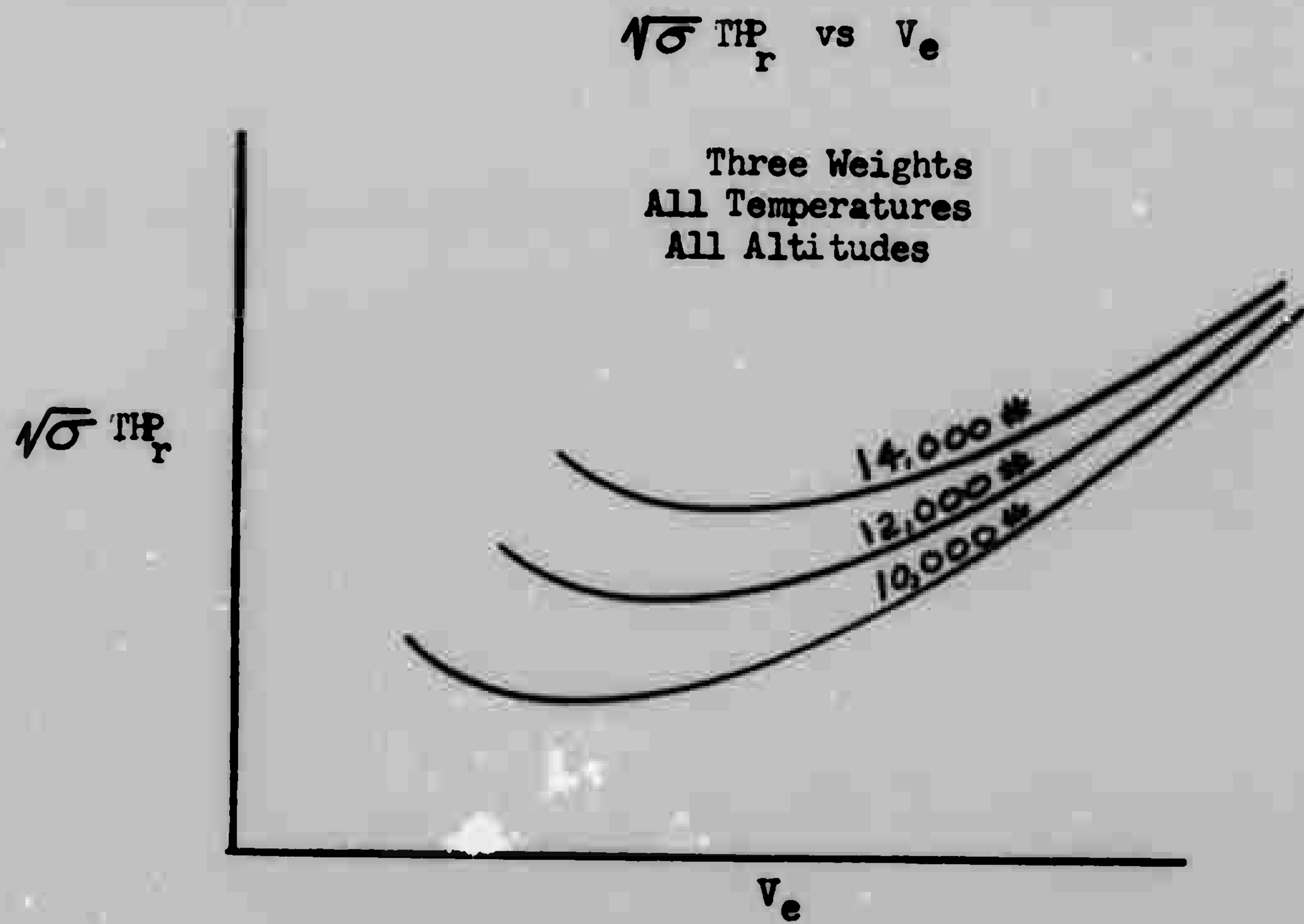


Figure 3.2.16

3.2.2.1 PIW-VIW CURVE

It is desirable to be able to obtain one curve for the power required for all weights. This type of plot would enable the engineer to use all the flight test points from a level flight performance test to determine one power required curve. It is apparent from the previous figure that test points obtained at differenct weights would give much scatter on a plot of $\sqrt{\sigma}$ THP vs V_e. Many test points must be determined for each of the curves and all the points must be obtained at the same airplane weight, so an excessive amount of flight time would be required. If it were possible to obtain one curve to represent the power required data for all weights, we could use each level flight

test point to help determine this curve. This is done on a plot called the PIW-VIW curve. We will derive the expression for the PIW-VIW plot by continuing with the previous derivation. It has been shown that:

$$\sqrt{\sigma}\ THP_r = K_1 \cdot V_e^3 + \frac{K_3\ n^2\ W^2}{V_e}$$

If we consider n = 1 (straight and level flight) and divide both sides of the equation by $W^{3/2}$, we obtain:

$$\frac{\sqrt{\sigma}\ THP_r}{W^{3/2}} = K_1 \cdot \frac{V_e^3}{W^{3/2}} + K_3 \cdot \frac{W^{\frac{1}{2}}}{V_e}$$

$$= K_1 \cdot \left(\frac{V_e}{W^{\frac{1}{2}}} \right)^3 + \frac{K_3\ W^{\frac{1}{2}}}{V_e}$$

We can consider the weight used above as the test weight (W_t) of the airplane. It is advantageous to use weight ratios to obtain power dimensions for the PIW scale and velocity dimensions for the VIW scale. We will introduce an airplane weight into the equation, a weight which we call the standard weight (W_s). This weight may be any reasonable weight but is usually taken as the weight of the airplane with a full fuel load.

We can multiply both sides of the equation by $W_s^{3/2}$

$$\frac{\sqrt{\sigma}\ THP_r}{(W_t/W_s)^{3/2}} = K_1 \cdot \left(\frac{V_e}{(W_t/W_s)^{\frac{1}{2}}} \right)^3 + \frac{K_3\ W_s^{3/2}\ W_t^{\frac{1}{2}}}{V_e}$$

but $W_s^{3/2} \; W_t^{\frac{1}{2}} = W_s^{2} \left(\frac{W_t}{W_s} \right)^{\frac{1}{2}}$

so $$\frac{\sqrt{\sigma}\,THP_r}{(W_t/W_s)^{3/2}} = K_1 \left[\frac{V_e}{(W_t/W_s)^{\frac{1}{2}}} \right]^3 + \frac{K_3 W_s^2}{V_e} \left(\frac{W_t}{W_s} \right)^{\frac{1}{2}}$$

Since W_s is a constant we can include it with the constant K_3.

Thus $$\frac{\sqrt{\sigma}\,THP_r}{(W_t/W_s)^{3/2}} = K_1 \left[\frac{V_e}{(W_t/W_s)^{\frac{1}{2}}} \right]^3 + \frac{K_4}{V_e} \left(\frac{W_t}{W_s} \right)^{\frac{1}{2}}$$

where: $K_4 = K_3 \; W_s^2$

We now have the equation in a form where we have only one parameter in the left hand side of the equation, that is $\frac{V_e}{(W_t/W_s)^{\frac{1}{2}}}$

Thus, we may write the functional expression:

$$\frac{\sqrt{\sigma}\,THP_r}{(W_t/W_s)^{3/2}} = f \left(\frac{V_e}{(W_t/W_s)^{\frac{1}{2}}} \right) \qquad \text{Equation 3.2.4}$$

The parameter $\frac{\sqrt{\sigma}\,THP_r}{(W_t/W_s)^{3/2}}$ is called PIW, and the parameter, $\frac{V_e}{(W_t/W_s)^{\frac{1}{2}}}$ is called VIW. The PIW vs VIW plot gives the desired one curve of power required information for all weights, temperatures and altitudes. The standard weight used in determining the PIW and VIW parameters should be noted on the plot. This curve is shown in Figure 3.2.17.

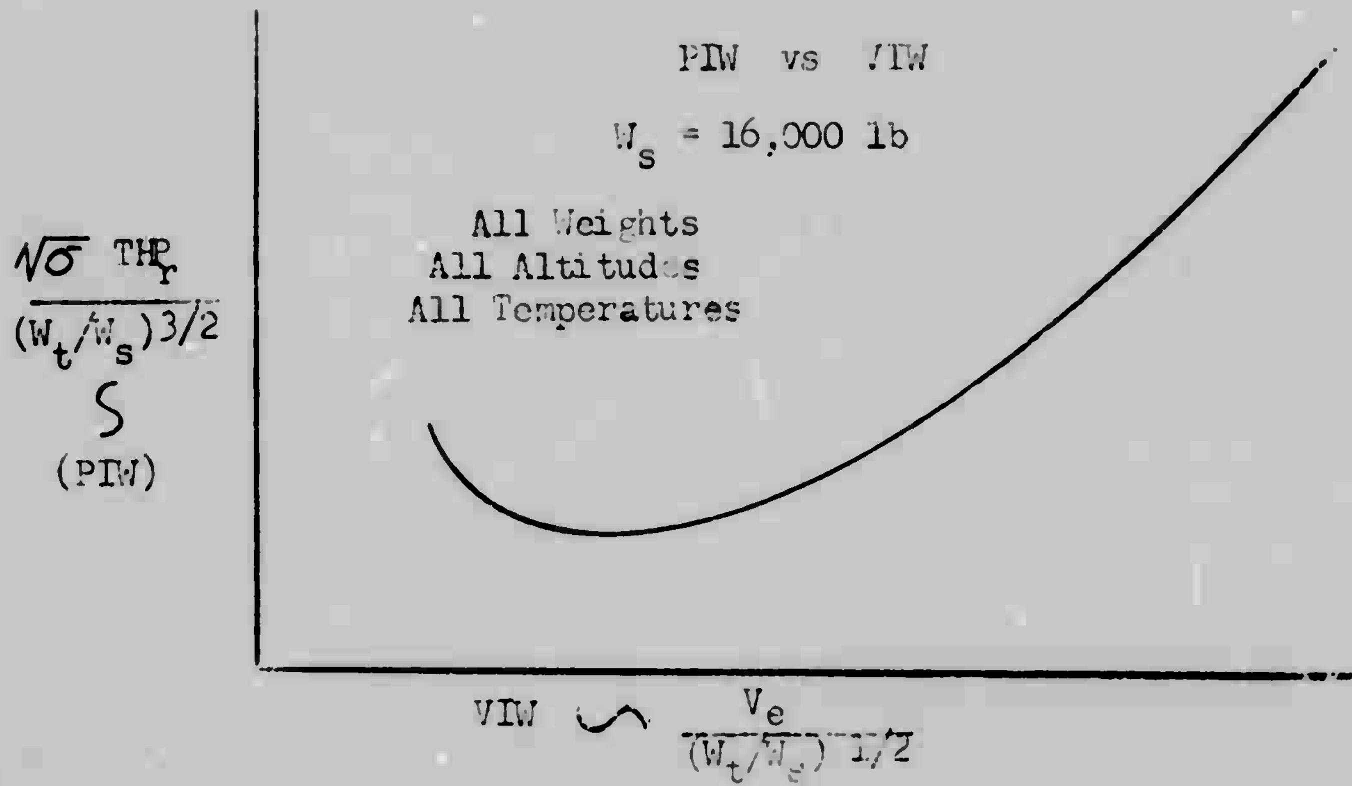


Figure 3.2.17

From the PIW-VIW plot, the THP_r can be determined directly for any true airspeed for the standard weight airplane at sea level on a standard day. Since the weight ratio Wt/Ws and σ are equal to one for these conditions, the PIW-VIW plot will be the plot of THP_r vs V_t for the conditions mentioned above. Note that this plot may be determined from level flight test data points which are not obtained at the standard weight or sea level standard density.

With the PIW-VIW plot we can determine the THP_r vs V_t plot for any weight, altitude and temperature. Note that this discussion so far applies to subsonic speeds only where the total drag is the sum of the parasite and induced drags.

The determination of the THP of a reciprocating engine-propeller combination at various power settings involves the use of engine charts or a torquemeter plus the propeller efficiency charts for the airplane. At the Test Pilot School the propeller efficiency charts are not used, so the THP_a of the power plant is not determined but the BHP delivered is obtained from the appropriate

engine charts. The PIW parameter plotted at the School involves BHP instead of THP_a, so some small degree of scatter may be expected between neighboring points on the PIW-VIW plot if their test altitudes and weights differ greatly. The factors which determine the propeller efficiency will be discussed later in the course.

The reasons for the use of the symbols, PIW and VIW may be appreciated at this time. The equivalent airspeed, V_e, was called indicated airspeed many years in the past. Therefore, we have the "VI" of VIW meaning indicated airspeed. Since the $\sqrt{\sigma}$ THP part of the PIW parameter is of the same form as $V_e = (\sqrt{\sigma}\, V_t)$, this quantity is called indicated horsepower, "PI". The W in PIW and VIW means weight corrected.

3.2.2.3 DETERMINATION OF DRAG POLARS

Once the PIW-VIW curve is determined the plot of C_L vs C_D can be obtained. The following derivation is used to show how C_D enters into the PIW-VIW relationship.

$$PIW = \frac{THP_r \sqrt{\sigma}}{(W_t/W_s)^{3/2}} = \frac{BHP\, \eta_p \sqrt{\sigma}}{(W_t/W_s)^{3/2}}$$

$$PIW = \frac{D \times V_t \sqrt{\sigma}}{550 (W_t/W_s)^{3/2}} = \frac{C_D \frac{1}{2} \rho_a V_t^2 S V_e}{550 (W_t/W_s)(W_t/W_s)^{\frac{1}{2}}} = \frac{C_D \rho_a V_t^2 S \, VIW}{550 \times 2 \, (W_t/W_s)}$$

$$PIW = \frac{C_D \rho_0 V_e^2 S \, VIW}{550 \times 2 \, (W_t/W_s)} = \frac{C_D \rho_0 S \, VIW^3}{550 \times 2}$$

$\therefore C_D = \dfrac{PIW \times 1100}{\rho_o (VIW)^3 S}$ Equation 3.2.5

and $C_L = \dfrac{2\ Ws}{\rho_o (VIW)^2 S}$ Equation 3.2.6

From these equations C_L and C_D can be obtained at each data point. The following plot can be made.

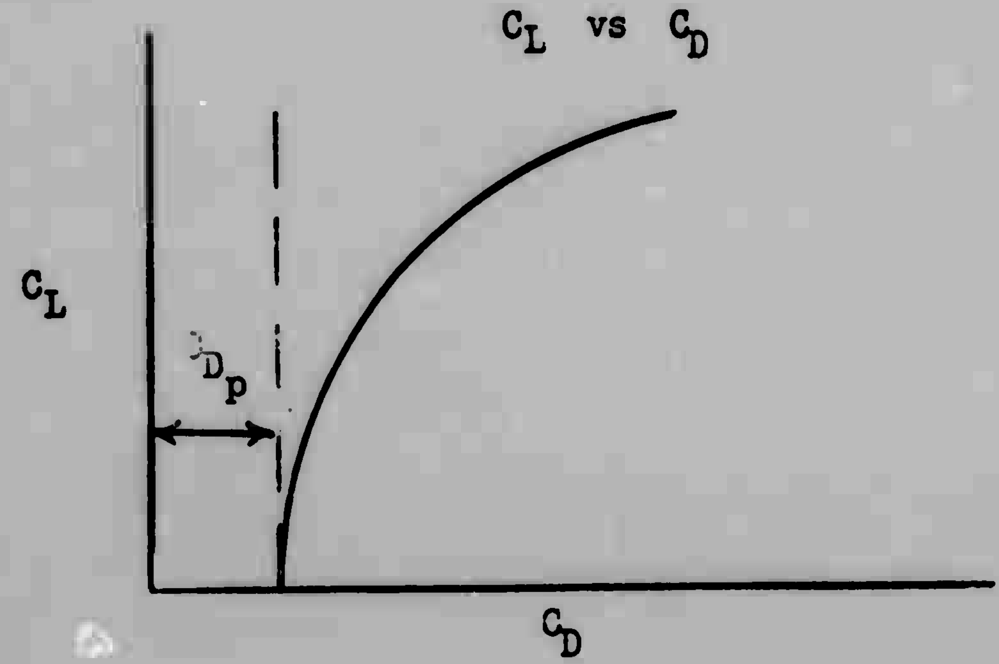


Figure 3.2.18

A linearized drag polar can now be constructed for the same data points.

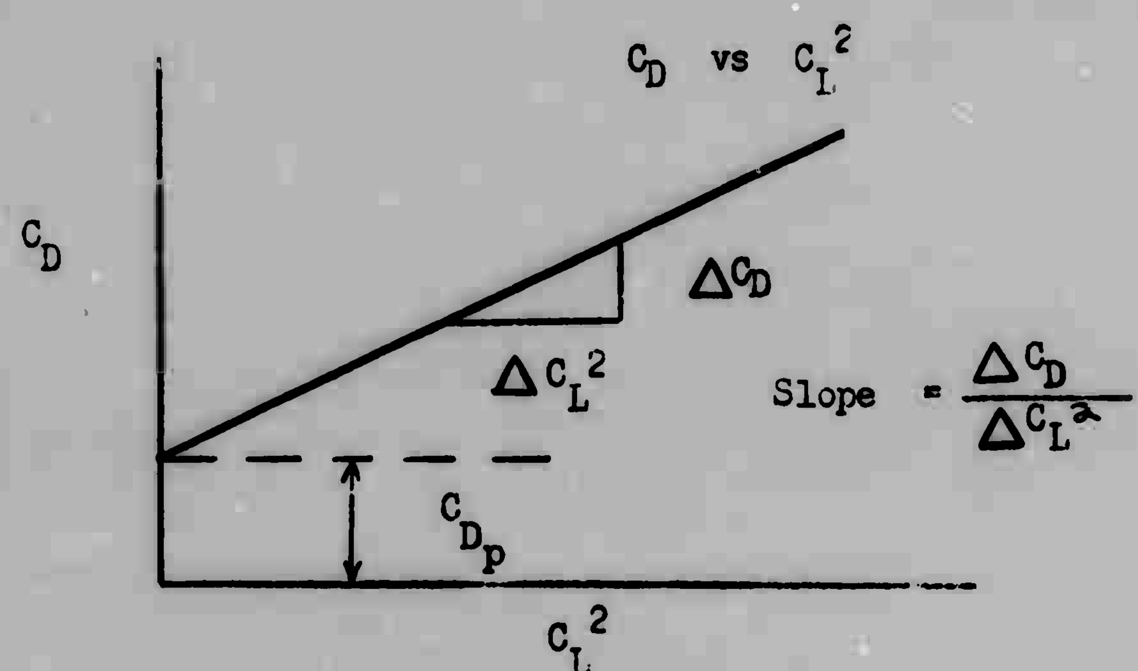


Figure 3.2.19

The slope of the line is $\frac{\Delta C_D}{\Delta C_L{}^2} = m$

but $C_{D_i} = \frac{C_L{}^2}{\pi AR e} \therefore \frac{\Delta C_{D_i}}{\Delta C_{L_2}} = \frac{1}{\pi AR e} = m$

This is due to the fact that the straight line portion **occurs** where

$\Delta C_D = \Delta C_{D_i}$ and $e = \frac{1}{\pi AR m}$

3.2.3 JET ENGINE AIRFRAME PERFORMANCE

3.2.3.1 DRAG CURVES AT TRANSONIC SPEEDS:

So far we have discussed the drag and power required characteristics of an airplane at subsonic speeds where the total drag is comprised of the parasite and induced drags. At high Mach numbers (.70 to .90 depending on the airplane design) the drag starts to rise rapidly due to the formation of shock waves about the airplane. This additional drag is called the compressibility drag. It is non-existant at subsonic speeds but occurs at transonic speeds (speeds where the local flow about some parts of an airplane is sonic or supersonic).

Since the local velocity of the flow about most points on an airplane is different from the free stream velocity, the local velocity at some point can be equal to or greater than the speed of sound when the velocity of the airplane is less than the speed of sound (or when the airplane is flying at a Mach number less than one). The lowest free stream Mach No. where sonic flow first occurs at any point about the airplane is called the Critical Mach number.

All speeds below the critical Mach number are called subsonic as the local velocities about all points on the airplane are subsonic (or less than sonic velocity). At free-stream speeds just above the critical Mach number the local flow is supersonic over some points on the airplane and subsonic over other points. This condition defines transonic speeds or transonic Mach numbers. At higher speeds (starting at about $M = 1.2$) the local flow about most of the airplane is supersonic so the airplane is considered to be in the supersonic speed regime.

Shock waves will form whenever the local flow becomes supersonic and at transonic speed the shock waves will induce separation of the boundary layer thus creating a large increase in the drag. This drag (compressiblility drag) primarily depends on the free stream Mach number and will increase rapidly as the transonic speeds are increased due to the stronger shock waves. At high transonic speeds the greater portion of the total drag of the airplane is this compressibility drag. Figure 3.2.20 gives the drag coefficient due to compressibility (C_{D_M}) as compared with the induced and parasite drag coefficients.

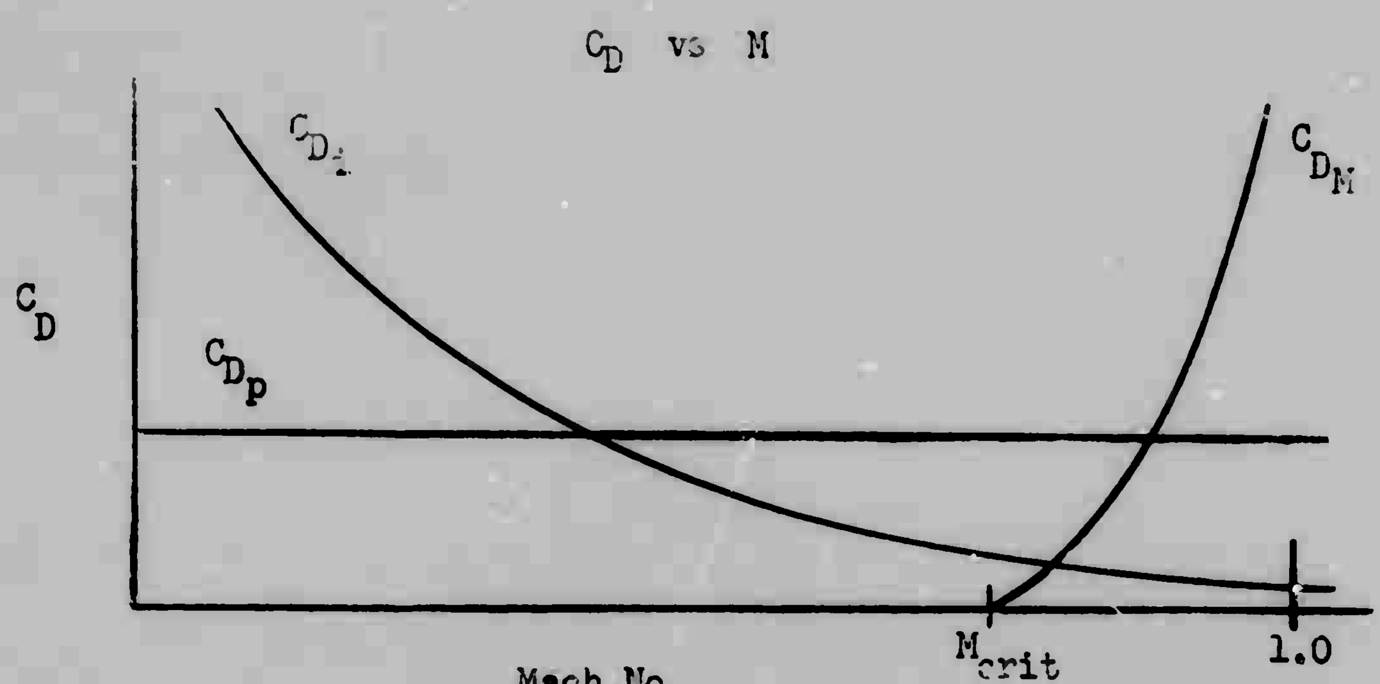


Figure 3.2.20

3.2.3.2 W/δ DRAG THEORY

We may express lift and drag in terms of Mach number instead of velocity. Considering the familiar lift equation we may easily derive the desired relationship.

$$L = \tfrac{1}{2} C_L \rho V_t^2 S$$

$$M = \frac{V_t}{a} \quad \text{where } a = \text{speed of sound}$$

$$\therefore V_t^2 = M^2 a^2$$

$$\text{but } a = \sqrt{\frac{\gamma P}{\rho}}$$

$$\text{so } V_t^2 = M^2 \left(\frac{\gamma P}{\rho}\right)$$

$$\therefore L = \tfrac{1}{2} C_L \rho \frac{M^2 \gamma P}{\rho} S$$

$$L = \tfrac{1}{2} C_L \gamma P M^2 S$$

If we multiply by $\frac{P_o}{P_o}$ we obtain

$$L = \tfrac{1}{2} C_L \gamma \frac{P}{P_o} P_o M^2 S \quad \text{and } \delta = P/P_o$$

$$\text{So: } L = \tfrac{1}{2} \gamma P_o C_L \delta M^2 S$$

$$L = \tfrac{1}{2} \times 1.4 \times 2116 \; C_L \delta M^2 S$$

$$L = 1481 \; C_L \delta M^2 S \qquad \text{Equation 3.2.7}$$

The equation for lift coefficient then becomes:

$$C_L = \frac{L}{1481\ \delta M^2\ S} = \frac{n\ W}{1481\ \delta M^2\ S}$$

If we only consider straight and level flight the load factor, n is unity.

$$C_L = \frac{W/\delta}{1481\ M^2\ S} \qquad \text{Equation 3.2.8}$$

now we may write the functional equation:

$$C_L = f\ (W/\delta\ ,\ M)$$

which means that if the Mach number and ratio of weight to pressure ratio is known, the lift coefficient is determined .

Similarly the drag equation may be obtained.

$$D = 1481\ C_D\ \delta\ M^2\ S$$

$$D/\delta = 1481\ C_D\ M^2\ S$$

or $\quad D/\delta = f\ (C_D, M)$

Instead of considering drag alone, we are working with the ratio of drag divided by the pressure ratio. We obtain the simple relationship that drag/δ depends only on C_D and Mach number.

In the subsonic theory the total drag coefficient varies only with the lift coefficient since $C_D = C_{D_p} + \frac{C_L^2}{\pi AR\ e}$ and C_{D_p} is a constant. This fact allows us to construct a drag polar of a single curve which will apply for all subsonic conditions. When considering transonic speeds the total drag coefficient must include the compressibility drag coefficient, so a

different drag polar curve must be considered for each Mach number above the critical Mach number. The drag polar for an airplane capable of flying at transonic speeds is shown on Figure 3.2.21.

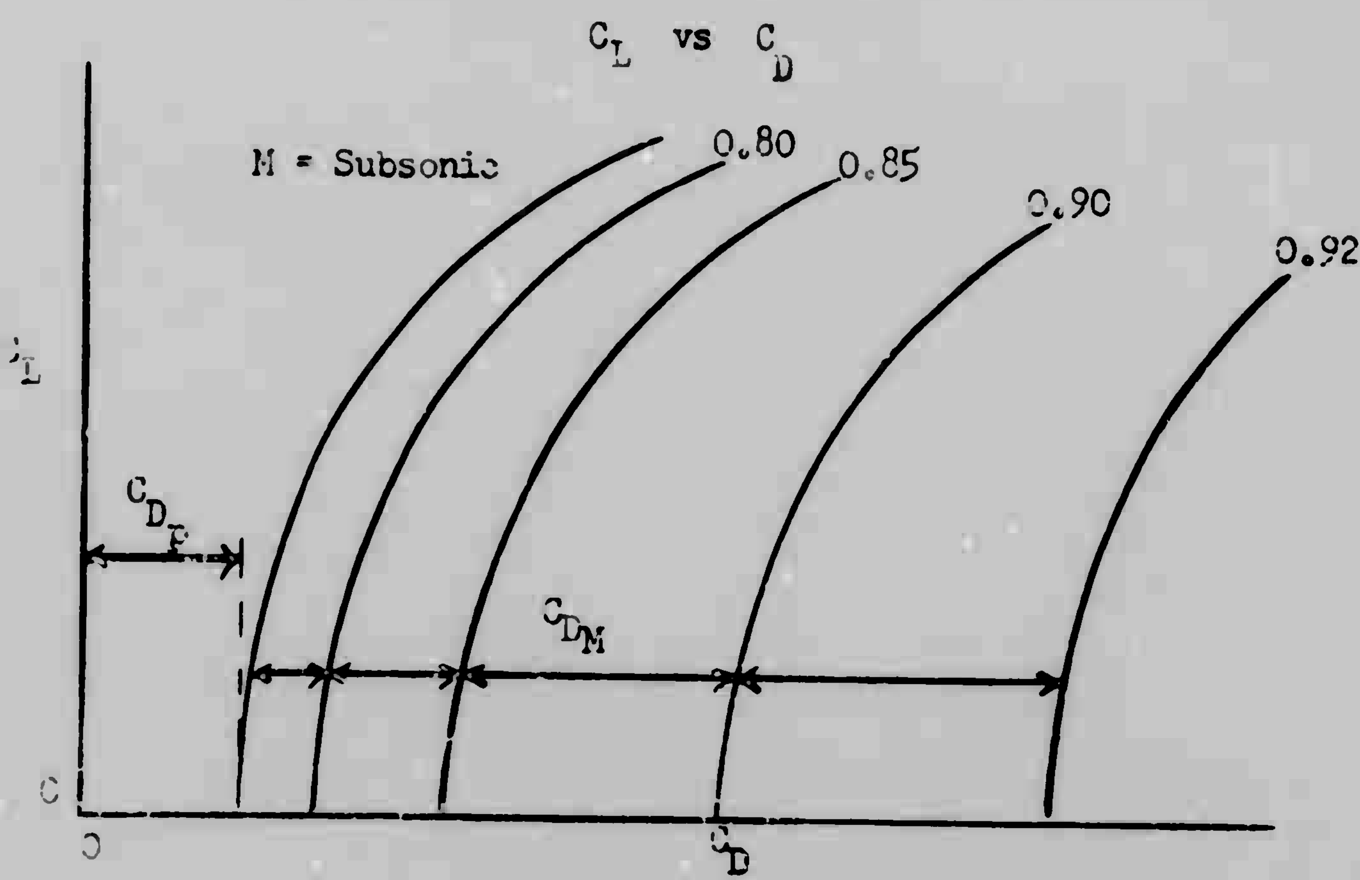


Figure 3.2.21

Note that the curves only differ by the compressible drag coefficient. The variation of C_{D_M} with Mach number is shown on Figure 3.2.20.

From this drag polar it is evident that the total drag coefficient at transonic speeds depends on both the Mach number and lift coefficient. Therefore, this fact allows us to write the following functional equation.

$$C_D = f(C_L, M)$$

since $C_L = f(W/\delta, M)$

we can conclude that

$$C_D = f(W/\delta, M)$$

But from our drag equation we obtain the relationship,

$$D/\delta = f(C_D, M)$$

since: $C_D = f(W/\delta, M)$

the following relationship is true:

$$D/\delta = f(W/\delta, M) \qquad \text{Equation 3.2.9}$$

We cannot write the algebraic equation for D/δ with W/δ and Mach number as the only variable since we cannot express the variation of C_{D_M} with Mach number by an equation. The compressibility drag cannot be derived but must be determined experimentally from wind tunnel tests or flight tests. Nevertheless, the functional equation for D/δ is very useful as it allows us to plot D/δ vs Mach number with curves at constant values of W/δ as shown in Figure 3.2.22

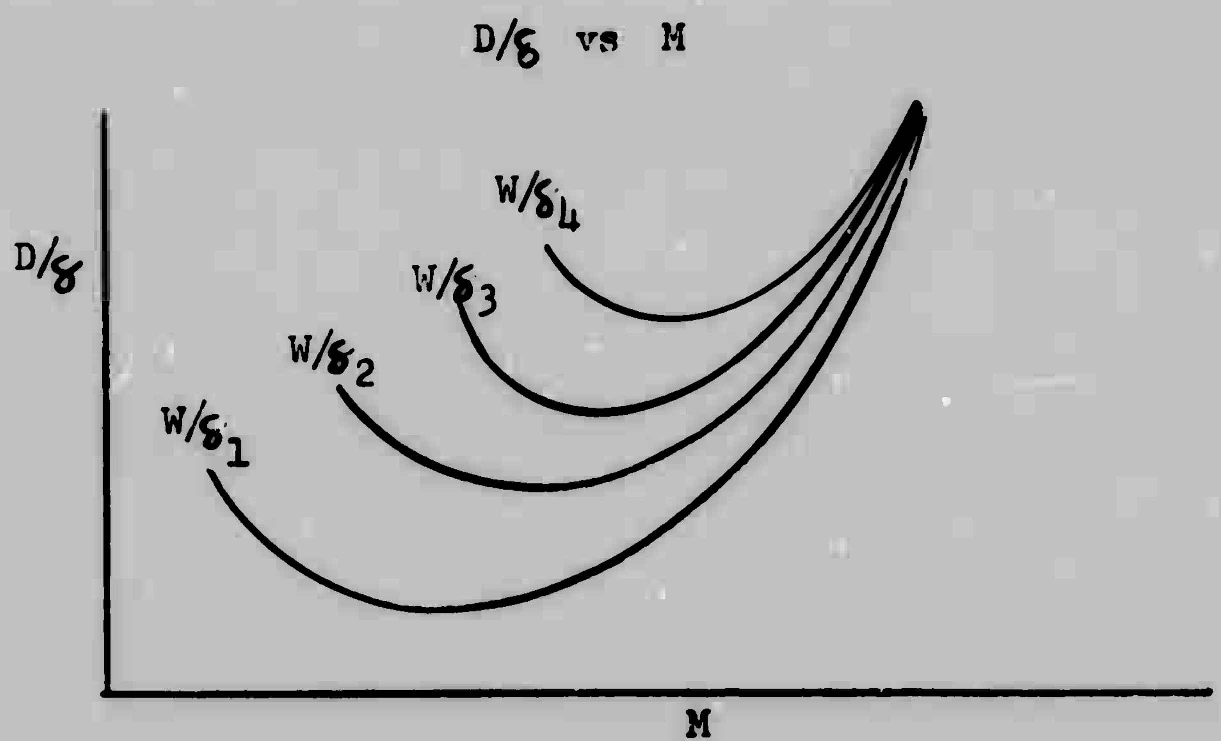


Figure 3.2.22

These curves will apply for all altitudes, temperatures and weights in the same manner as the PIW - VIW plots. If the drag data at a particular altitude is desired, the value of drag and weights which apply to the D/δ and W/δ values on the plots can be determined by multiplying these parameters by the pressure ratio, δ, for the altitude in question. Thus, plots of drag vs Mach number can be obtained for any combinations of weights or altitudes.

This plot is extremely useful if it can be obtained. Unfortunately drag is not easily obtained for jet aircraft. Measurement of drag is generally made by measuring thrust in stabilized level flight and equating it to the drag. These measurements require the use of engine manufacturers data or tail-pipe probes to measure total static pressure. Either method requires a somewhat involved data reduction process to obtain acceptable answers. Therefore, an alternative presentation in terms of more easily measured engine parameters (RPM, EGT, pressure ratio, etc) is desired. In order to determine the proper parameters and their functional relationships it is necessary to resort to dimensional analysis.

3.2.3.3 DIMENSIONAL ANALYSIS FOR TURBOJET ENGINES

The most convenient method of dimensional analysis for the determination of the performance parameters is the Buckingham π Theorem which will give them in nondimensional form. Consider a simple single spool jet engine. The independent variables which affect the performance of a simple jet engine are engine speed, N, ambient pressure, P_a, exhaust nozzle area, A, and compressor inlet total pressure and temperature, P_{t_2} and T_{t_2}. The numerical

subscripts refer to the standard engine station notation shown on the diagram below.

SINGLE SPOOL TURBOJET ENGINE WITH AFTERBURNER

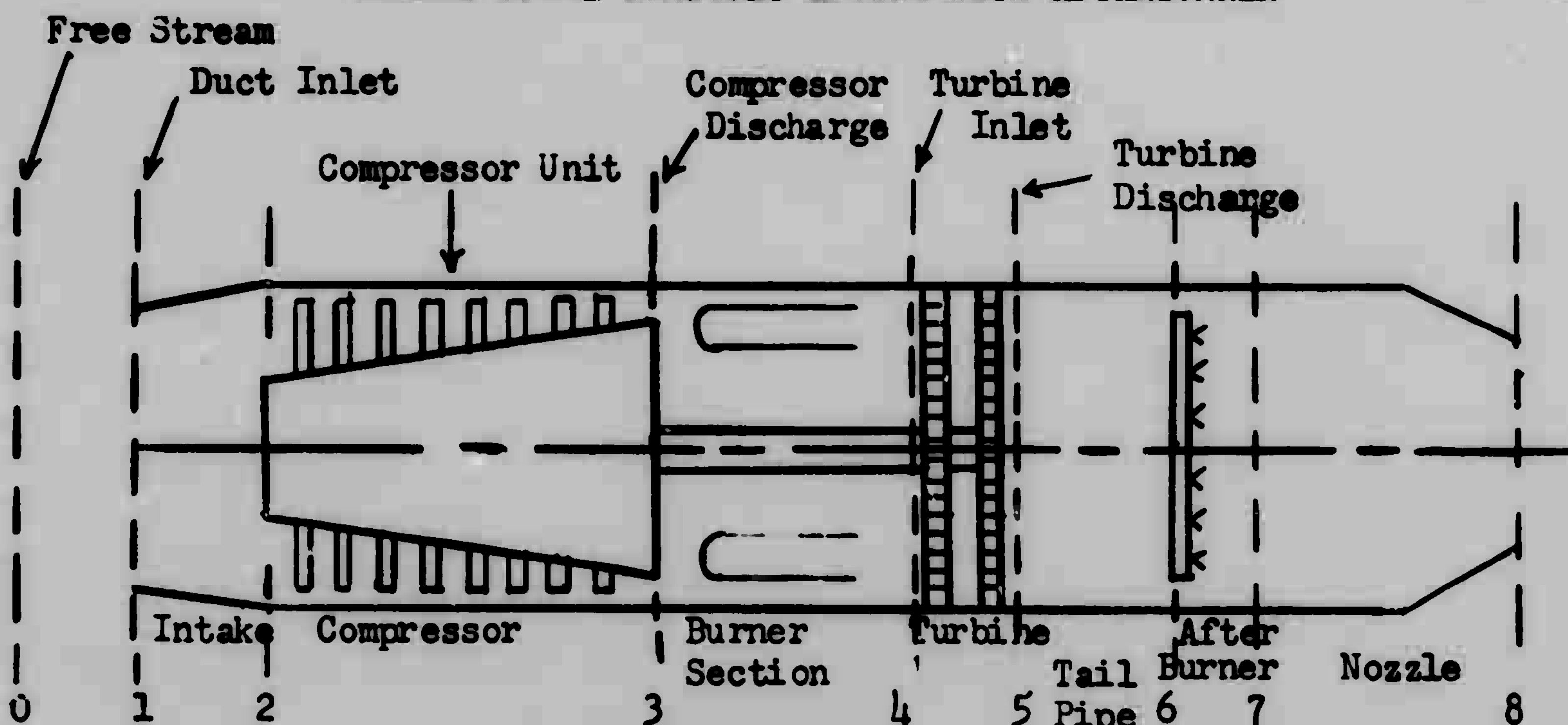


Figure 3.2.23

The subscript t refers to total or stagnation pressure and temperature.

The performance parameters of interest which depend on the above variables are thrust, F_g, fuel flow, W_f, air flow, W_a and turbine discharge temperature EGT, T_{t_5}. The functional relationships are shown in the following equations:

THRUST: $F_g = f\,(N, P_a, A, P_{t_2}, T_{t_2})$

FUEL FLOW: $W_f = f\,(N, P_a, A, P_{t_2}, T_{t_2})$

AIR FLOW: $W_a = f\,(N, P_a, A, P_{t_2}, T_{t_2})$

TEMPERATURE: $T_{t_5} = f\,(N, P_a, A, P_{t_2}, T_{t_2})$

Note that all of the dependent variables (F_g, W_f, W_a, T_{t_5}) are functions of the same five independent variables. We will perform the dimensional analysis on the thrust relationship to develop the dimensionless parameters.

$$f\ (F_g,\ N,\ P_a,\ A,\ P_{t_2},\ T_{t_2}) = 0$$

Variables		Dimensions
F_g	Thrust, lbs	F
N	RPM, rad/sec	1/t
P_{t_2}	Inlet Pressure, lb/ft^2	F/L^2
T_{t_2}	Inlet Temperature, energy unit mass	L^2/t^2
P_a	Ambient Pressure, lb/ft^2	F/L^2
A	Exhaust Noz. Area, Ft^2	L^2

L = length F = force t = time

Observe that there are six variables and three basic dimensions. Therefore, n - m = 3, indicating that three dimensionless parameters will be obtained and the relationship will be of the form

$$\pi = f\ (\pi\ 1, \pi\ 2, \pi\ 3) = 0$$

Expressing the original relationship in exponential form

$$\pi = F_g^{\ a}\ N^b\ P_{t_2}^{\ c}\ T_{t_2}^{\ d}\ P_a^{\ e}\ A^f = \text{constant}$$

Any one of the exponents may be set equal to one without changing the final result; however, this still leaves a set of three equations with five unknowns when the exponents of like units are equated.. In order to obtain a solvable set of equations (3 equations and 3 unknown) we can set any two exponents equal to zero and solve for $\pi 1$. This is permissible as long as these same two variables are not set equal to zero when solving for $\pi 2$ and $\pi 3$.

<u>Parameter $\pi 1$</u>

let a = 1 and e & d = 0

$$\pi 1 = F_g \; N^b \; P_{t_2}{}^c \; A^f = 0$$

substituting the basic dimensions

$$\pi 1 = (F)^1 (1/t)^b (F/L^2)^c (L^2)^f$$

equating the exponents of the same dimensions and solving

F: $1 + c = 0$ $\qquad$ $c = -1$

L: $-2c + 2f = 0$ $\qquad$ $f = -1$

$-b = 0$

therefore, substituting into the expression for $\pi 1$

$$\pi 1 = F_g / (P_{t_2} \; A)$$

In the same manner $\pi 2$ & $\pi 3$ are obtained.

Parameter $\pi 2$

let b = 1 a = 0 c = 0

$$\pi_2 = N \; T_{t_2}^{\,d} \; P_a^{\,e} \; A^f = 0$$

$$\pi_2 = (1/t)\;(L^2/t^2)^d\;(F/L^2)^e\;(L^2)^f$$

equating exponents and solving

F: e = 0

L: 2d - 2e + 2f = 0

t: -1 -2d = 0   d = - 1/2

f = e - d = - d = 1/2

substituting into original expression for $\pi 2$

$$\pi 2 = N \; T_{t_2}^{-\frac{1}{2}} \; A^{\frac{1}{2}} = NA^{\frac{1}{2}}/T_{t_2}^{\frac{1}{2}}$$

Parameter $\pi 3$

let c = 1   a = 0   b = 0

$$\pi 3 = P_{t_2} \; T_{t_2}^{\,d} \; P_a^{\,e} \; A^f$$

$$\pi 3 = (F/L^2)\;(L^2/t^2)^d\;(F/L^2)^e\;(L^2)^f$$

equating exponents and solving

F: 1 + e = 0   e = - 1

L: - 2 + 2d = 2e + 2f = 0

t: - 2d = 0   d = 0

f = e = d + 1 = - 1 + 0 + 1 = 0

substituting into the original express for $\pi 3$

$$\pi 3 = P_{t_2} \; P_a^{-1} = P_{t_2}/P_a$$

therefore,

$$\pi = f(\pi_1, \pi_2, \pi_3) = 0$$

$$\pi = f(F_g/P_{t_2} A,\ NA^{\frac{1}{2}}/\sqrt{T_{t_2}},\ P_{t_2}/P_a)$$

Note: All of the parameters are dimensionless.

For a constant area engine that is one without inlet guide vanes or variable exhaust nozzle, the functional relationship may be written

$$\pi = f(F_g/P_{t_2},\ N/\sqrt{T_{t_2}},\ P_{t_2}/P_a)$$

or $F_g/P_{t_2} = f(N/\sqrt{T_{t_2}},\ P_{t_2}/P_a)$ Fixed Geometry Engine

Temperatures and pressures are generally referred to sea level in the following way:

$$\delta_{t_2} = P_{t_2}/P_{a_{SL}} ;\ \theta_{t_2} = T_{t_2}/T_{a_{SL}}$$

Substituting

δ_{t_2} for P_{t_2} and θ_{t_2} for T_{t_2} gives

$$F_g/\delta_{t_2} = f(N/\sqrt{\theta_{t_2}},\ P_{t_2}/P_a)$$

Note that the parameters are no longer dimensionless but they are still the primary variables which describe the performance of a constant area jet engine.

The pressure ratio P_{t_2}/P_a is expressed in terms of δ's since

$$\frac{\delta_{t_2}}{\delta_a} = \frac{\dfrac{P_{t_2}}{P_{a_{SL}}}}{\dfrac{P_a}{P_{a_{SL}}}} = \frac{P_{t_2}}{P_a}$$

thus, nothing is gained by doing so since the numerical value is unchanged.

A similar analysis of fuel flow (W_f), air flow (W_a), and turbine discharge temperature (T_{t_5}) give similar functional relationships

$$W_f/\delta_{t_2}\sqrt{\theta}_{t_2} = f\left(N/\sqrt{\theta}_{t_2},\ P_{t_2}/P_a\right)$$

$$W_a\sqrt{\theta}_{t_2}/\delta_{t_2} = f\left(N/\sqrt{\theta}_{t_2},\ P_{t_2}/P_a\right)$$

$$T_{t_5}/\theta_{t_2} = f\left(N/\sqrt{\theta}_{t_2},\ P_{t_2}/P_a\right)$$

Thus far, we have considered only the independent variables (P_{t_2}, T_{t_2}, N, P_a, etc) which have a primary effect on performance. Other factors such as viscous effects, combustion efficiency, ratio of specific heats, have secondary effects on performance particularly at high altitudes and high Mach numbers. Engine manufacturers frequently publish correction curves to be used in conjunction with non-dimensional performance curves which account for these secondary effects. Errors which result by neglecting these effects at present day speeds and altitudes are minor, and are within the accuracies of flight test data. However, as speed and altitude of future aircraft capabilities increase, secondary effects may have to be considered.

3.2.3.4 ENGINE PARAMETERS

Dimensional analysis of engine parameters have shown us that both F_g/δ_{t_2} and $W_f/\delta_{t_2}\sqrt{\theta_{t_2}}$ are functions of $N/\sqrt{\theta_{t_2}}$ and P_{t_2}/P_a

Noting that

$$P_{t_2}/P_a = P_{t_2}/P_{t_o} \times P_{t_o}/P_a = P_{t_2}/P_{t_o} \; (1 + .2M^2)^{3.5}$$

$$\text{where } P_{t_o}/P_a = (1 + .2M^2)^{3.5}$$

and that $P_{t_2}/P_{t_o} = f \text{ (inlet efficiency)} = f\,(M)$

Since P_{t_2}/P_a is a function of Mach number we may replace P_{t_2}/P_a

$$F_g/\delta_{t_2} = f\,(N/\sqrt{\theta_{t_2}},\; M)$$

$$W_f/\delta_{t_2}\sqrt{\theta_{t_2}} = f\,(N/\sqrt{\theta_{t_2}},\; M)$$

A typical graphical solution of these relationships is shown in the following plots.

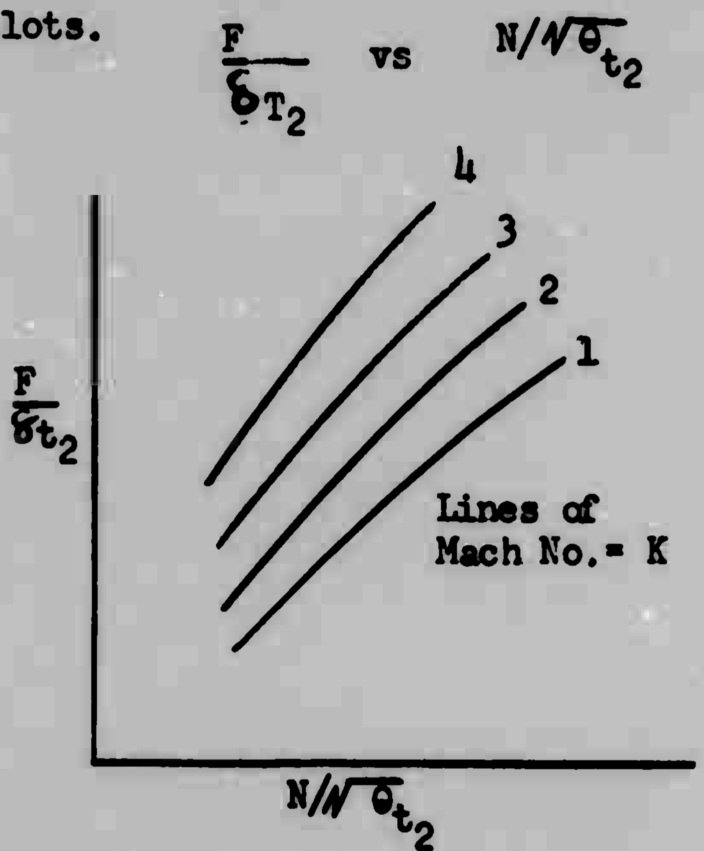


Figure 3.2.24a

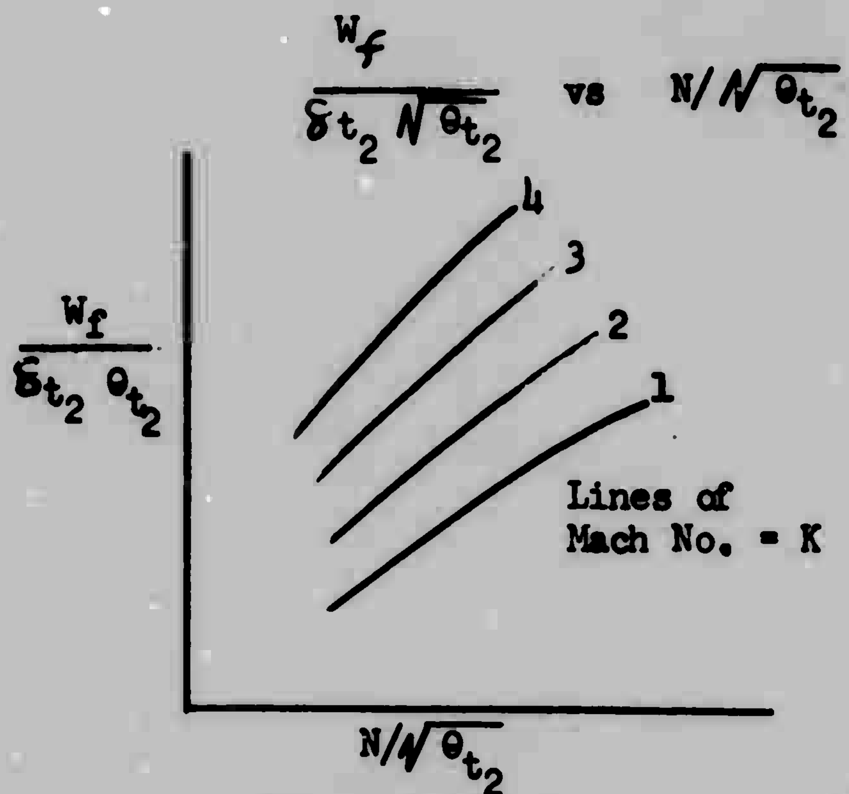


Figure 3.2.24b

Note that these plots represent the engine performance at all altitudes, temperatures and Mach numbers.

3.2.3.5 ENGINE SPEED PARAMETER

From earlier discussions, it was shown that $D/\delta = f\,(W/\delta\,,\,M)$ also, it was stated that drag was not easily obtained. We now can show that jet aircraft performance can be expressed in terms of $N/\sqrt{\theta}$ as an alternative to D/δ. For stabilized flight $D/\delta = F/\delta$.

$$D/\delta = f\,(W/\delta\,,\,M) = F/\delta = f\,(N/\sqrt{\theta}\,,\,M)$$

or $N/\sqrt{\theta} = f\,(W/\delta\,,\,M)$ **Equation 3.2.10**

Note: The subscripts t_2 have been dropped since the relationships between pressures and temperatures at the compressor face and free stream are functions of Mach number and, therefore, do not change the functional relationship above.

The variables in this relationship are easy to measure and $N/\sqrt{\theta}$ is as valid a parameter as D/δ.

Typical plots of these two variables show their similarity

$D/\delta = f\,(W/\delta\,,M)$

D/δ vs M

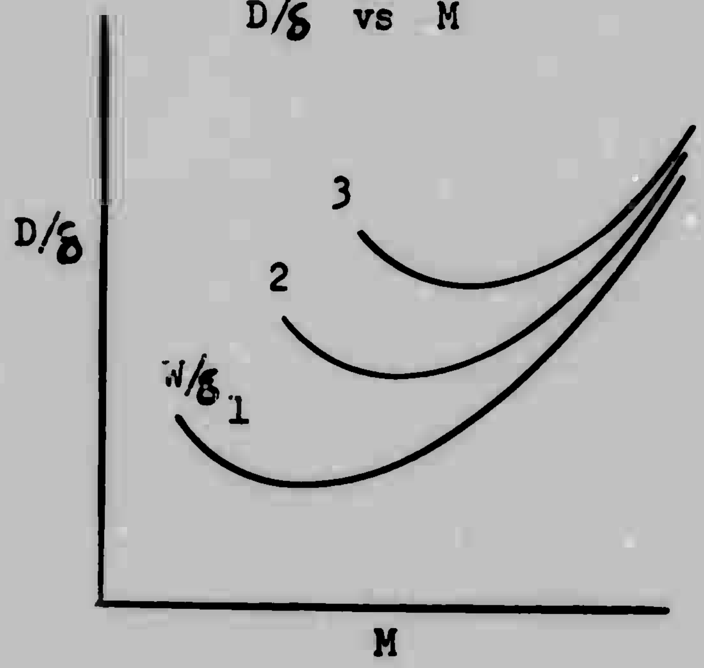


Figure 3.2.25a

$N/\sqrt{\theta} = f\,(W/\delta\,,\,M)$

$N/\sqrt{\theta}$ vs M

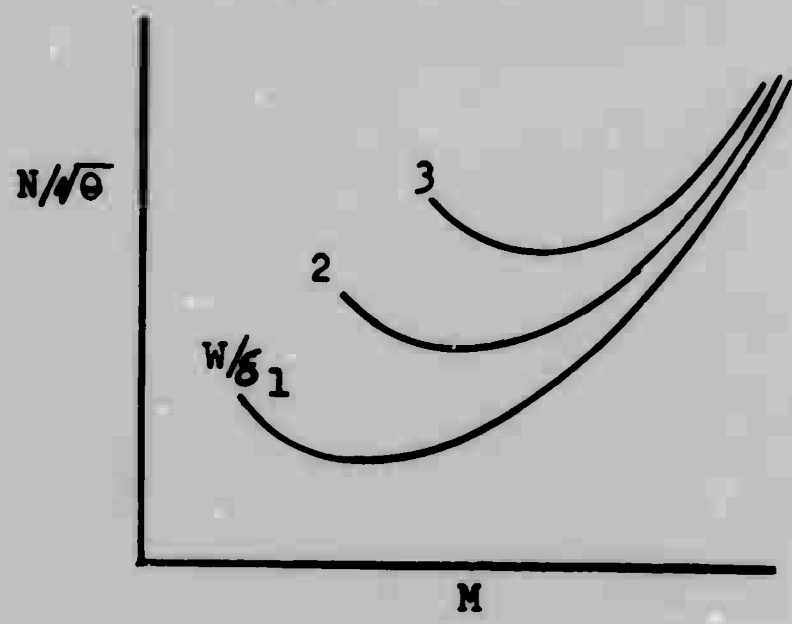


Figure 3.2.25b

Both of these plots are valid for all speeds, altitudes and weights and temperatures.

3.2.3.6 FUEL FLOW PARAMETER

Engine analysis yielded $W_f/\delta\sqrt{\theta}$ as a function of $N/\sqrt{\theta}$ and Mach number. Further analysis of the airplane, that is, equating heat energy input as a function of the airplane drag energy yield the following fuel flow parameter

$W_f/\delta\sqrt{\theta} = f\,(W/\delta\ ,\ M)$ Equation 3.2.11

or $= f\,(N/\sqrt{\theta}\ ,\ W/\delta\ )$ Equation 3.2.12

3.2.3.7 TURBOJET LEVEL FLIGHT PERFORMANCE

With the preceeding information, level flight performance of jet aircraft may be presented in two generalized plots. These plots are shown below.

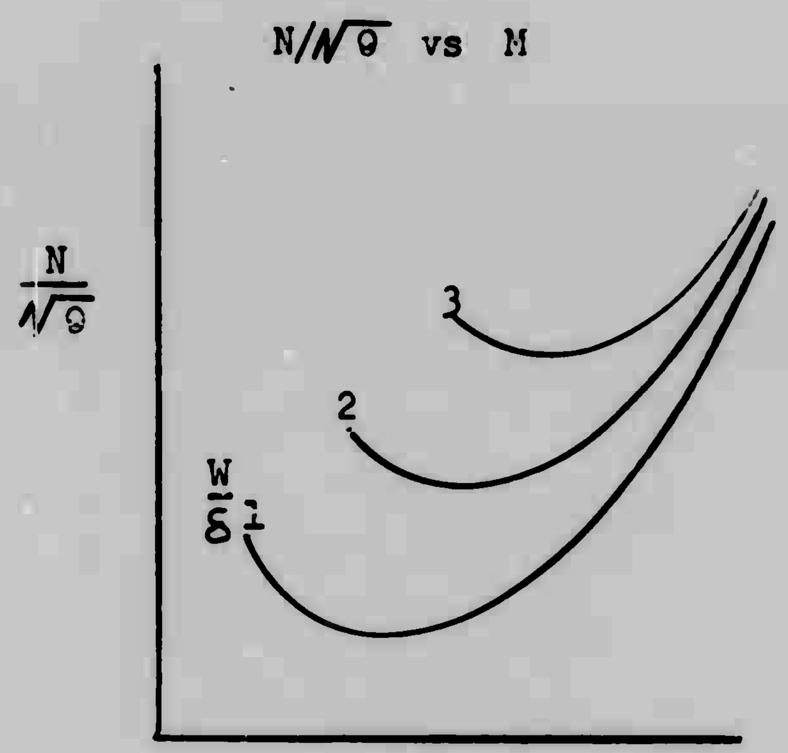


Figure 3.2.26A

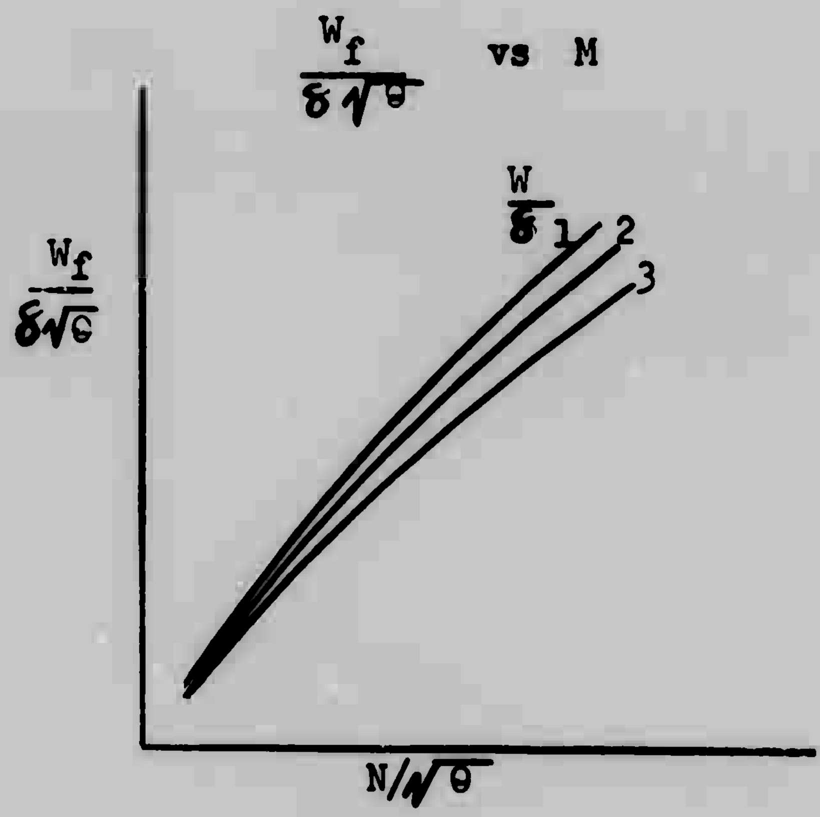


Figure 3.2.26b

These plots, though containing all of the information necessary to describe the aircrafts performance, are not easily used in this form. The information has been so cleverly disguised as to be unusable to the uninitiated pilot. To translate the data presented in these plots to something that can be read on cockpit instruments, one need only determine the Mach number and W/δ at which you are flying and read the corresponding value of $N/\sqrt{\theta}$ and $W_f/\delta\sqrt{\theta}$ off of the plots. The <u>test day</u> RPM and fuel flow may be obtained by multiplying by $\sqrt{\theta}$ or $\delta\sqrt{\theta}$ to eliminate the denominator of the parameters.

$$RPM_{test\ day} = N/\sqrt{\theta} \times \sqrt{\theta}_{test}$$

$$W_{f_{test\ day}} = W_f/\delta\sqrt{\theta} \times \delta\sqrt{\theta}_{test}$$

From W_f and V_t the cruise performance (specific range) can be obtained, where

specific range = V_t/W_f nautical miles per pound.

This information is normally presented for a <u>standard day</u> in the back of the flight manual in a form similar to that shown in the following figures.

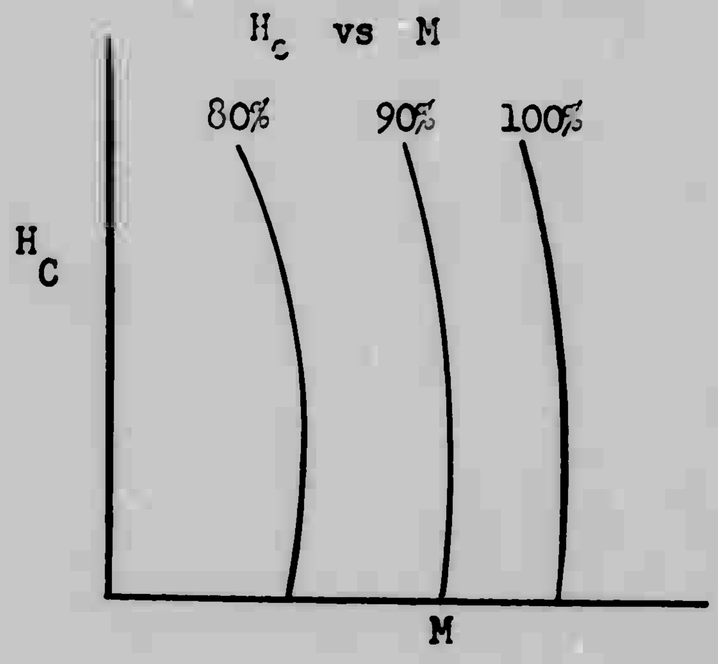


Figure 3.2.27a

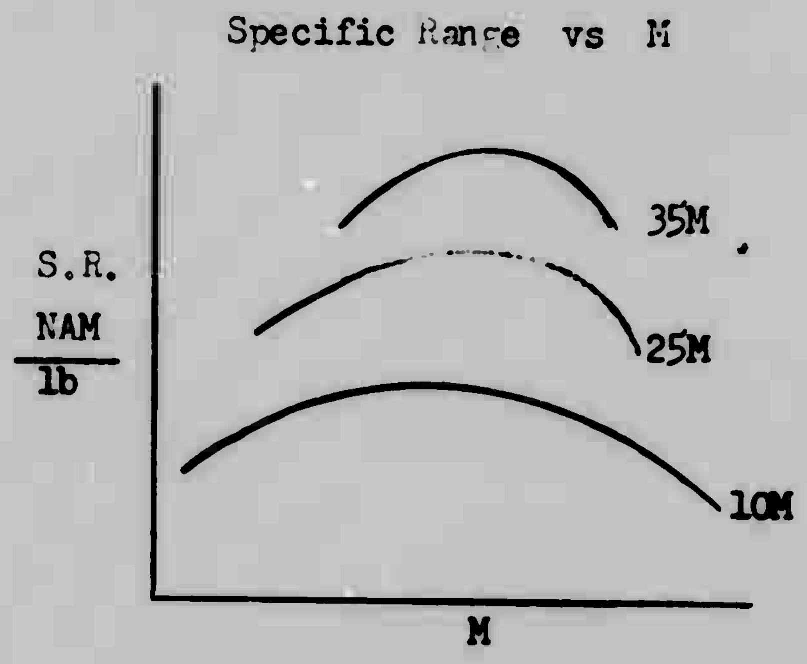


Figure 3.2.27b

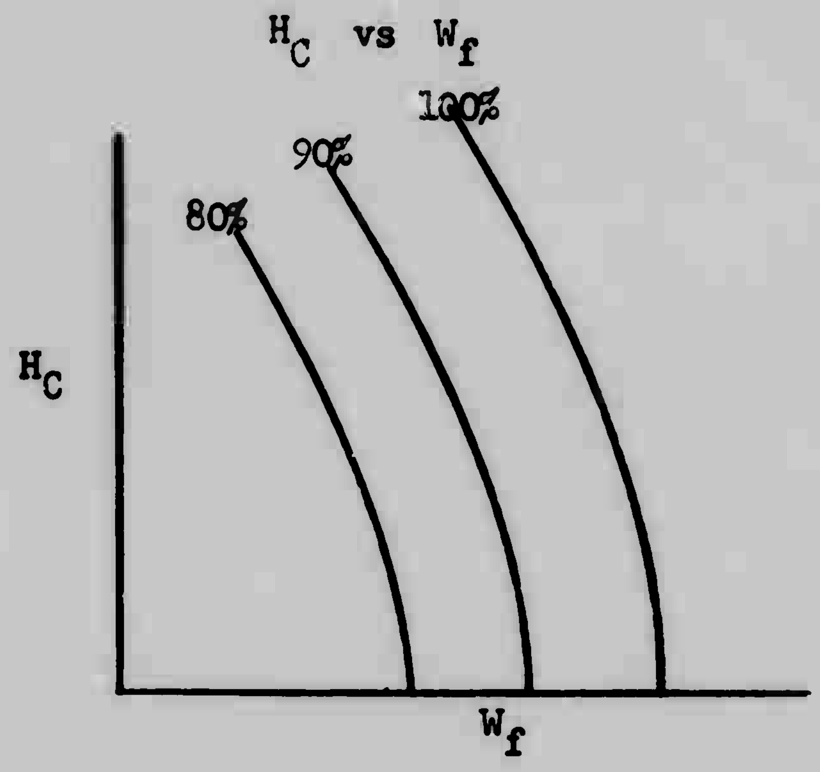


Figure 3.2.27c

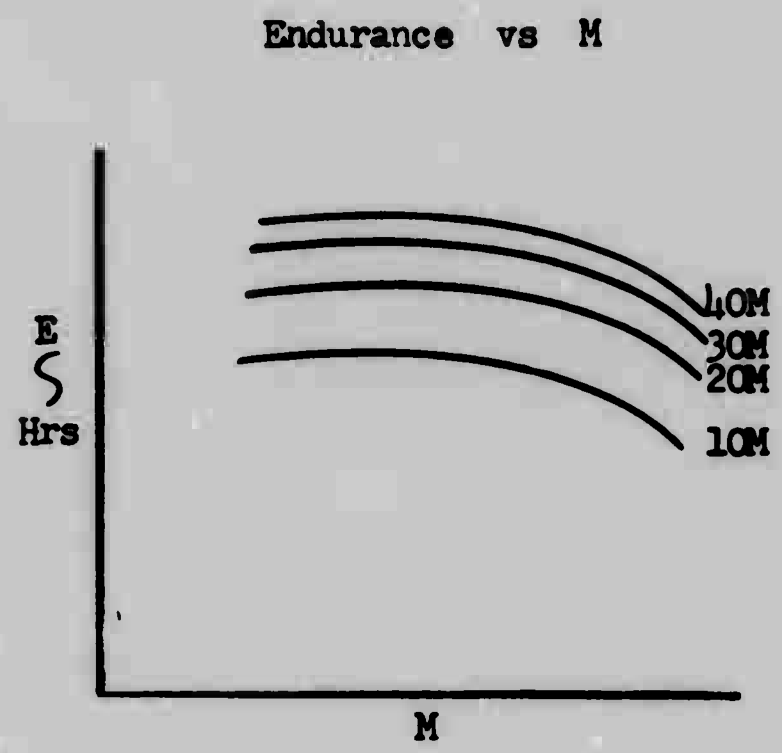


Figure 3.2.27d

Note: These plots are good for a standard day only. If data is required for other temperature conditons another set of plots would be required.

3.2.3.8 CONCLUSION

The purpose of this section was to show how basic aerodynamic theory can be applied to flight testing when it is desired to determine standard performance from test day results. Also, it was shown that jet performance could be expressed in terms of parameters more easily obtained during flight tests. There are a few assumptions used which should be discussed. First, it was assumed that the parasite drag coefficient is a constant for a given aircraft configuration. The parasite drag coefficient actually varies with Reynolds number but the changes are small so in most cases this assumption is valid. At the same speed, Reynolds number decreases with altitude so the results obtained at low altitude are not absolutely correct at high altitude and vise versa due to the change in parasite drag coefficient. Jet engine performance also varies with Reynolds number as well as combustion efficiency

and the ratio of specific heats, however, these effects were ignored.

The results obtained for transonic speeds may apply to supersonic flight testing but the reasoning used to verify these results is not applicable to supersonic flight. The concept of induced drag (or drag due to lift) as used for subsonic flight does not apply to supersonic flight. Other new concepts of drag at supersonic speeds, such as wave drag, must be considered to completely understand the drag characteristics of supersonic airplanes. However, the plots presented in this section will have applications in stabilized supersonic flight.

3.2.4 RANGE AND ENDURANCE

3.2.4.1 INTRODUCTION

The following derivations of the Breguet equations are used to show how various parameters effect the range and endurance of an aircraft. The equations will be separated with respect to the type of propulsion.

3.2.4.2 RECIPROCATING ENGINE - PROPELLER COMBINATION

The computation of range and endurance for a reciprocating engine aircraft can be simplified if the specific fuel consumption, c_r, and the propulsive efficiency, η_p, are assumed to be constant for the cruising speeds under consideration.

A. ENDURANCE:

The fuel weight will decrease with respect to time and is expressed by the following equation:

$$dW = -c_r\,P\,dt$$

where: dW = change of weight, pounds

dt = change of time, either min. or hrs.

P = brake horsepower output

c_r = specific fuel consumption $\frac{\#\ \text{fuel}}{\text{Bhp - hr}}$

Rearranging, the equation becomes $dt = -\dfrac{dW}{c_r\,P}$ Equation 3.2.13

$$E = \text{endurance} = \int dt = -\int_{W_o}^{W_1} \frac{dW}{c_r\,P}$$

now by rearranging $E = \displaystyle\int_{W_1}^{W_o} \frac{dW}{c_r\,P}$

where W_o = initial gross weight

$W_1 = W_o - W_f$ = final gross weight

For stabilized flight $P \times \eta_p = \dfrac{D \times V}{550}$

Substituting for P in the endurance equation

$E = \displaystyle\int_{W_1}^{W_o} \frac{\eta_p\,dW\,550}{c_r\,VD}$ and multiplying by

W/W the equation becomes $E = 550 \displaystyle\int_{W_1}^{W_o} \frac{\eta_p\,dW}{c_r\,VD}\,\frac{W}{W}$

For stabilized flight L = W and the equation can now be rewritten as

$$E = \frac{550\,\eta_P}{c_r}\frac{C_L}{C_D}\int_{W_1}^{W_0}\frac{dW}{W\,V}$$

also $V = \sqrt{\frac{2\,W}{\rho S\,C_L}}$ or $\left(\frac{2\,W}{\rho S\,C_L}\right)^{\frac{1}{2}}$

Substituting this value of V in the equation

$$E = \left(\frac{\eta_P}{c_r}\right)\frac{C_L^{3/2}}{C_D}\left(\frac{\rho\,S}{2}\right)^{\frac{1}{2}} 550\int_{W_1}^{W_0}(W)^{-3/2}\,dW$$

Intergrating the equation becomes

$$E = \left(\frac{\eta_P}{c_r}\right)\frac{C_L^{3/2}\sqrt{2\rho S}}{C_D}\left(\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}}\right)550$$

The constants are combined and the final equation becomes:

$$E = 778\left(\frac{\eta_P}{c_r}\right)\frac{C_L^{3/2}\sqrt{\rho S}}{C_D}\left(\frac{1}{\sqrt{W_1}} - \frac{1}{\sqrt{W_0}}\right) \qquad \text{Equation 3.2.14}$$

Thus it is seen to obtain maximum endurance for a reciprocating engine aircraft it is required:

1. That the aircraft fly at a speed where $\frac{C_L^{3/2}}{C_D}$ is a maximum value.

2. The specific fuel consumption be at a minimum.

3. That the density, ρ, be a maximum, i.e., fly at the lowest possible altitude.

4. The largest possible fuel load be carried.

In actual flight the specific fuel consumption for a reciprocating engine decreases slightly with an increase in altitude. The same manifold pressure at a high altitude will produce more BHP than the HP at a lower altitude. This is due to decreased back pressure and increased density of the air due to a lower temperature. An altitude compromise must be made where $\frac{\sqrt{\rho}}{c_r}$ is a maximum.

B. RANGE:

Working with the time and distance relationship $R = \int ds = V\int dt =$ range, substituting $dt = -\frac{dW}{c_r P}$

in the equation $R = -V\int_{W_o}^{W_1}\frac{dW}{c_r P}$ and $P = \frac{DV}{\eta_P 550}$

$$\therefore R = \frac{\eta_P}{c_r}\int_{W_1}^{W_o}\frac{550\ dW}{D}$$ now multiplying by W/W

and L = W the equation becomes

$$R = \frac{\eta_P 550}{c_r}\int_{W_1}^{W_o}\frac{L}{D}\frac{dW}{W} = 550\left(\frac{\eta_P}{c_r}\right)\left(\frac{C_L}{C_D}\right)\int_{W_1}^{W_o}\frac{dW}{W}$$

$$R = \left(\frac{\eta_P}{c_r}\right)\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_o}{W_1}\right)(550)$$

Expressed in statute miles and c_r in lb/BHP-hr the equation becomes

$$R = 375\left(\frac{\eta_P}{c_r}\right)\left(\frac{C_L}{C_D}\right)\ln\left(\frac{W_o}{W_1}\right) \qquad \text{Equation 3.2.15}$$

Thus it can be seen from the investigation of the above equation to obtain maximum range with respect to each parameter that:

1. The specific fuel consumption be at a minimum.

2. The flight be conducted at a speed where L/D or C_L/C_D is a maximum.

3. The ratio of W_o/W_1 be as large as possible, i.e., the aircraft be designed so that the fuel weight be as large as possible in comparison to the empty gross weight.

4. It is interesting to note that altitude does not enter directly into the range equation for this type power plant. However, a decrease in specific fuel consumption with increase in altitude would increase the range. The reason for this decrease has been discussed in the endurance part of this section. This is the primary reason for a reciprocating engine aircraft to cruise at altitude, disregarding terrain clearances and weather conditions.

Due to weight limitations and limitations of the airframe, engine, and propeller efficiency, the range of this type aircraft is definitely limited. In actual practice, due to the pilots inability to fly the aircraft at the required C_L/C_D ratio and due to no allowance being made for the type of mission flown, the operational values should never be taken as greater than 80-85% of the theoritical value.

C. ANALYSIS OF C_L/C_D RATIOS:

$$C_D = C_{D_p} + C_L^2/\pi AR\, e$$

Investigate first the endurance equation. Divide both sides by $1/C_L^{3/2}$

$$\frac{C_L^{3/2}}{C_D} = \frac{C_L^{3/2}}{C_{D_p} + C_L^2/\pi AR\, e}$$

The condition for maximum endurance may be found by setting the derivative of

$$\frac{C_L^{3/2}}{C_D} = 0 \text{ and solving}$$

$$\frac{\partial}{\partial C_L}\left(\frac{C_L^{3/2}}{C_D}\right) = 0 = \frac{\left(C_{D_p} + C_L^2/\pi AR\, e\right)\left(3/2\, C_L^{\frac{1}{2}}\right) - \left(C_L^{3/2}\right)\left(2\, C_L/\pi AR\, e\right)}{\left(C_{D_p} + C_L^2/\pi AR\, e\right)^2}$$

Multiply both sides by $(C_{D_p} + C_L^2/\pi AR\, e)^2$ we have $(C_{D_p} + C_L^2/\pi AR\, e)\, 3/2 = C_L\, (2\, C_L/\pi AR\, e)$

$$\therefore\; C_{D_p} = C_L^2/3\pi AR\, e = C_{D_i}/3$$

Therefore, to obtain best endurance the aircraft is required to fly at an angle of attack or a speed where the parasite drag is 1/3 of the induced drag.

Investigation of the range equation is shown below. Divide the coefficient of drag oquation by $1/C_L$ and it becomes

$$\frac{C_L}{C_D} = \frac{C_L}{C_{D_p} + C_L^2/\pi AR\, e}$$

The condition for maximum range may be found by setting the derivative of $C_L/C_D = 0$ and solving

$$\frac{\partial}{\partial C_L}\left|\frac{C_L}{C_D}\right| = 0 = \frac{\left(C_{D_p} - C_L^2/\pi AR\, e\right) - C_L\ (2\, C_L/\pi AR\, e)}{\left(C_{D_p} + C_L^2/\pi AR\, e\right)^2}$$

Multiplying both sides by $(C_{D_p} + C_L^2/\pi AR\ e)^2$

$$C_{D_p} = C_L^2/\pi AR\ e = C_{D_i}$$

Therefore, to obtain maximum range it is necessary to fly at an angle of attack or a speed where $C_{D_p} = C_{D_i}$, or the point of minimum drag.

Figure 3.2.28a shows the relationship of D versus V_t for a typical reciprocating engine aircraft. Figure 3.2.28b shows the relationship of THP_r versus V_t for the same aircraft.

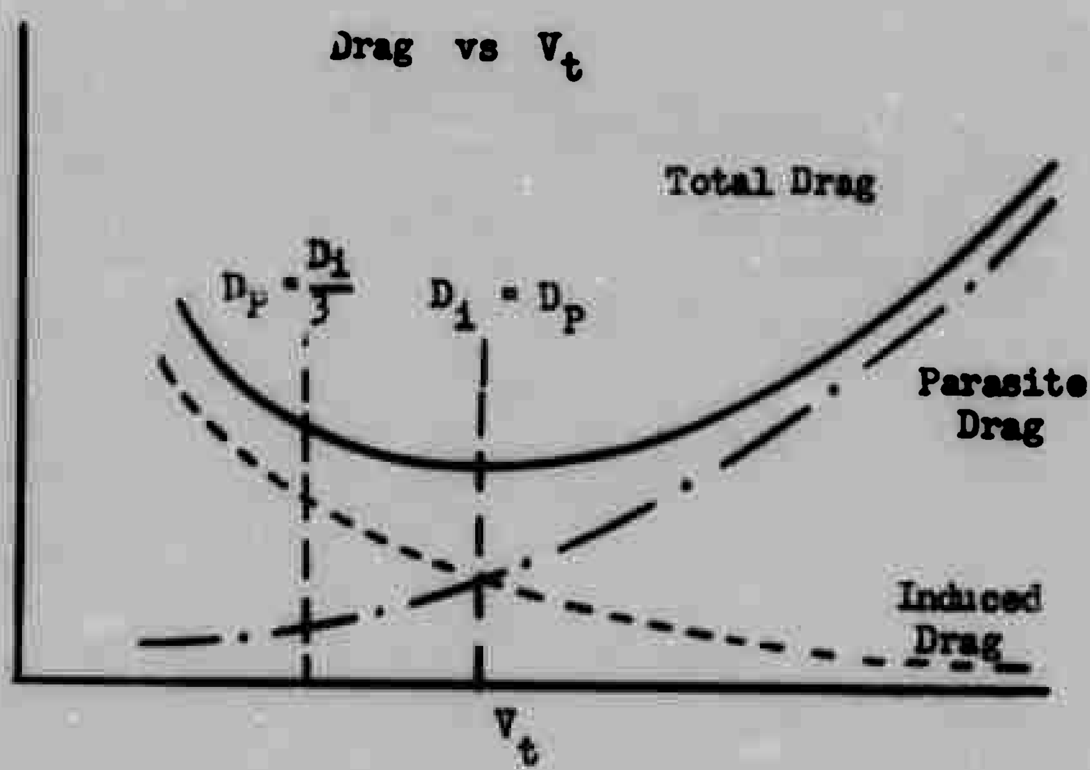


Figure 3.2.28a

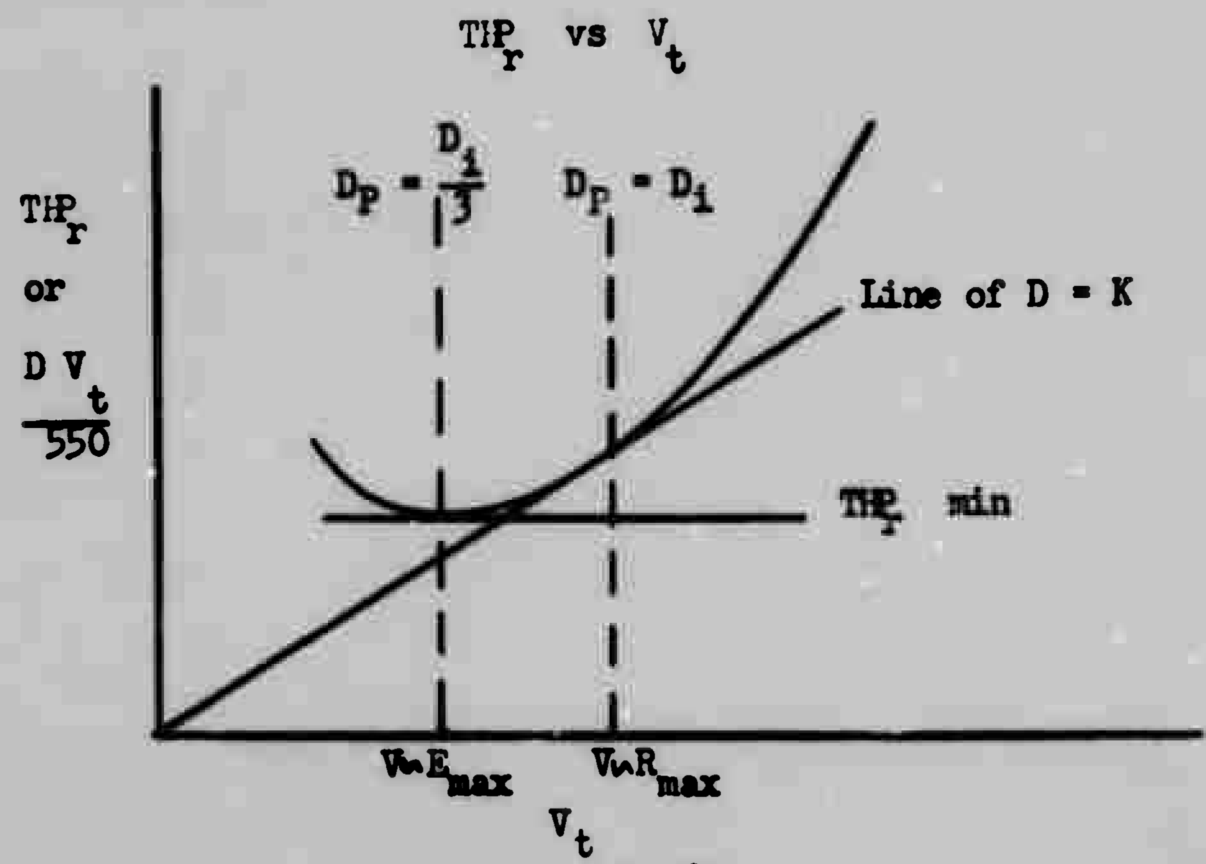


Figure 3.2.28b

The speed for maximum endurance is the speed where minimum THP_r occurs or where the horizontal target touches the curve. The speed for maximum endurance is also the speed for the minimum #fuel/hr. The speed for maximum range occurs where the tangent to the curve is drawn from the origin. The maximum range speed can also be referred to as the speed for the maximum ratio of V_t/#fuel-hr.

D. HEADWIND AND TAILWIND EFFECTS:

The thrust horsepower required curve is drawn in reference to true airspeed. When a steady wind acts on the aircraft the shape of the curve is not changed,but the true airspeed is no longer equal to the ground speed. The endurance remains the same but the range is effected.

The velocity for maximum range with a headwind or tailwind may be determined by locating new origins on the abscissa. These origins are displaced from the first origin by the amount of either the headwind or tailwind component. Tangents are drawn from these new origins to the same curve and the speed for maximum range determined, reference Figure 3.2.29.

The figure indicates, in order to obtain maximum range in a headwind the true airspeed should be increased slightly and decreased slightly in case of encountering a tailwind. In both cases the amount of true airspeed changed will be less than the wind component encountered.

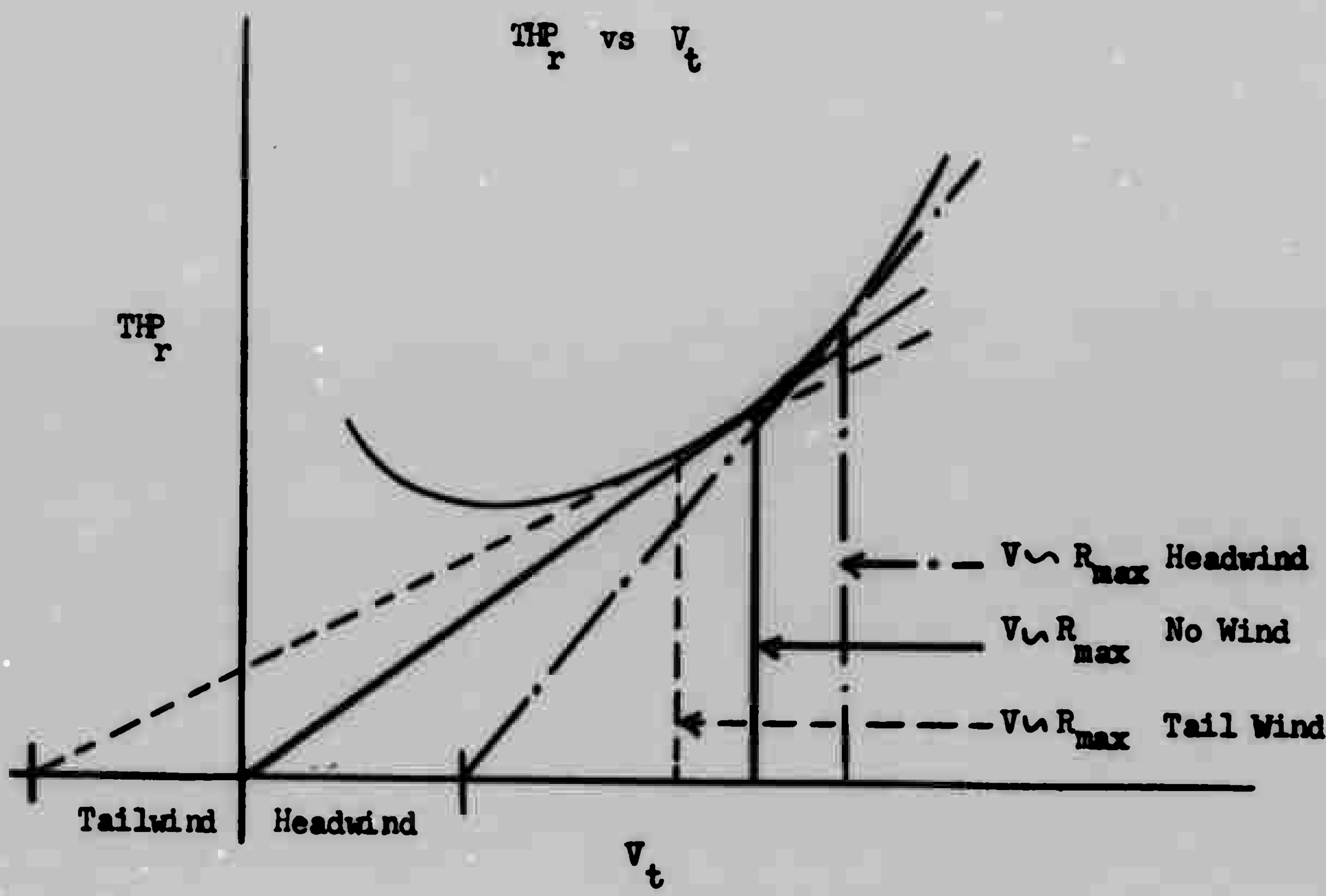


Figure 3.2.29

3.2.4.3 TURBOJET ENGINE POWERED AIRCRAFT

The turbojet engine specific fuel consumption, C_J, is dependent on the following parameters; thrust developed, free air temperature, true airspeed and ambient air pressure. The units are lb-fuel/(lb-thrust)(hr). The variation of the specific fuel consumption at a specified altitude and Mach number does not vary greatly throughout the cruising range of the flight region covered and is assumed to be constant for this development.

A. ENDURANCE

The weight of fuel on board decreases with respect to time.

$$dW = - C_J \; T \, dt \qquad \text{Equation 3.2.16}$$

$$\text{Endurance} = E = \int dt = - \int_{W_o}^{W_1} \frac{dW}{C_J \, T}$$

In stabilized level flight T = D and L = W

$$E = \int_{W_1}^{W_o} \frac{dW}{C_J \, D} \frac{W}{W} = \frac{L}{C_J \, D} \int_{W_1}^{W_o} \frac{dW}{W}$$

$$E = \frac{1}{C_J}\left(\frac{L}{D}\right) \ln\left(\frac{W_o}{W_1}\right) \qquad \text{Equation 3.2.17}$$

This final equation gives the total endurance in hours.

In order to obtain maximum endurance for a turbojet powered aircraft the following parameter magnitudes are listed.

1. The aircraft must fly at a speed where L/D or C_L/C_D is a maximum

2. The empty gross weight be as light as possible and the fuel load be as large as possible.

3. Altitude does not directly appear as a factor in the equation. However, some slight decrease in specific fuel consumption can often be made by going to a higher altitude. The amount of decrease depends upon the individual power plant.

B. RANGE

Again using the time and distance formulas

$$R = \int ds = V\int dt \qquad \text{and } dt = -\frac{dW}{C_J\,T}$$

$$R = -\int_{W_o}^{W_1} \frac{V\,dW}{C_J\;T}$$

In stabilized level flight L = W and T = D

$$V = (2\,W/\rho C_L\;S)^{1/2}$$

$$\therefore R = \int_{W_1}^{W_o} \frac{dW}{C_J}\left(\frac{2W}{\rho C_L\,S}\right)^{1/2}\left(\frac{C_L}{W\,C_D}\right)$$ Now arranging for R in nautical miles

$$R = \frac{1.929}{C_J\sqrt{\rho S}}\left(\frac{C_L^{1/2}}{C_D}\right)\left(\sqrt{W_o} - \sqrt{W_1}\right) \qquad \text{Equation 3.2.18}$$

Examination of this equation gives the following conclusions in obtaining maximum range.

1. The aircraft should be flown at a speed corresponding to $(C_L^{1/2}/C_D)_{max}$. However, this could be a speed where drag divergence due to critical Mach occurs. Both aspects of this should be investigated prior to setting up the cruise speed. In actual practice a Mach number is picked corresponding to a W/δ and this Mach number held constant.

2. The range will increase with decreasing density, ρ, or increasing altitude. However, an altitude limit can be reached where the range will be decreased or the aircraft is flying above its optimum altitude for a given weight. This is in essence flying at too high a W/δ. Some aircraft can be limited from reaching their optimum altitude by having the cruise speed approaching the one "g" stall or stall warning buffet. In other cases they are thrust limited and cannot reach the optimum W/δ.

4. An increase of the ratio $\sqrt{(W_o - W_1)/S}$ or fuel loading to wing area will increase the range of the aircraft.

C. ANALYSIS OF C_L/C_D RATIOS:

$$C_D = C_{D_p} + C_L^2/\pi AR\, e$$

The endurance equation evaluation of C_L/C_D to obtain a maximum value is the same as the reciprocating range equation evaluation. It was found to occur at the minimum drag point where $C_{D_p} = C_{D_i}$. It must be remembered for jet aircraft we deal in terms of thrust and not thrust horsepower.

The range equation will now be evaluated to find where the ratio $C_L^{1/2}/C_D$ is a maximum. Substitute the value of C_D into the ratio of $C_L^{1/2}/C_D$ and differentiate with respect to C_L. Set the derivative equal to 0 and solve to find the maximum value. It is found that the maximum condition occurs when $C_{D_p} = \frac{3\,C_L^2}{\pi AR\,e}$, or when the parasite drag is three times the induced drag.

The figure drawn below is a graphic solution for the endurance and range of a turbojet aircraft.

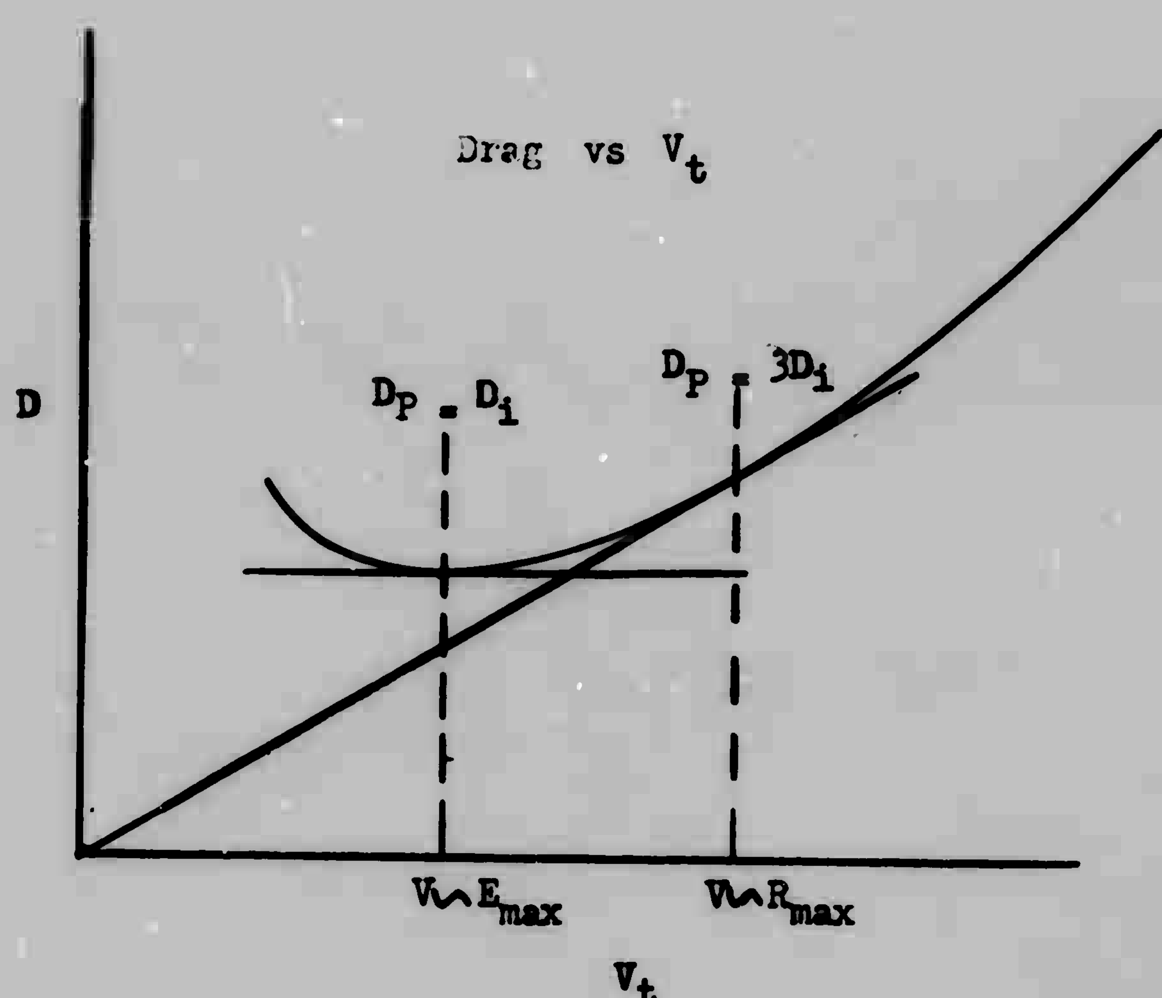


Figure 3.2.30

The solution of maximum range for a headwind or tailwind is similar to that employed for a reciprocating engine aircraft. However, the plot of thrust or drag versus V_t is used instead of THP_r vs V_t. The endurance is unaffected by wind.

3.2.4.4 MISCELLANEOUS:

The turboprop powerplant has characteristics that are combinations of the reciprocating engine propeller and turbojet powerplants. The rated output of the engine is in equivalent shaft horsepower - hr which is the sum of the propeller horsepower and the exhaust thrust of the engine. The specific fuel consumption C_{T_p}, lb-fuel/equiv - S-hp-hr is not constant. The speed to obtain maximum range is between those for the reciprocating engine and turbojet aircraft. This speed is then between V at $(C_L/C_D)_{max}$ and $(C_L^{1/2}/C_D)_{max}$. The range obtainable is increased as altitude is increased. At high altitude parts of the propeller obtain sonic velocity and result in a decreased η_p, which results in a decrease in both endurance and range. Thus, it is seen that the turboprop aircraft has a lower ceiling and lower V_{max} than turbojet aircraft. The design of an efficient supersonic propeller can increase both of these parameters.

The development of a range equation for the ramjet and rocket powered aircraft is given below. A turbojet engine with an afterburner is considered to be a combination of a turbojet and a ramjet.

$$dR = V\int dt \text{ and } dt = -\frac{dW}{cT} = -\frac{dW}{cD}$$

Multiplying by W/W and L = W the equation becomes

$$dR = -\frac{V}{c}\frac{L}{D}\int_{W_o}^{W_1}\frac{dW}{W}$$

$$R = \left(\frac{V}{c}\right)\left(\frac{L}{D}\right)\ln\left(\frac{W_o}{W_1}\right) \qquad \text{Equation 3.2.19}$$

Equation 3.2.19 is applied for any type of aircraft. The units of c will be required to be changed appropriate to the type power plant installed and will result in a slight modification of the final equation. Thus it is seen that the maximum range is obtained where the product of $\left(\frac{V}{c}\right)\left(\frac{L}{D}\right)$ is a maximum. The basic investigation of weight W_o/W_1 is the same as before.

Below is a typical diagram of L/D versus Mach Number.

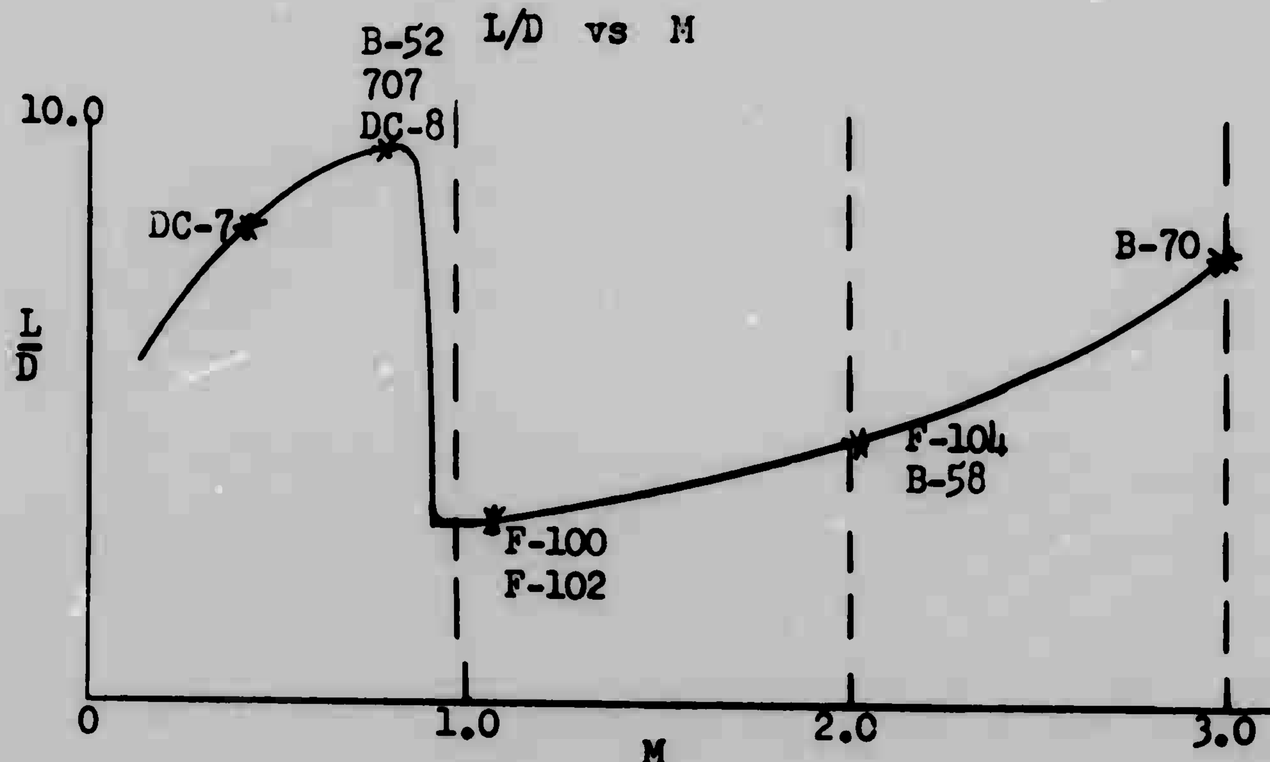


Figure 3.2.31

It is seen that the L/D ratio increases until drag divergence is reached near 0.9 Mach number. From 0.9 it falls rapidly to 1.0 and remains at a low value until Mach 2.0. This explains in part the short range of our present day supersonic fighters and the B-58 when operating in this region. New breakthroughs in aerodynamics such as afterbody compression effect has

raised the L/D ratio in the Mach 3.0 region to a value where it is feasible to cruise at these Mach numbers. However, the L/D ratio still is not as great as 0.8 - 0.9.

The use of honeycomb steel to withstand the temperatures and pressures at Mach 3.0 also results in an increase in the ratio of W_o/W_1 and will help increase the range.

The total range at Mach 3.0 can be a large value due to the V terms of the equation $\left(\frac{V}{c}\right)\left(\frac{L}{D}\right)$. Thus, if L/D is not as large as at M - 0.9 the V term will help to bring the total range up to a reasonable value.

3.3 CLIMB PERFORMANCE

The climb performance of an aircraft is proportional to the excess thrust horsepower. This relationship is found directly from a force balance of the aircraft in steady climbing flight.

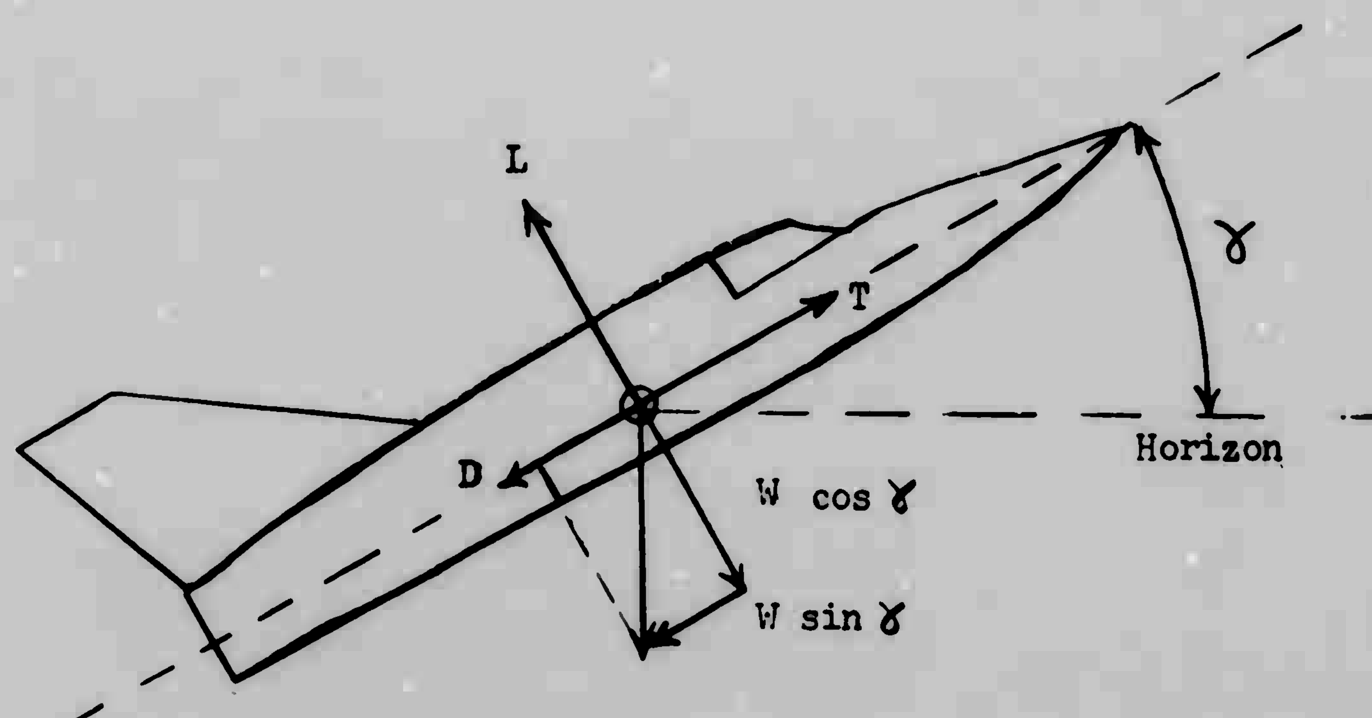


Figure 3.3.1 Force diagram for an aircraft in steady climbing flight

$\sum Z = 0: \quad W \cos \gamma - L = 0$ Equation 3.3.1

$\quad \text{or } L = nW = W \cos \gamma$

$\sum X = 0: \quad F - D - W \sin \gamma = 0$ Equation 3.3.2

multiplying by V

$FV_t - DV_t - W V_t \sin \gamma$

but $V_t \sin \gamma$ is the vertical velocity or rate of climb, dh/dt.

Solving we get

$$V_t \sin \gamma = \frac{dh}{dt} = \frac{(F - D) \, V_t}{W}$$ Equation 3.3.3

or in terms of thrust horsepower

$$\frac{dh}{dt} = \frac{THPa - THPr}{W} \times 33000 \qquad \text{Equation 3.3.4}$$

While the basic climb performance equation is simple in principle there are many factors which can occur under test conditions which significantly effect the performance and therefore, must be taken into account. The most important of these is the temperature; others are wind gradients, acceleration along the flight path and weight. It is the purpose of this section to discuss each of the above effects and provide the theory for making suitable corrections for flight test data.

3.3.1 TEMPERATURE EFFECTS

Variation in temperature causes differences in climb performance in three different ways.

1. Pressure altitude to tapeline altitude conversion: The pressure altitude read on the cockpit instruments reads the true tapeline altitude only on a standard day because temperature effects the pressure through the equation of state; $P = \rho gRT$. Thus, a conversion is required for this effect.

2. Effects on true speed: $V_t = k\,M\sqrt{T}$ since the true speed varies as the square root of the temperature and is multiplied by the drag and thrust to obtain excess thrust horsepower a correction to the true speed portion of the thrust horsepower is required.

3. Effects on thrust or horsepower available: Since $\rho = P/RT$ the effect of increased temperature is to decrease the amount of air which is to be utilized by the engine resulting in decreased thrust and power available at any given altitude. This then is the third correction that must be considered.

3.3.1a PRESSURE ALTITUDE TO TAPELINE ALTITUDE CONVERSION

For flight test we use a pressure altitude system, that is, the altimeter senses a pressure and converts it to a standard pressure altitude. So regardless of whether test or standard conditions are encountered

$$P_{test} = P_{standard}$$

and the altimeter will read the same under both conditions. Note, however, that the indicator reads tape line altitude <u>only</u> on a standard day, $h = Hc$.

For a given density

$$\frac{dh}{dp} = -\frac{1}{\rho g}$$

$$\Delta h = -\frac{\Delta p}{\rho g}$$

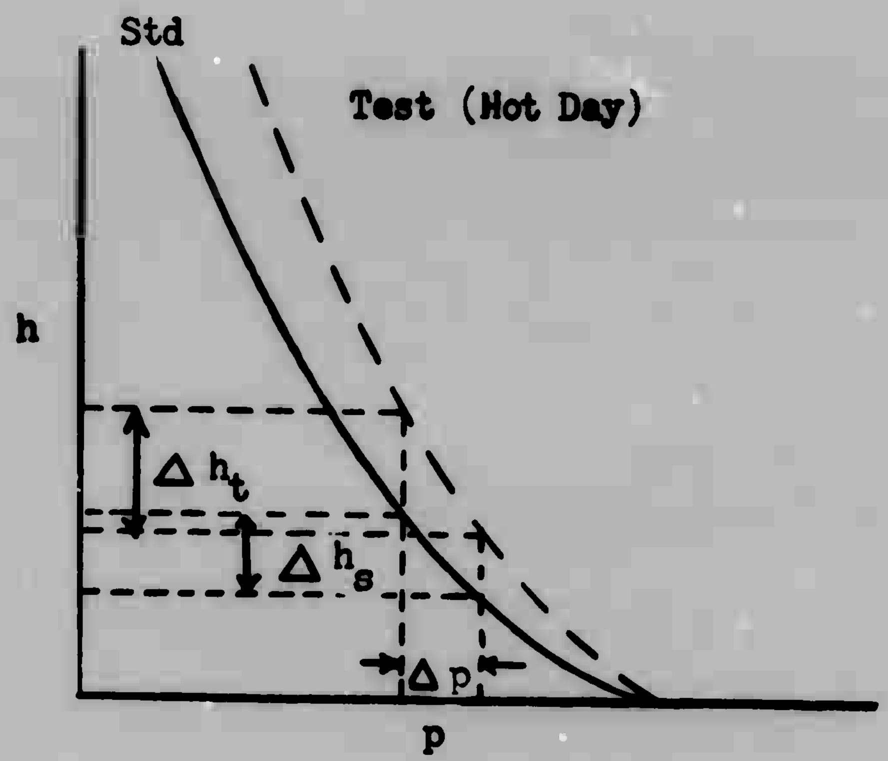


Figure 3.3.2a Tapeline Altitude to Pressure Relation

Also, note from Figure 3.3.2a that for a hot day the change in tapeline altitude is greater for a given ΔP than for standard (or colder) day.

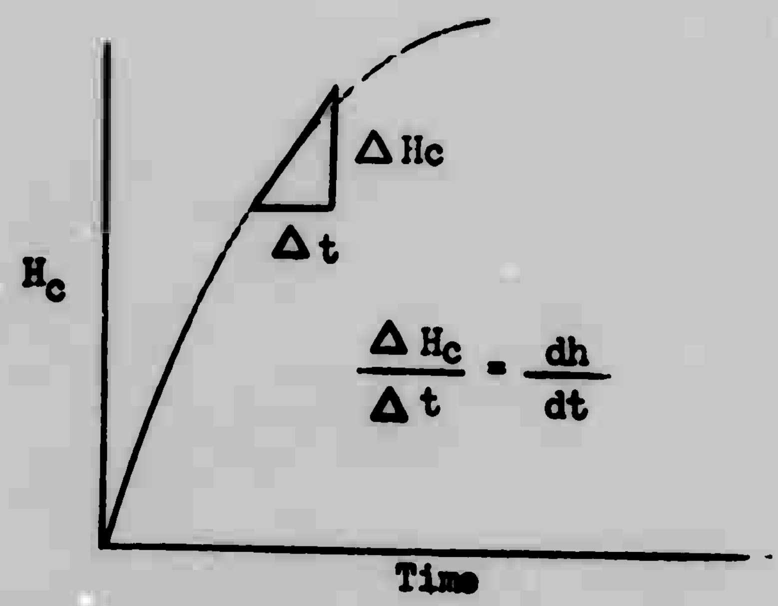


Figure 3.3.2b Apparent R/C on Test Day

From this it is seen that the apparent rate of climb, $\frac{dHc}{dt}$, may be considerably different from the tape line rate which would be obtained on a standard day. Remember that apparent rate of climb equals the tapeline rate of climb only on a standard day, Figure 3.3.2b. We wish to find some means of correcting apparent rate of climb to tapeline rate of climb.

<u>Derivation</u>

Using the gas law, $\frac{P}{\rho} = gRT$, we develop a few simple relationships. Subscripts t and s refer to test and standard conditions.

$$\frac{\frac{P_t}{\rho_t}}{\frac{P_s}{\rho_s}} = \frac{g\,RT_t}{g\,RT_s}$$

but $P_t = P_s$ for the pressure altitude system, so

$$\frac{\rho_s}{\rho_t} = \frac{T_t}{T_s}$$

For a given change in pressure (ΔP) the indicated or apparent change in altitude is

$$\Delta H_c = - \frac{\Delta P}{\rho_s\, g}$$

The change in tapeline or actual change in altitude for the same ΔP is

$$\Delta h = -\frac{\Delta P}{\rho_t\, g}$$

Rewriting these two equations and equating ΔP 's

$$\Delta P = \Delta Hc\, \rho_s\, g = \Delta h\, \rho_t\, g$$

or

$$\frac{\Delta h}{\Delta Hc} = \frac{\rho_s}{\rho_t} = \frac{T_t}{T_s}$$

writing this in terms of rates of climb

$$\frac{dh}{dt} \Big/ \frac{dHc}{dt} = \frac{T_t}{T_s}$$

$$\frac{dh}{dt} = \frac{T_t}{T_s} \cdot \frac{dHc}{dt} = R/C_s$$

Tapeline or standard day R/C at test day V_t and THP_a

Equation 3.3.5

This says that the tapeline rate of climb is equal to the product of the ratio of the test to standard temperatures and the apparent rate of climb. Note that this is the standard day rate of climb since dh/dt = dHc/dt on a standard day.

3.3.2b TEMPERATURE CORRECTION TO THP_a and THP_r

Since the effects of temperature on true speed and engine output are not immediately separable they will both be considered in this section. It is our ultimate objective to obtain a correction equation of the form

$$R/C_s = R/C_t \sqrt{\frac{T_s}{T_t}} + \Delta R/C_1$$

where $R/C_t \sqrt{\frac{T_s}{T_t}}$ corrects for V_t variation with temperature and $\Delta R/C_1$ corrects for the thrust variation.

First let us recall the variation of THP_a and THP_r with temperature by referring to Figure 3.3.3

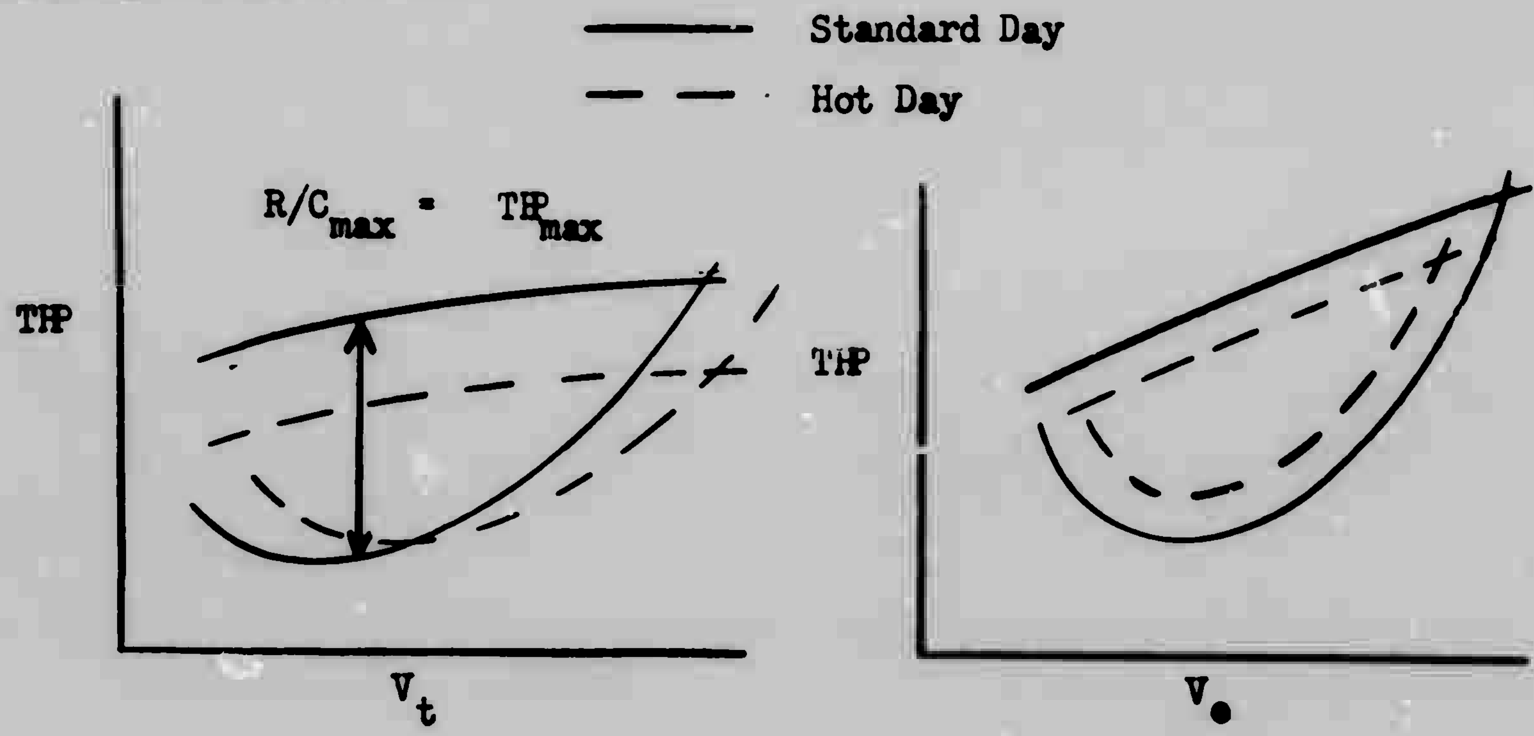


Figure 3.3.3a

Figure 3.3.3b

Thrust Horsepower Available and Required Curves

The two reasons why R/C changes with temperature are 1) THP_a changes due to V_t and thrust variations and 2) THP_r changes due to changes in V_t only since drag is unaffected by temperature variation. This fact may be seen from the curve of Figure 3.3.4 which is good for all temperatures.

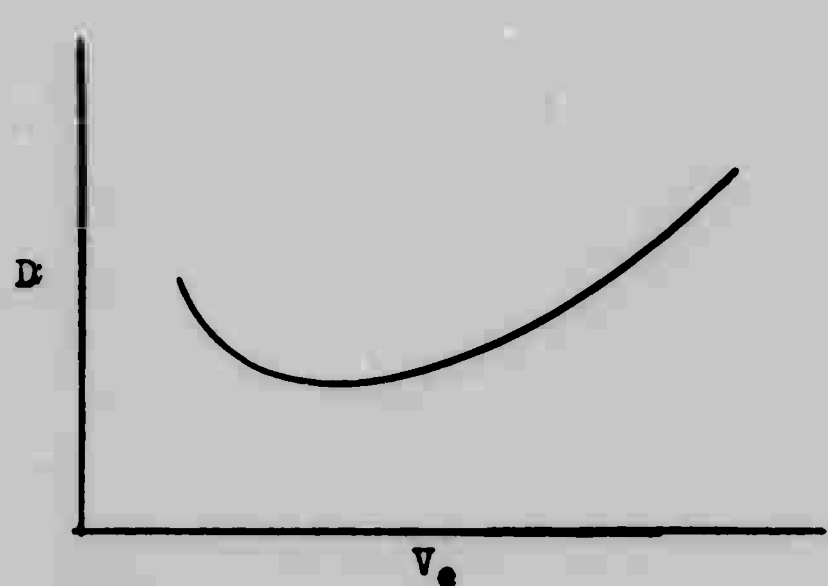


Figure 3.3.4 Drag Versus Equivalent Airspeed

Thus it is seen that for any given equivalent airspeed that $D_s = D_t$.

From the definition of THP_r the following equation can be obtained

$$\frac{THP_{r_t}}{THP_{r_s}} = \frac{\frac{D_t \times V_{t_t}}{550}}{\frac{D_s \times V_{t_s}}{550}} = \frac{V_{t_t}}{V_{t_s}}$$

Since $V_t = K M \sqrt{T_a}$ this equation can be reduced to

$$\frac{THP_{r_t}}{THP_{r_s}} = \frac{K M \sqrt{T_t}}{K M \sqrt{T_s}} = \frac{\sqrt{T_t}}{\sqrt{T_s}}$$

or $THP_{r_s} = THP_{r_t} \sqrt{\frac{T_s}{T_t}}$ Equation 3.3.6

Using a similar analysis for THP_a

$$\frac{THP_{a_s}}{THP_{a_t}} = \frac{\frac{F_s V_{t_s}}{550}}{\frac{F_t V_{t_t}}{550}} = \frac{F_s V_{t_s}}{F_t V_{t_t}} = \frac{F_s}{F_t} \sqrt{\frac{T_s}{T_t}}$$

so that

$THP_{a_s} = \frac{F_s}{F_t} \sqrt{\frac{T_s}{T_t}} \; THP_{a_t}$ Equation 3.3.7

F_s and F_t cannot be eliminated, as previously done, for drag since temperature effects the thrust or power output of the engine. Substituting the THP_{a_s} and THP_{r_s} into the performance equation 3.3.4

$$R/C_s = \left(\frac{THP_{a_s} - THP_{r_s}}{W}\right) \times 33000 = \left[\frac{F_s}{F_t}\sqrt{\frac{T_s}{T_t}}\ THP_{a_t} - \sqrt{\frac{T_s}{T_t}}\ THP_{r_t}\right]\frac{33000}{W}$$

Equation 3.3.8

This is a good correction equation but it is not in usable form since we may not know the test and standard thrust. We will now set out to simplify this equation into a more useable form for both jet and propeller type propulsion systems.

To simplify the above equation let

$$\Delta F = F_s - F_t$$

so that

$$\frac{F_s}{F_t} = \frac{F_t + \Delta F}{F_t} = 1 + \frac{\Delta F_t}{F_t}$$

substituting this in Equation 3.3.8 we obtain

$$R/C_s = \left[THP_{a_t}\sqrt{\frac{T_s}{T_t}}\left(1 + \frac{\Delta F}{F_t}\right) - THP_{r_t}\sqrt{\frac{T_s}{T_t}}\right]\frac{33000}{W}$$

this may be rewritten in the following form

$$R/C_s = \sqrt{\frac{T_s}{T_t}}\left(THP_{a_t} - THP_{r_t}\right)\frac{33000}{W} + \left(THP_{a_t}\sqrt{\frac{T_s}{T_t}}\ \frac{\Delta F}{F_t}\right)\frac{33000}{W}$$

Equation 3.3.9

Note that the first term is $R/C_t\sqrt{\frac{T_s}{T_t}}$ and the second term is the thrust correction and is termed $\Delta R/C_1$. The thrust correction in this form is inconvenient to use on aircraft for either jet or reciprocating engine aircraft because it involves both THP and thrust. Let us first develop an equation for $\Delta R/C_1$ which is conveniently used for propeller type power plants (reciprocating and turboprop).

$$\Delta R/C_1 = \left(THP_{a_t}\sqrt{\frac{T_s}{T_t}}\ \frac{\Delta F}{F_t}\right)\frac{33000}{W}$$

Equation 3.3.10

since

$$\frac{\Delta F}{F_t} = \frac{F_s - F_t}{F_t} = \frac{F_s}{F_t} - 1$$

$$\Delta R/C_1 = \left(THP_{a_t} \sqrt{\frac{T_s}{T_t}} \frac{F_s}{F_t} - THP_{a_t} \sqrt{\frac{T_s}{T_t}} \right) \frac{33000}{W}$$

substituting

$$\sqrt{\frac{T_s}{T_t}} = \frac{V_{t_s}}{V_{t_t}}$$

$$\Delta R/C_1 = \left(THP_{a_t} \frac{V_{t_s}}{V_{t_t}} \frac{F_s}{F_t} - THP_{a_t} \sqrt{\frac{T_s}{T_t}} \right) \frac{33000}{W}$$

$$\Delta R/C_i = \left(THP_{a_s} - THP_{a_t} \sqrt{\frac{T_s}{T_t}} \right) \frac{33000}{W} \qquad \text{Equation 3.3.11}$$

The complete temperature correction equation is:

$$R/C_s = R/C_t \sqrt{\frac{T_s}{T_t}} + \left(THP_{a_s} - THP_{a_t} \sqrt{\frac{T_s}{T_t}} \right) \frac{33000}{W} \qquad \text{Equation 3.3.12}$$

The reciprocating or turboprop power plant output, THP_{a_s} and BHP_{a_t}, are obtained from calibrated engine charts and/or torque meters. The THP_{a_s} and THP_{a_t} are obtained from the propeller efficiency

$$THP_a = BHP_a \times \eta_p$$

If the propeller efficiency is the same on test and standard day equation 3.3.11 can be written

$$R/C_s = R/C_t \sqrt{\frac{T_s}{T_t}} + \left(BHP_s - BHP_t \sqrt{\frac{T_s}{T_t}} \right) \frac{\eta_p \times 33000}{W} \qquad \text{Equation 3.3.13}$$

A convenient form of the thrust correction is obtained for jet aircraft by the following procedure.

$$\Delta R/C_1 = \left(THP_{a_t} \sqrt{\frac{T_s}{T_t}} \; \frac{\Delta F}{F_t} \right) \frac{33000}{W} \qquad \text{Equation 3.3.10}$$

$$\Delta R/C_1 = \frac{F_t V_{t_t}}{550} \sqrt{\frac{T_s}{T_t}} \; \frac{(F_s - F_t)}{F_t} \; \frac{33000}{W}$$

substituting

$$\sqrt{\frac{T_s}{T_t}} = \frac{V_{t_s}}{V_{t_t}}$$

$$\Delta R/C_1 = \frac{V_{t_t}}{550} \; \frac{V_{t_s}}{V_{t_t}} \; \frac{(F_s - F_t)}{W} \; 33000$$

$$\Delta R/C_1 = \frac{60 V_{t_s}}{W} (F_s - F_t)$$

where V_{t_s} is in ft/sec

or when V_{t_s} is in knots

$$\Delta R/C_1 = \frac{101.3 V_{t_s}}{W} (F_s - F_t) \qquad \text{Equation 3.3.14}$$

The correction for jets is then

$$R/C_s = R/C_t \sqrt{\frac{T_s}{T_t}} + \frac{101.3 V_t}{W} (F_s - F_t) \qquad \text{Equation 3.3.15}$$

The term $\Delta R/C_1$ for a turbojet aircraft can also be expressed in terms of Mach number

$$\Delta R/C_1 = \frac{101.3 V_{t_s} (F_s - F_t)}{W}$$

$$\Delta R/C_1 = \frac{101.3 M \sqrt{\gamma g R T_s} (F_s - F_t)}{W} \times \left(\frac{P_a}{P_a}\right)\left(\frac{P_o}{P_o}\right)$$

$$\Delta R/C_1 = \frac{101.3 \times 38.92\, M \sqrt{T_s}\ \ \Delta F\ \ P_a}{W\ \delta\ \ P_o}$$

since $P_o = 29.92$ "Hg

$$\frac{\Delta R/C_1}{\Delta F/\delta} = \frac{101.3 \times 38.92\, M \sqrt{T_s}\ \ P_a}{29.92\ \ \ \ W}$$

$$\frac{\Delta R/C_1}{\Delta F/\delta} = \frac{131.9\, M \sqrt{T_s}\ \ P_a}{W}$$ Equation 3.3.16

This parameter can be computed and is found in Charts A-72 to A-76 of the Pilots Handbook for Performance Flight Testing.

The value of $\Delta R/C_1$ can then be found by the below relationship

$$\Delta R/C_1 = \frac{\Delta R/C_1}{\Delta F/\delta} \times \Delta F/\delta$$

The final form of temperature correction equation is obtained by substituting equation (3.3.5) previously obtained into the general form of the R/C equation. This gives

$$R/C_s = \left(\frac{dHc}{dt}\ \frac{T_t}{T_s}\right)\sqrt{\frac{T_s}{T_t}} + \Delta R/C_1$$

$$R/C_s = \frac{dHc}{dt}\sqrt{\frac{T_t}{T_s}} + \Delta R/C_1$$ Equation 3.3.17

where the $\Delta R/C_1$ equation used depends on the power plant, equation 3.3.11 or 3.3.14.

Frequently it is desirable to take into account other variables which affect climb performance. These variables are weight, vertical wind gradients, and climb path acceleration.

3.3.2 WEIGHT EFFECTS AND CORRECTIONS

Weight directly effects the climb performance of an aircraft. Increased weight not only increases the work required to raise an aircraft to a given altitude but also it increases the induced drag. Hence, there are two separate weight corrections to be made to the climb performance. Weight corrections are normally kept to a minimum or eliminated completely in flight test by proper flight planning, that is, the climb test being made immediately after takeoff where the engine start weight is the same in every case. Even though the initial weights were the same, variation will occur as the aircraft goes to altitude since nonstandard temperatures change the fuel flow and, therefore, change the weight slightly. This amount may be quite significant for small jet type aircraft with high fuel flows as are encountered with afterburner operating, since a weight change may represent a large percentage of the total weight of the vehicle.

To set up an analytical expression with which to make weight corrections it is necessary to make a few assumptions. These are that the atmospheric, velocity and power conditions are the same for both test and standard weights. For this reason, weight corrections are applied to standard day climb data (i.e., weight corrections should be made last, after temperature, wind gradients, acceleration, etc. corrections have been made). It should also be noted that the following derivation does not take into account the effects of transonic drag changes with lift (or angle of attack). Corrections of this sort would have to be evaluated for each particular aircraft if their climb schedule requires climbing at speeds greater than the drag rise or drag divergence Mach number. In the supersonic speed regime ($M > 1.3$) corrections may be approximated by linearized theory. When the weight correction is small, as it usually is, this correction is not normally considered.

BLANK PAGE

First, let us consider the subsonic incompressible relations and defer transonic and supersonic considerations until later. In either case the basic climb equation is

$$R/C_s = \frac{\left(T_s - D_{test}\right) V_{ts}}{W_t}$$ Corrected for standard temperature and test weight

$$T = \frac{W}{V} R/C_s + D$$

differentiating with respect to weight

$$\frac{dT}{dW} = \frac{Wt}{V} \cdot \frac{d\,(R/C_s)}{dW} + \frac{R/C_s}{V} + \frac{dD}{dW}$$

The assumption is made that the thrust available is unaffected by weight and its associated angle of attack change, $\frac{dt}{dW} = 0$

$$\frac{d\,(R/Cs)}{dW} = -\left[\frac{R/C_s}{W_t} + \frac{dD}{dW} \cdot \frac{V}{W_t}\right]$$

$$\Delta R/C_{weight} = -\left[R/C_s \frac{dW_t}{W_t} + \frac{dD \cdot V}{W_t}\right]$$ Equation 3.3.18

where $dW = W_s - W_t$ and W = test weight.

Note that the term in brackets shows that there are really two weight corrections as previously mentioned. The first term represents the additional increment of inertia requiring power to raise the aircraft to a given altitude. The second term is the additional power required to overcome the increment of drag; more specifically, induced drag which is caused by an increase in weight. Since the aircraft power available is fixed, these terms represent a reduction in performance and, therefore, have a negative sign. Each correction will be considered separately.

3.3.2a INDUCED DRAG CORRECTION

$$D = C_{D_p}\, q\, S + C_{D_i}\, q\, S = C_{D_p}\, q\, S + \frac{C_L^2}{\pi AR e}\, q\, S$$

substituting

$$C_L = n\, W/q\, S$$

$$D = C_{D_p}\, q\, S + \frac{(n W)^2}{\pi AR\, e\, (q\, S)}$$

Parasite drag is not effected by weight change, therefore, the change in drag due to weight change is

$$dD = \Delta Di = \frac{\Delta (n\, W)^2}{\pi AR_e\, (q\, S)} = \frac{.000675}{\pi b^2\, e M^2}\, \Delta \left(\frac{(NW)^2}{\delta} \right)$$

$$dD = \frac{.00675}{\pi b^2\, e\, M^2} \left(\frac{N_t^2\, W_t^2}{\delta_t} - \frac{N_s^2\, W_s^2}{\delta_s} \right)$$ Equation 3.3.19

Multiplying the right side by P_a/P_a and P_o/P_o and simplyfing the equation becomes

$$dD = \frac{.006429}{P_{a_s}\, b^2\, M^2\, e} \left(\frac{N_t^2\, W_t^2}{} \frac{\delta s}{\delta t} - \frac{N_s^2\, W_s^2}{} \right)$$

Substituting this value into the second term of equation 3.3.18 the induced drag correction becomes

$$\Delta R/C_3 = dD \frac{V_{t_s}}{W_t} = \frac{.006429}{P_{a_s}\, b^2\, M^2\, e} \left(\frac{N_t^2\, W_t^2}{} \frac{\delta s}{\delta t} - N_s^2\, W_s^2 \right) \frac{V_t}{W_t}$$

where V_t is in feet per second and subtracting

$$V_{t_s} = 38.98\, M\sqrt{T_s} \times 101.33 \text{ ft/min}$$

$$R/C_3 = \frac{25.38\sqrt{T_s}}{P_{a_s}\, b^2\, M\, e} \left[\frac{(N_t^2\, W_t^2)\, \delta s/\delta t - N_s^2\, W_s^2}{W_t} \right]$$ Equation 3.3.20

3.3.2b INERTIA CORRECTION

Examination of the first term of equation 3.3.18 gives

$$\Delta R/C_2 = R/C_s \frac{dWt}{W_t}$$

$$\Delta R/C_2 = \frac{dh}{dt} \frac{\Delta W}{W}$$ Equation 3.3.21

where $\Delta W = W_t - W_s$

The two weight corrections are now added to equation 3.3.17 and the equation for the standard rate of climb now becomes

$$R/C_s = \frac{dHc}{dt} \sqrt{\frac{T_t}{T_s}} + \Delta R/C_1 + \Delta R/C_2 + \Delta R/C_3$$

Equation 3.3.22

NOTE: If wind gradient or acceleration corrections are to be made, these would ordinarily be included in the standard rate of climb used for $\Delta R/C_2$ computation.

3.3.3 ACCELERATION DURING CLIMB

If an aircraft not only climbs but accelerates, the performance picture changes slightly.

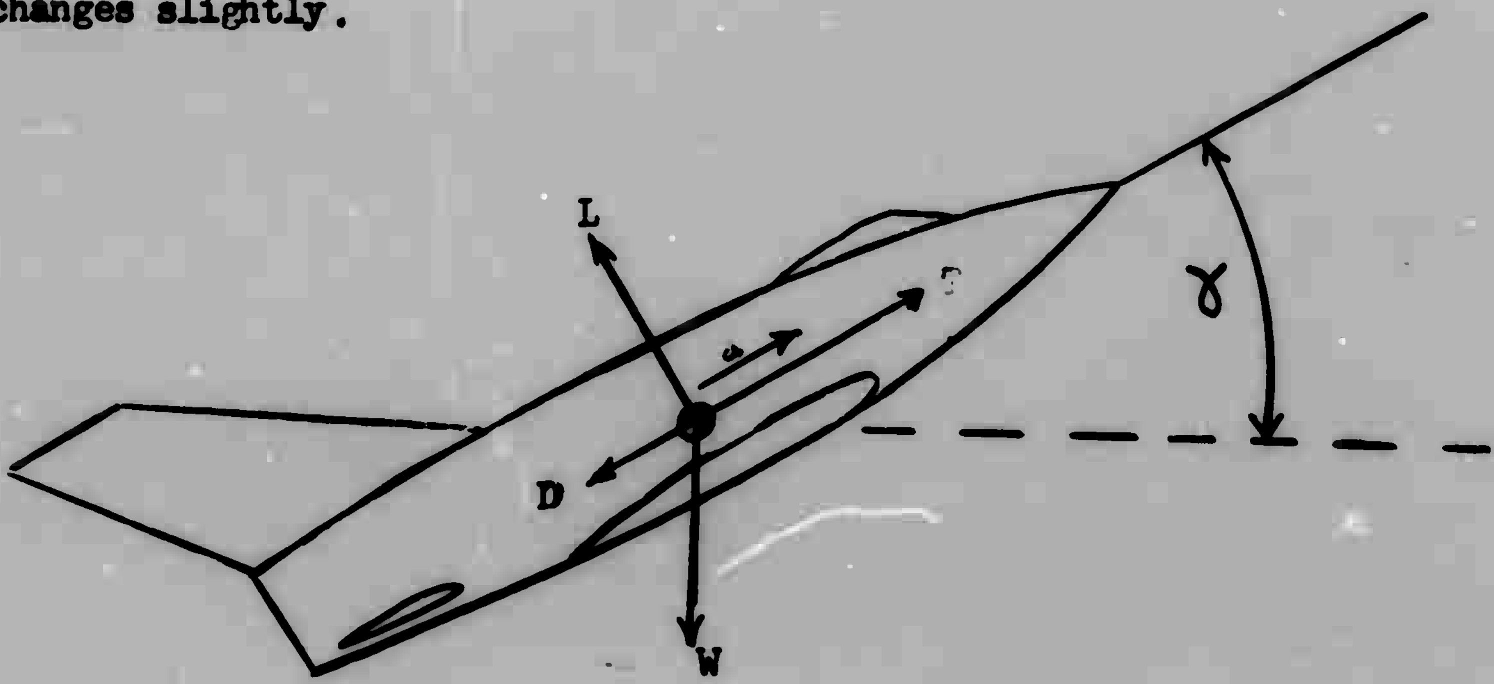


Figure 3.3.5

Writing the force equation as before gives

$$T - D - W \sin \gamma = \frac{W}{g} a$$

multiplying by V_t as before

$$W (V_t \sin \gamma) = (T - D) V_t - \frac{W}{g} V_t a$$

or $$R/C = \frac{(T - D) V_t}{W} - \frac{V_t}{g} \frac{dV}{dt}$$

$$R/C_a = R/C - \frac{V_t}{g} \frac{dV}{dt} \qquad \text{Equation 3.3.23}$$

where

R/C = rate of climb with no acceleration

R/C_a = rate of climb with acceleration

This indicates that as the aircraft accelerates while climbing it does not achieve its maximum rate of climb since some of the thrust produced by the power plant is being used to accelerate the aircraft.

For comparison purposes and for determination of climb schedules it is generally desired to correct the accelerated rate of climb such as is obtained at constant indicated airspeed or Mach number to zero acceleration. This is done by algebraicly manipulating equation 3.3.23 to a more convenient form

$$R/C = R/C_a + \frac{V_t}{g} \frac{dV}{dt}$$

$$R/C = R/C_a + \frac{V_t}{g} \frac{dV}{dh} \frac{dh}{dt}$$

where

$$\frac{dh}{dt} = R/C_a$$

$$R/C = R/C_a \left[1 + \frac{V_t}{g} \frac{dV}{dh} \right] \qquad \text{Equation 3.3.24a}$$

The term in brackets is called the acceleration factor and is determined once the true speed and the change in true speed with altitude are determined for the particular altitude being considered. An incremental correction is obtained by using only the second term of the acceleration factor. That is

$$\Delta R/C_a = \left[\frac{V_t}{g} \frac{dV}{dh} \right] R/C_a \qquad \text{Equation 3.3.24b}$$

Some more expanded forms of these equations and some charts for simplified computation are found in AFTR No 6273, Flight Test Engineering Manual.

3.3.4 VERTICAL WIND GRADIENTS

Wind velocities vary significantly with altitude because of jet streams and other meteorological phenomena. A vertical wind gradient is defined as the change in the horizontal wind velocity, V_w, with altitude.

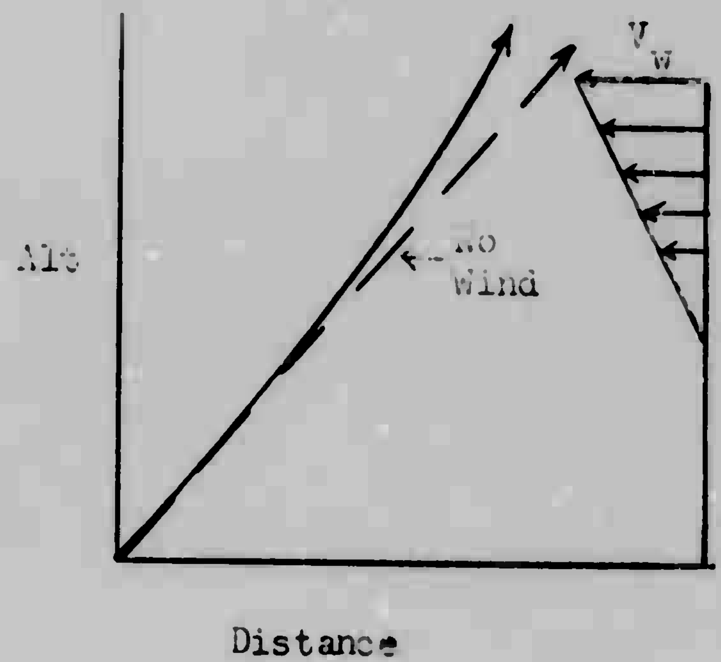
$$\text{Wind gradient} = \frac{dV_w}{dh}$$

The wind gradient is positive if the wind velocity increases with altitude. As a matter of convention a headwind is taken as positive and a tailwind is negative.

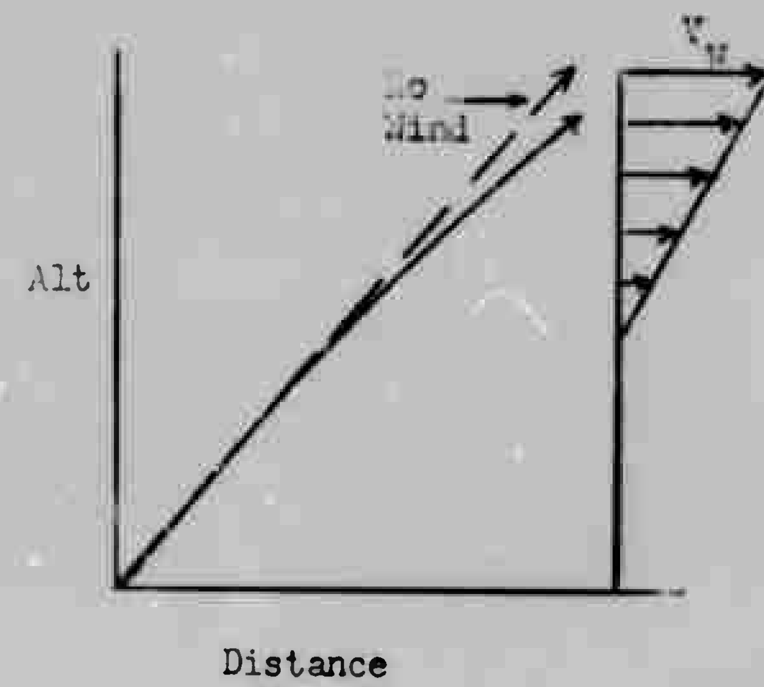
Climbing through a vertical wind gradient has a very significant effect on the performance of an aircraft. Consider an aircraft climbing into a

positive wind gradient at a given indicated speed, Figure 3.3.6a. Under zero or steady wind conditions the aircraft would follow the flight path indicated by the dotted line. However, when flying along this path into a vertical wind gradient, as shown, the aircraft appears to accelerate; that is, the indicated speed increases. Since the pilot is flying a climb schedule, (either airspeed or Mach number) he will convert the apparent acceleration into increased rate of climb and follow some steeper flight path indicated by the solid line. Thus, an aircraft flying into a positive wind gradient accelerates relative to the air mass. This acceleration can be and is converted to climb performance with the indicated climb speed is held constant.

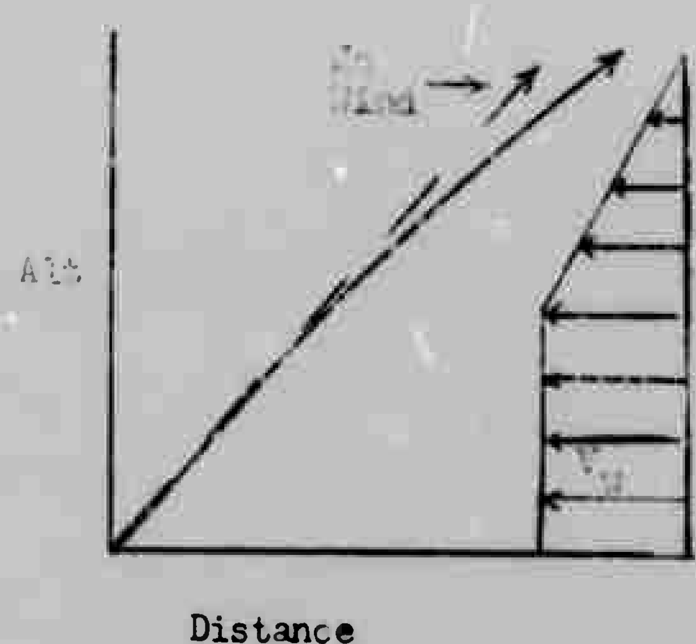
When the wind gradient or the direction of flight relative to the wind gradient changes, the overall effect on the climb performance changes. All possible effects of wind gradients on flight path and rate of climb are qualitiatively summarized in Figure 3.3.6.



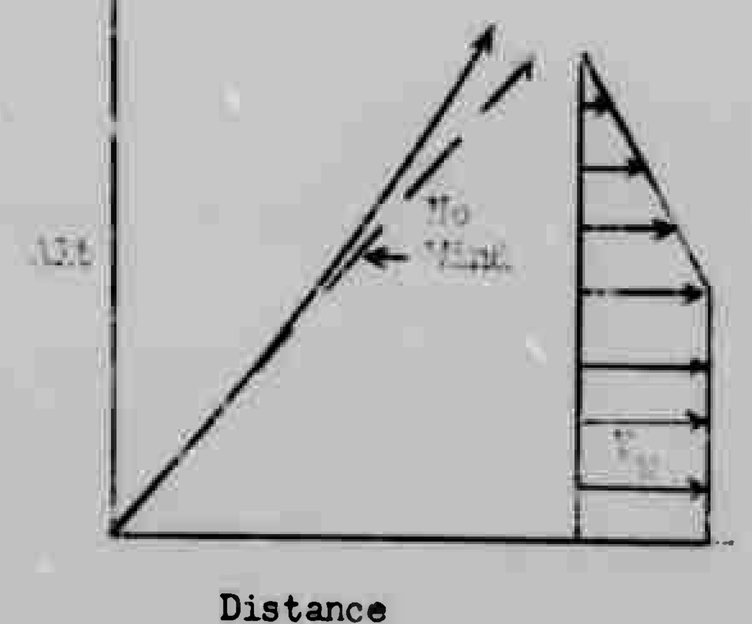
a) Increasing headwind results in higher rate of climb than with no wind



b) Increasing tailwind results in lower rate of climb than with no wind



c) Decreasing headwind results in lower rate of climb than with no wind



d) Decreasing tailwind results in higher rate of climb than with no wind

Figure 3.3.6

Note in each case where the climb path steepens the aircraft is essentially tending to accelerate due to the changing wind velocity with altitude. This

acceleration tendency is converted to rate of climb by pilot reaction in maintaining climb schedule.

Because flight test data must be presented for standard zero wind conditions it is necessary to account for the effects of wind gardients and make appropriate corrections. Since flying through a wind gradient is essentially an acceleration or deceleration, the wind gradient correction can be handled in the same way as the acceleration correction except that the sign of the second term of equation 3.3.24 must be changed.

Acceleration Correction

$$R/C = R/C_a + \frac{V_t}{g}\frac{dV}{dh}\ R/C_a \qquad \text{Equation 3.3.24}$$

Wind Gradient Correction

$$R/C = R/C_w - \frac{V_t}{g}\left(\frac{dV_w}{dh}\right) R/C_w$$

where R/C is the rate of climb without a wind gradient

R/C_w is the rate of climb with a wind gradient

$\frac{dV_w}{dh}$ is the wind gradient

This equation is arrived at from the fact that the wind gradient produces a change in the true speed with respect to the ground or air mass which results in a change in kinetic energy. If the airspeed is held constant the change in energy must appear as potential energy. Since the total energy of the aircraft remains constant at any given instant

$$E_t = W\,h + \frac{W\,V^2}{2g} = K$$

Differentiating with respect to time

$$\frac{dE_t}{dt} = W\left(\frac{dh}{dt}\right)_w + \frac{W\,V_t}{g}\left(\frac{dV_w}{dt}\right) = 0 \qquad \text{Equation 3.3.25}$$

An aircraft traversing a vertical wind gradient will undergo an acceleration or deceleration which is given by

$$a_w = \frac{dV_w}{dt} = \left(\frac{dV_w}{dh}\right)\left(\frac{dh}{dt}\right)_w$$

This value of $\frac{dV_w}{dt}$ is substituted into equation 3.3.25 in order to convert the acceleration, a_w, into an equivalent rate of climb increment

$$W\,\Delta\left(\frac{dh}{dt}\right)_{wind} = -\frac{W\,.\,V_t}{g}\left(\frac{dV_w}{dh}\right)\left(\frac{dh}{dt}\right)_a$$

$$\Delta\left(\frac{dh}{dt}\right)_w = -\frac{V_t}{g}\left(\frac{dV_w}{dh}\right)\left(\frac{dh}{dt}\right)_w = \Delta R/C_w \qquad \text{Equation 3.3.26a}$$

where $\left(\frac{dh}{dt}\right)_w$ is R/C_g obtained form equation 3.3.17. The unaccelerated rate of climb can be written as

$$R/C = R/C_w + \Delta R/C_w$$

$$\frac{dh}{dt} = \left(\frac{dh}{dt}\right)_w - \frac{V_t}{g}\left(\frac{dV_w}{dh}\right)\left(\frac{dh}{dt}\right)_w \qquad \text{Equation 3.3.26b}$$

Note the similarity between this and the acceleration equation 3.3.24.

Equation 3.3.26a and b are two forms of the correction equation for vertical wind gradients. They should be modified slightly and explained in order to avoid misinterpretation. Equation 3.3.26a gives the increment which must be added to the rate of climb corrected for temperature effects, equation 3.3.17. This equation should be rewritten as

$$\Delta R/C_w = -\frac{K V_t}{g}\left(\frac{dV_w}{dh}\right)\left(\frac{dH_c}{dt}\sqrt{\frac{T_t}{T_s}} + \Delta R/C_1\right) \qquad \text{Equation 3.3.27}$$

where K is plus 1.0 for a headwind and minus 1.0 for a tailwind, $\frac{dV_w}{dh}$ is positive for increasing wind velocity with altitude and negative for decreasing wind with increasing altitude

Thus,

$$R/C_{\substack{\text{std temp}\\ \text{no wind}}} = \frac{dH_c}{dt}\sqrt{\frac{T_t}{T_s}} + \Delta R/C_1 + \Delta R/C_w \qquad \text{Equation 3.3.28a}$$

Equation 3.3.26b should likewise be rewritten to avoid misinterpretation

$$R/C_{\substack{\text{std temp}\\ \text{no wind}}} = \left(\frac{dH_c}{dt}\sqrt{\frac{T_t}{T_s}} + \Delta R/C_1\right)\left[1 + \frac{K V_t}{g}\left(\frac{dV_w}{dh}\right)\right] \qquad \text{Equation 3.3.28b}$$

where the signs convention of $\frac{dV_w}{dh}$ and K are the same as before.

It is noted that the horizontal wind gradient due to updrafts and downdrafts has been ignored. Such weather phenomena is extremely difficult to measure and presents a problem only in local areas such as a thunderstorm or near mountain ranges. With little care these areas can generally be avoided.

3.3.5 FINAL CORRECTION EQUATION

The order in which climb corrections are made is important. If all of the forementioned corrections are to be made the final equation is

$$R/C = \frac{dH_c}{dt}\sqrt{\frac{T_{a_t}}{T_{a_s}}} + \Delta R/C_1 + \Delta R/C_w + \Delta R/C_a + \Delta R/C_2 + \Delta R/C_3$$

Equation 3.3.29

3.4 DESCENT PERFORMANCE

Descent performance is analized in the same manner as climb performance. As a matter of fact the performance equations derived for climbs under accelerated and unaccelerated flight conditions (equation 3.3.3 and 3.3.23) apply equally well to descents since a negative rate of climb is obviously a rate of descent.

Unaccelerated:

$$R/D = -R/C = \frac{(T - D)\,V_t}{W} \qquad \text{Equation 3.3.3}$$

Accelerated:

$$R/D_a = -R/C_a = \frac{(T - D)\,V_t}{W} - \frac{V_t}{g}\frac{dV}{dt} \qquad \text{Equation 3.3.23}$$

In order for an aircraft to descend, the thrust, T, must be less than the drag. Generally the thrust is very nearly zero in which case the above equations can be modified to the following form

Unaccelerated:

$$R/D = \frac{D\,V_t}{W} \qquad \text{Equation 3.4.1a}$$

Accelerated:

$$R/D_a = \frac{D\,V_t}{W} + \frac{V_t}{g}\frac{dV}{dt} \qquad \text{Equation 3.4.1b}$$

From this we see that descent performance is a function of thrust, drag, speed and weight.

1. Thrust: Increasing the thrust decreases the rate of descent.

2. Drag: Increasing the drag increases the rate of descent by decreasing the lift-drag ratio. This is seen from the fact that

$$\sin \gamma = \frac{D}{W} \qquad \text{Equation 3.4.2}$$

3. Speed: The rate of descent will increase with increased speed.

4. Weight: The effects of a change in weight on rate of descent is not clearly defined. It depends on whether the glide angle or the indicated speed is held constant, and on the relative change of the forward component of the weight and the change in the induced drag with a change in weight. Thus, the rate of descent can increase, decrease or remain unchanged for an increase in gross weight. This may be qualitatively understood by examining the forces acting during descent

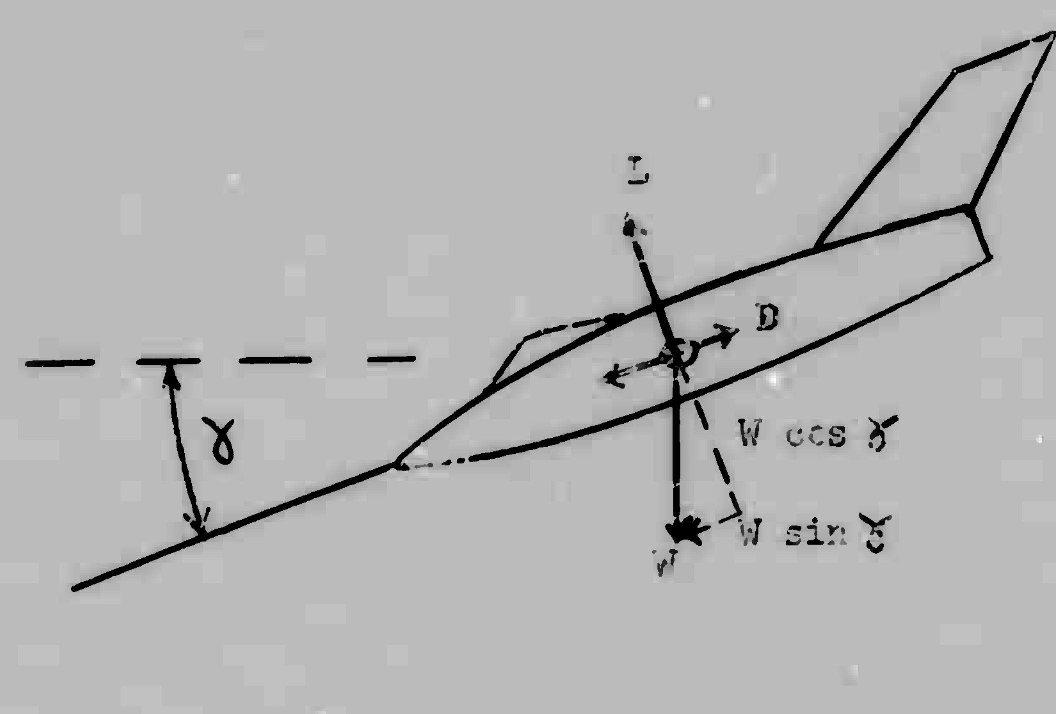


Figure 3.4.1

The addition of weight not only increases the forward component of weight, $W \sin \gamma$, but also it increases the induced drag. If the glide angle remains constant the net change in force,

$$\Delta F = \Delta W \sin \gamma - \Delta D$$

will cause the aircraft to accelerate or decelerate and consequently increase or decrease the rate of descent.

Generally, it is more important to know what happens to the rate of descent when the airspeed is held constant and the weight increased. When the speed is held constant the glide angle is allowed to change. Here again as in the previous example, where the glide angle remained constant the rate of descent can increase, decrease or remain the same. This is seen from equation 3.4.1a. If the velocity is constant the rate of descent depends directly on the ratio of D/W. That is, if the addition of weight results in no change in D/W

$$\frac{D}{W} = \frac{D + \Delta D}{W + \Delta W} = \text{constant}$$

there will be no change in the rate of descent. If as a result of the weight change

$$\frac{D}{W} > \frac{D + \Delta D}{W + \Delta W}$$

the rate of descent will decrease and conversely if

$$\frac{D}{W} < \frac{D + \Delta D}{W + \Delta W}$$

the rate of descent will increase. It may be shown that the speed for maximum L/D is very nearly the point where D/W remains constant for a change in weight. Therefore, little change in rate of descent should be expected for different weights when descending at the glide speed for maximum range.

3.4.2 CORRECTIONS FOR TEMPERATURE, WEIGHT, ACCELERATION AND WINDS

Temperature Corrections are made for the tapeline to pressure altitude relationship and for the thrust horsepower required. The correction for thrust horsepower available is not normally required since descents are usually performed with idle power (near zero thrust). The basic correction equation 3.3.17 is simplified to

$$R/D = \frac{dH_c}{dt}\sqrt{\frac{T_{a_t}}{T_{a_s}}} \qquad \text{Equation 3.4.3}$$

Weight corrections, $\Delta R/D_2$ and $\Delta R/D_3$, are similar to those derived in Section 3.3.2 for rate of climb except the sign of the inertia term, $\Delta R/D_2$, has been changed

$$\Delta R/D_2 = -\frac{dh}{dt}\frac{\Delta W}{W} \qquad \text{Equation 3.4.4}$$

$$\Delta R/D_3 = \frac{25.38\sqrt{T_s}}{P_{a_s} b^2 e M}\left[\frac{n_t^2 W_t^2 \delta s/\delta_t - n_s^2 W_s^2}{W_t}\right] \qquad \text{Equation 3.4.5}$$

Acceleration and wind gradient corrections are applied in the same way and with the same sign convention as they are in Section 3.3.3 and 3.3.4 remembering of course that negative rate of climb is rate of descent.

The final equation for the rate of descent is now given in the following form

$$R/D = \frac{dH_c}{dt}\sqrt{\frac{T_{a_t}}{T_{a_s}}} + \Delta R/D_w + \Delta R/D_a + \Delta R/D_2 + \Delta R/D_3 \qquad \text{Equation 3.4.6}$$

3.5 TURNING PERFORMANCE

3.5.1 INTRODUCTION:

This section will deal with the turning performance of an aircraft in a steady state or level thrust limited turning performance. This can also be stated as "the ability of an aircraft to turn while holding height and airspeed constant at a power setting". The thrust limited means that the aircraft is limited in speed and "n" by thrust available and not by aircraft structure. To state one example, the maneuver characteristics, dictated by performance limitations, will determine how and where a fighter should turn when attempting a high altitude intercept.

The turning performance of an aircraft is limited aerodynamically by:

(a) Thrust or power available

(b) Drag characteristics

(c) $C_{L_{max}}$

(d) Controllability

3.5.2 THEORY

Some of the assumptions which are normally made and accepted in estimating turbojet turning performance are:

(a) Reynolds number effects are negligible both for the engine and airplane.

(b) Effects of ambient pressure on the engine performance are negligible or the effects of ambient pressure on the combustion are disregarded.

(c) There are no effects due to varying load distribution, local velocity variations and the like. The drag coefficient in turning flight is identical with that obtained in level unaccelerated flight at the same angle of attack and Mach number.

During a steady turn, where the above assumptions hold, there is a simple relation for turning performance

$$n = \frac{\delta}{W} = f \left(M , N/\sqrt{\theta} \right) \qquad \text{Equation 3.5.1}$$

By looking at the above relation it can be seen that at a constant weight, Mach and $N/\sqrt{\theta}$, the normal load factor is only a function of the pressure ratio. This states that a steady level turn at one altitude may be simply related to the performance at any other altitude. With this in mind a simple plot of height or pressure ratio may be plotted versus Mach. On this plot is shown a line of maximum level flight Mach number for each height. This plot may now be used to determine the steady turning performance at various heights. This is shown in Figure 3.5.1.

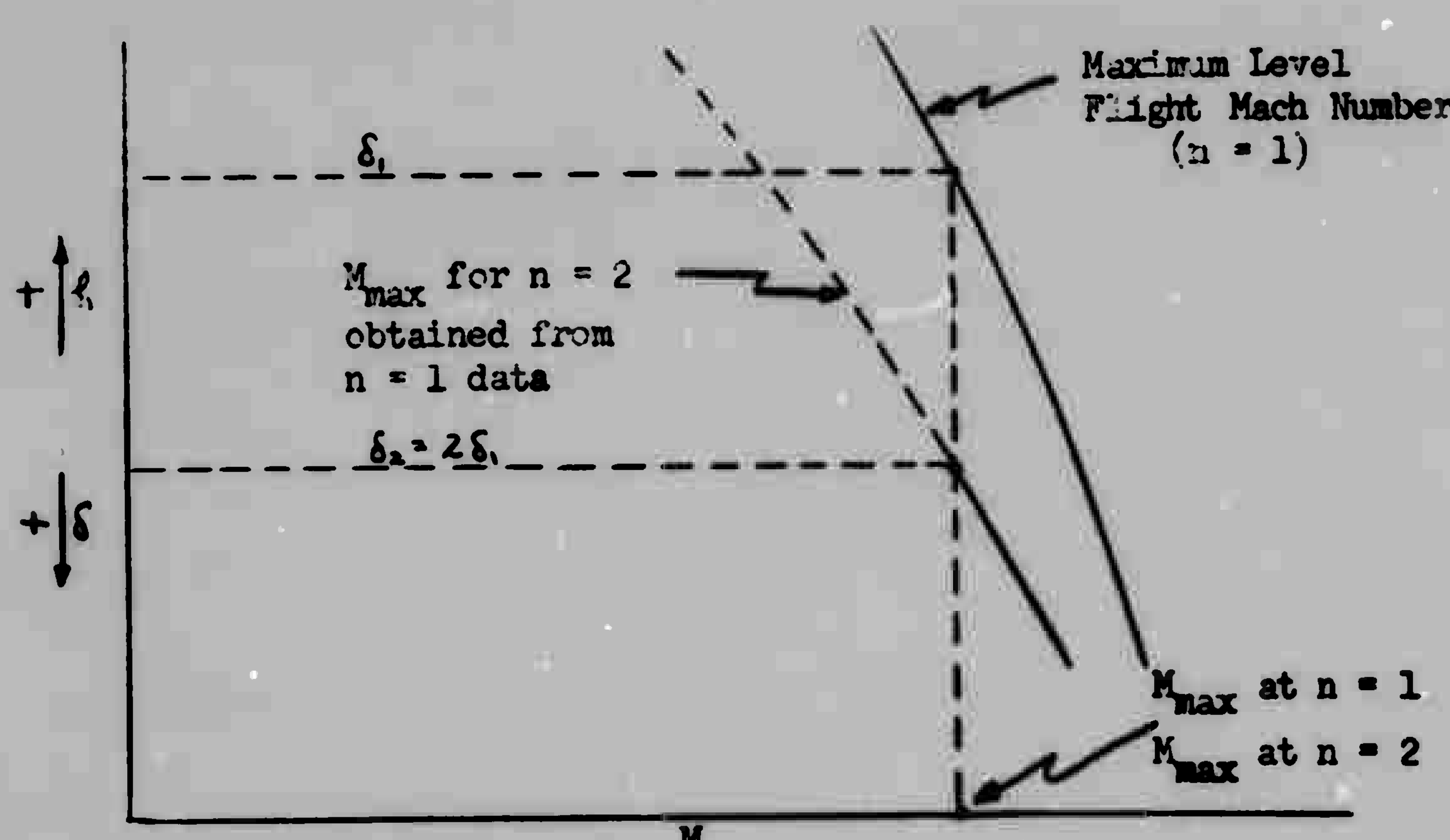


Figure 3.5.1

In many tests the generalized data mentioned above is not always obtained. The only reason, presumably, is due to Reynolds number effects on the effects of absolute pressure on the engine and aircraft. Even with the errors involved in this generalization it is well worth the effort to attempt to predict turning performance. The use of F_n/δ instead of $N/\sqrt{\theta}$ may avoid some of the difficulties since a large change of drag due to Reynolds number is hard to imagine.

Consider an aircraft in straight and level flight, the performance equation may be written as:

$$D_1 = (C_{D_p} + C_{D_i})\ q\ S$$

$$D_1 = (C_{D_p} \times \tfrac{1}{2}\rho_a V_t^2 S) + \left(\frac{C_{L_1}^2}{\pi \mathcal{R} e} \times \tfrac{1}{2}\rho_a V_t^2 S\right)$$

Taking the same aircraft at the same speed and height with, ng, accelerometer reading. This increase in g forces is an increase above level flight where n = 1.

$$D_2 = \left(C_{D_p} \times \tfrac{1}{2}\rho_a V_t^2 S\right) + \left(\frac{C_{L_1}^2}{\pi \mathcal{R} e} \times \tfrac{1}{2}\rho_a V_t^2 S\right) + \left(\frac{C_{L_1}^2}{\pi \mathcal{R} e}(n^2 - 1) \times \tfrac{1}{2}\rho_a V_t^2 S\right)$$

By comparing D_1 to D_2 it can be seen that an increase in drag in the last term of D_2, by increasing n, is the amount of additional thrust that will be required to maintain airspeed and height.

The change in induced drag due to turning (constant weight) is:

$$\Delta D_i = \left(\frac{W^2}{b^2 \pi q e}\right) \times (n_2^2 - n_1^2) = \frac{W^2 \left(n_2^2 - n_1^2\right)}{\pi \, ARe \; q \, S} \qquad \text{Equation 3.5.2}$$

and the following is true considering a weight change during a change of g.

$$\Delta D_i = \frac{\left(n_2^2 W_2^2\right) - \left(n_1^2 \; W_1^2\right)}{b^2 \pi q e} \qquad \text{Equation 3.5.3}$$

The next few curves are used to explain how drag changes with weight and temperature during maneuvering flight. Consider the first curve below.

Thrust and Drag vs V_t

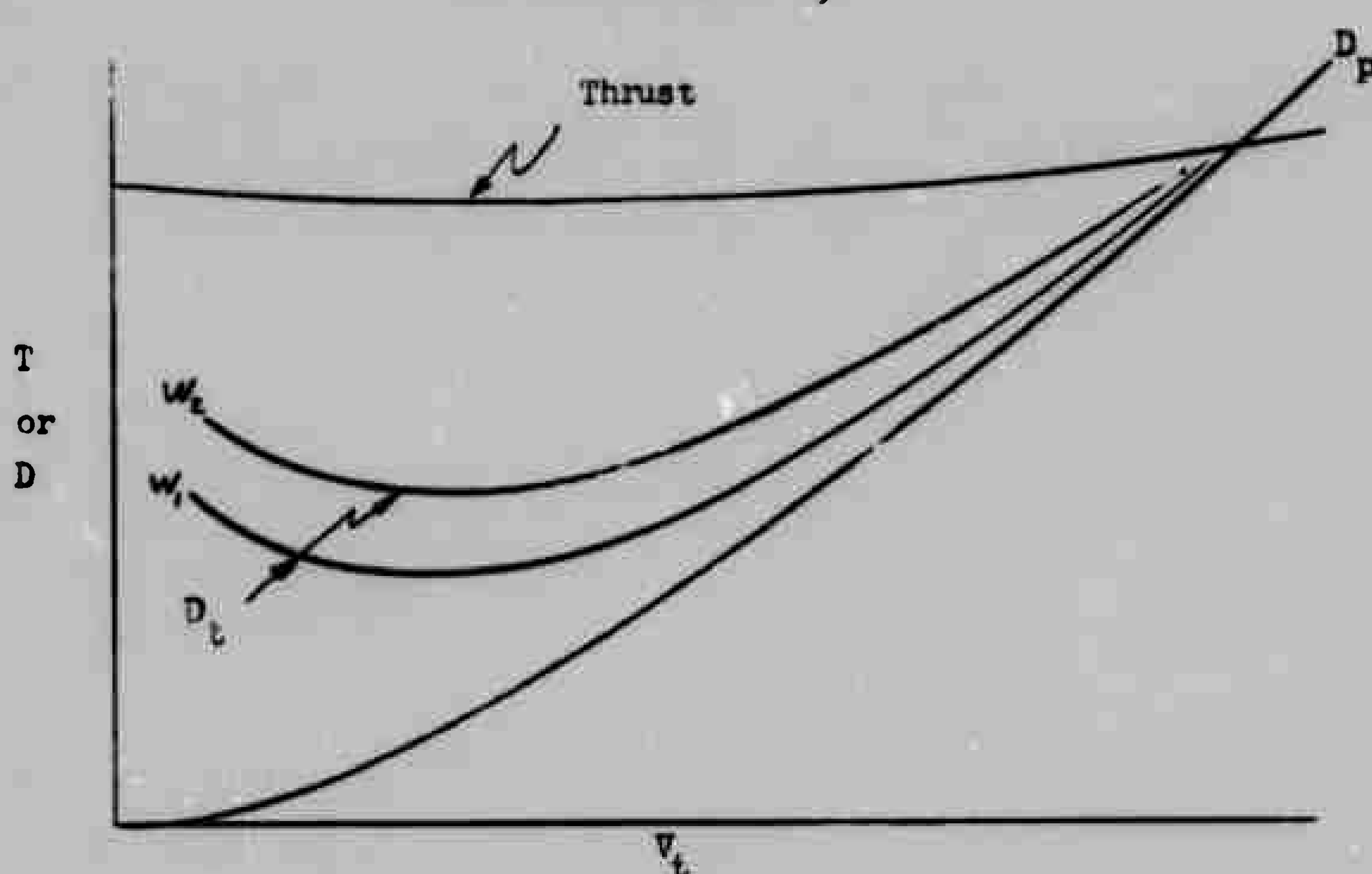


Figure 3.5.2

This curve considers one temperature but two different weights. Therefore, thrust is one line and so is D_p, but total drag is different for each weight due to change of induced drag. The different total drag lines can also be considered to be due to a change of nW as well as a change of weight. From this a change of drag can be noted and also a change of excess thrust.

To go further lets consider the following curve:

Thrust and Drag vs V_e

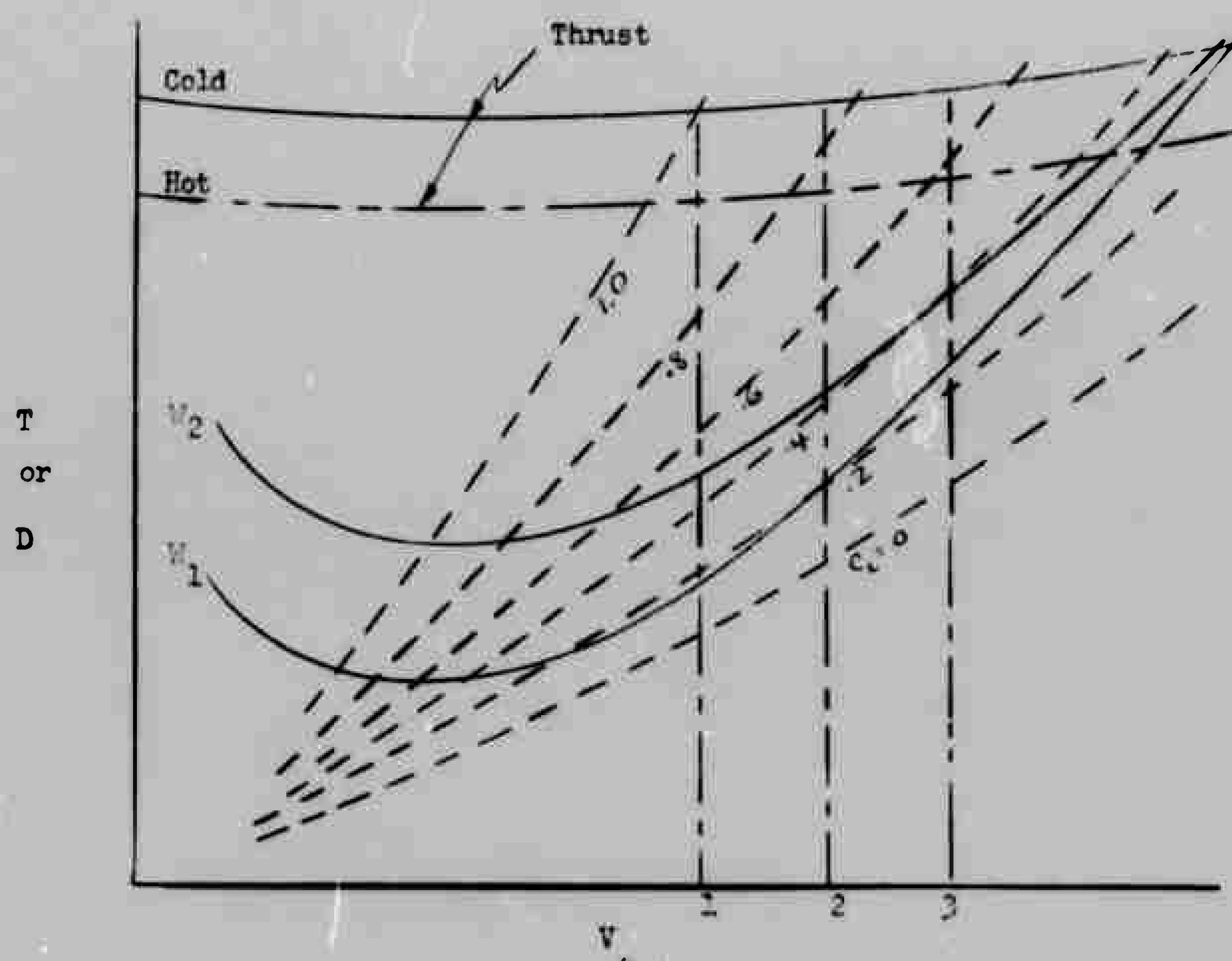


Figure 3.5.3

Figure 3.5.3 shows a thrust available curve on a hot and cold day, also two drag curves for two different weights, W_1 and W_2. The lines of constant C_L are also drawn versus equivalent airspeed.

To analyse this curve lets start with the drag lines, W_1 and W_2, and the C_L lines. It can be seen that to fly at W_1, the lower weight, takes a lower C_L than to fly at W_2, the heavier weight, at the same V_e. The curve also shows that on a hot day the thrust available is lower than on a cold day; therefore, the maximum C_L obtainable is lower. It is a proven fact that $D_t = D_s$ at the same V_e; therefore, if an increase in turning performance is desired the excess thrust must be increased. For instance in Figure 3. 5.3, if we consider an aircraft at $V_e = 1$ on a hot day, the maximum C_L obtainable is .9, but on a cold day it is 1.0.

Another version of Figure 3.5.3 is the Figure 3.5.4:

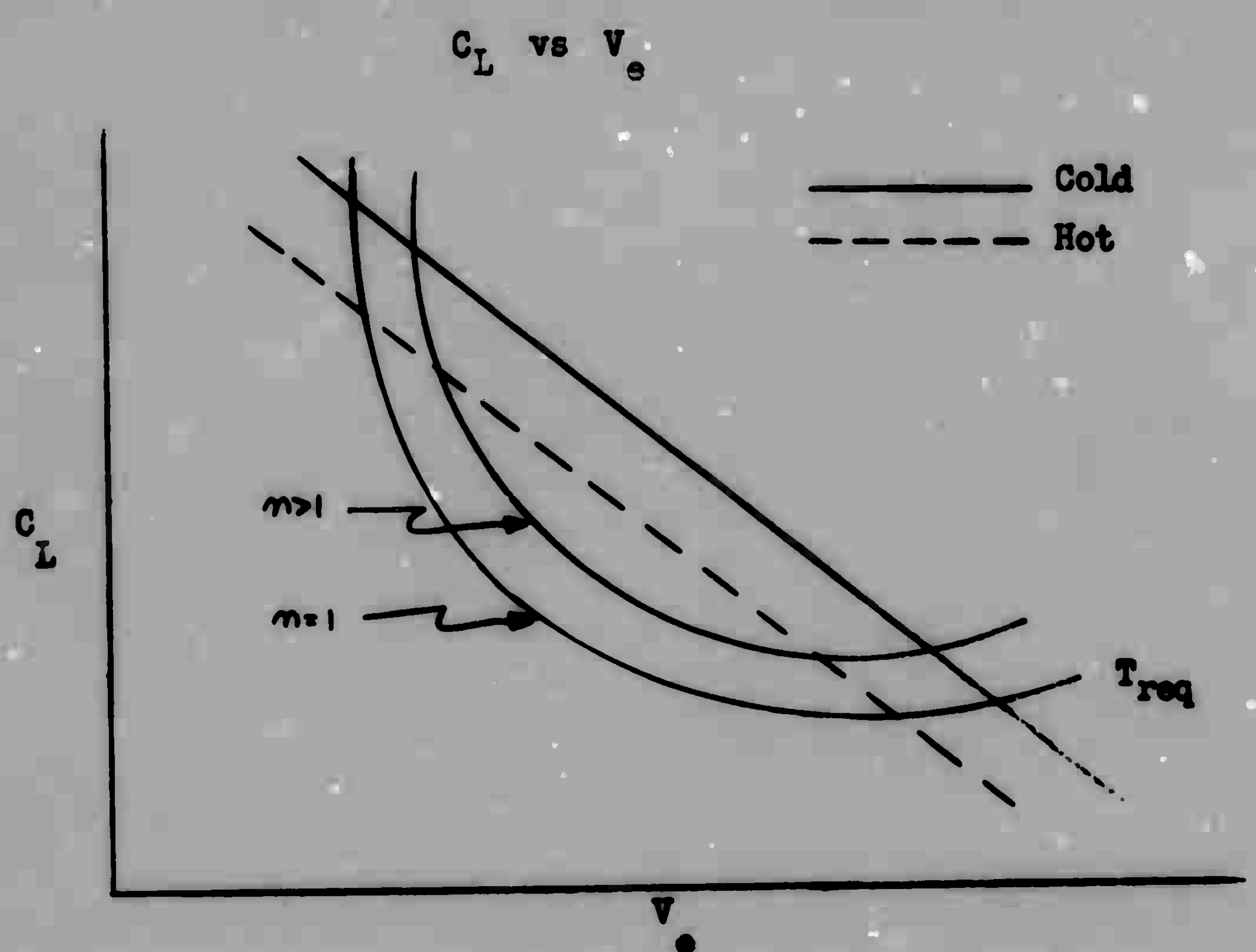


Figure 3.5.4

The $n = 1$ line is the thrust required or drag of an aircraft in stabilized level flight and line $n > 1$ is the same aircraft in a stabilized level turn. The thrust available lines are for a hot and a cold day. This indicates less thrust available on a hot day. It can be seen that as the load factor is increased the excess thrust decreases and if the load factor were increased enough there would be no excess thrust. This point where the thrust available and thrust required line are tangent is the point, maximum V_e, for best turning load factor. This plot can also be considered in terms of nW rather than n alone.

The temperature effect on turning performance can be corrected for by determining the change of thrust and by the following equations:

$$C_L = \frac{.000675}{M^2 S} \times \frac{nW}{\delta}$$

$$\Delta C_D = \frac{.000675}{M^2 S} \times \frac{Fn}{\delta} \qquad \text{Equation 3.5.4}$$

Where Fn/δ is determined from engine charts as in the check climb test.

By using the aircraft drag polar, C_L versus C_D, plotted at constant Mach numbers, the standard day conditions may be obtained. This is done by going into the drag polar at test conditions and computing ΔC_D as in the equation above, then the standard conditions may be obtained as indicated below:

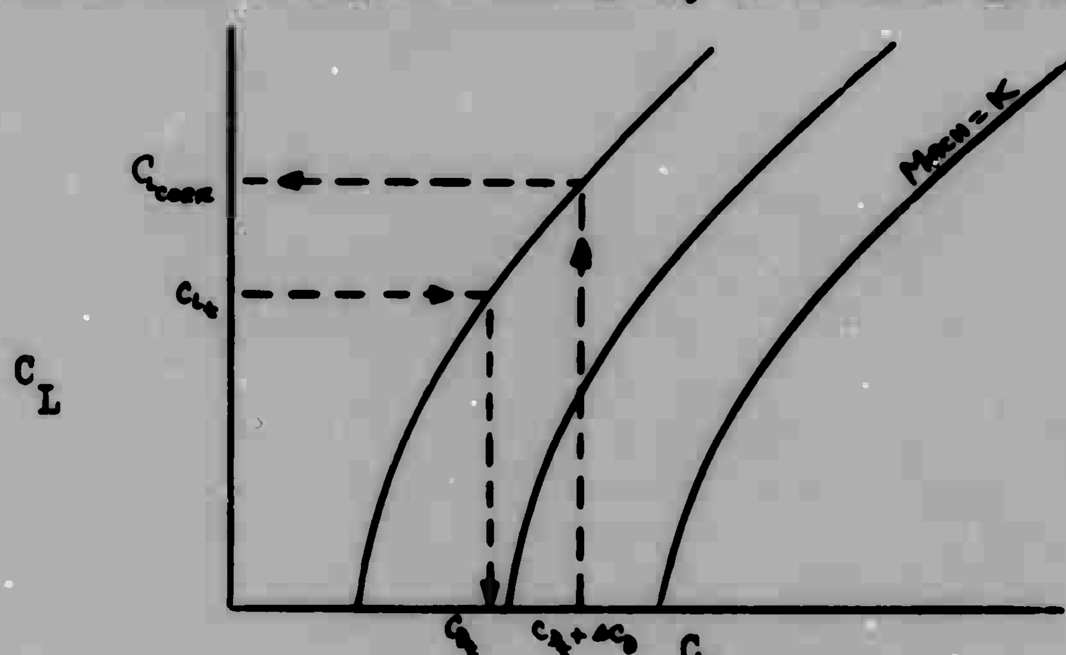


Figure 3.5.5

Now in order to correct n_t to n_s the following relationship is used:

$$n_s = \frac{C_{L_{corr}} M^2 S}{.000675\ (W/\delta\ )} \qquad \text{Equation 3.5.6}$$

where $C_{L_{corr}}$ is C_L corrected for temperature.

If the assumption is made that $C_{L_{corr}} = C_{L_t}$ then Equation 3.5.7

$$n_s = n_t \frac{W/\delta\ t}{W/\delta\ s}$$

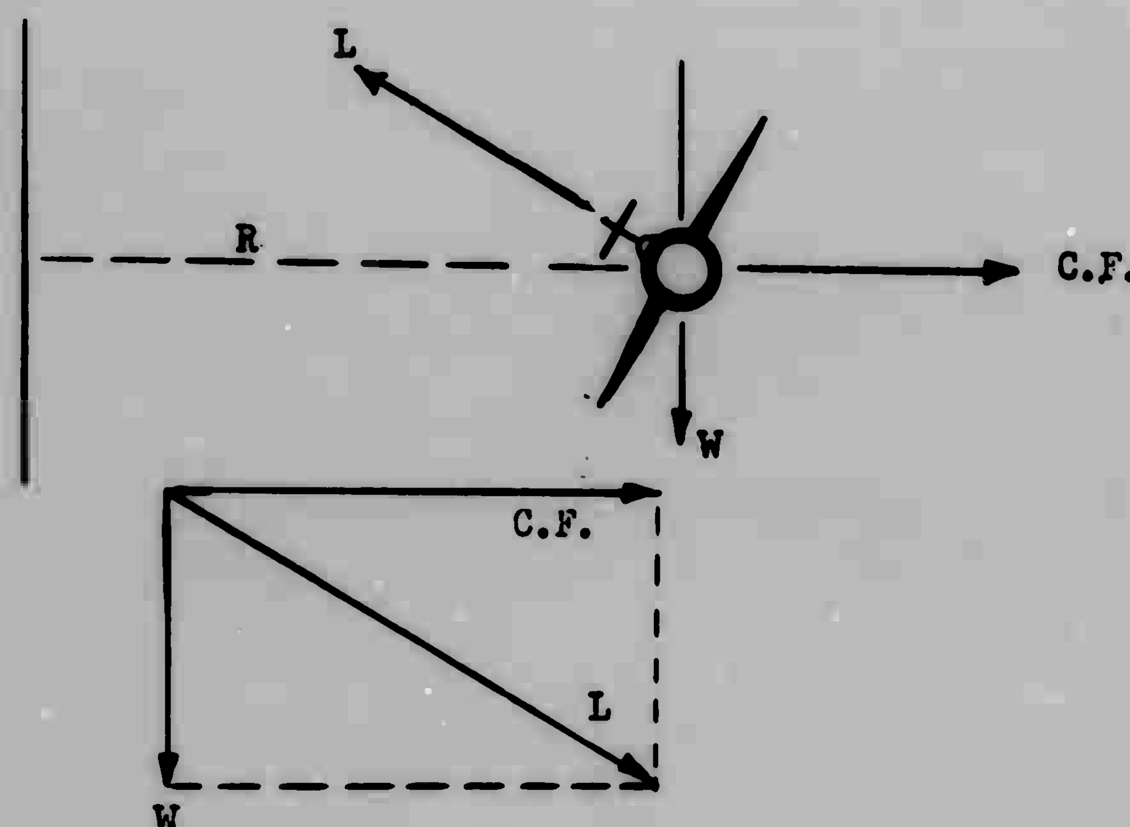


Figure 3.5.6

Before anymore is said, let's consider the forces acting on an aircraft in stabilized level turn of radious R. Consider Figure 3.5.6 above. From physics centrifugal force (C.F.) acts in the horizontal direction with weight (W) acting in the vertical direction. These forces are balanced by the lift (L) as shown. The radius of turn (R) in terms of speed and load factor can be derived as follows:

$$C.F. = Ma = \frac{W}{g}\frac{V^2}{R}$$ Equation 3.5.8

$$L^2 = \overline{C.F.}^2 + W^2$$

$$L^2 = \left(\frac{W}{g}\frac{V^2}{R}\right)^2 + W^2$$

$$L^2 = W^2\left(\frac{V^4}{g^2 R^2} + 1\right)$$

$$L = W\sqrt{\frac{V^4}{g^2 R^2} + 1}$$

now $n = \frac{L}{W}$

therefore $n = \sqrt{\frac{V^4}{g^2 R^2} + 1}$ Equation 3.5.9

$$n^2 = \frac{V^4}{g^2 R^2} + 1$$

Solving for the radius of turn (R)

$R = \frac{V_t^2}{g\sqrt{n^2 - 1}} =$ feet, where V_t = Ft/Sec Equation 3.5.10

$R = \frac{V_t^2}{11.3\sqrt{n^2 - 1}}$ = feet, where V_t = knots Equation 3.5.11

or for high altitude where speed of sound is a constant 575 knots:

$R = \frac{(575)^2 \; M^2}{11.3\sqrt{n^2 - 1}\,(6080)} = \frac{4.813 \; M^2}{\sqrt{n^2 - 1}}$ = N. M. Equation 3.5.12

In like manner, the rate of turn (W) can be derived:

$$\omega = \frac{V}{R} = \frac{V}{V_t^2/g \sqrt{n^2 - 1}} = \text{angular velocity} \qquad \text{Equation 3.5.13}$$

$$\omega = \frac{g \sqrt{n^2 - 1}}{V} = \text{rad/sec} \qquad \text{Equation 3.5.14}$$

or converting to nautical miles, knots and degrees per second

$$\omega = \frac{360 \quad 9.2 \sqrt{(n^2 - 1)}}{V_t} = \frac{.0159 \, V_t}{R} \qquad \text{Equation 3.5.15}$$

This equation can be plotted as shown in Chart A-74 and A-75 in AFFTC-TN-59-46. Knowing the V_t and n, and going into these charts, the radius and rate of turn can be determined. From these charts it is obvious that an aircraft which has a different weight but the same V_t and n as another will have the same turning performance.

The data which is obtained from turning performance is normally presented in the following form:

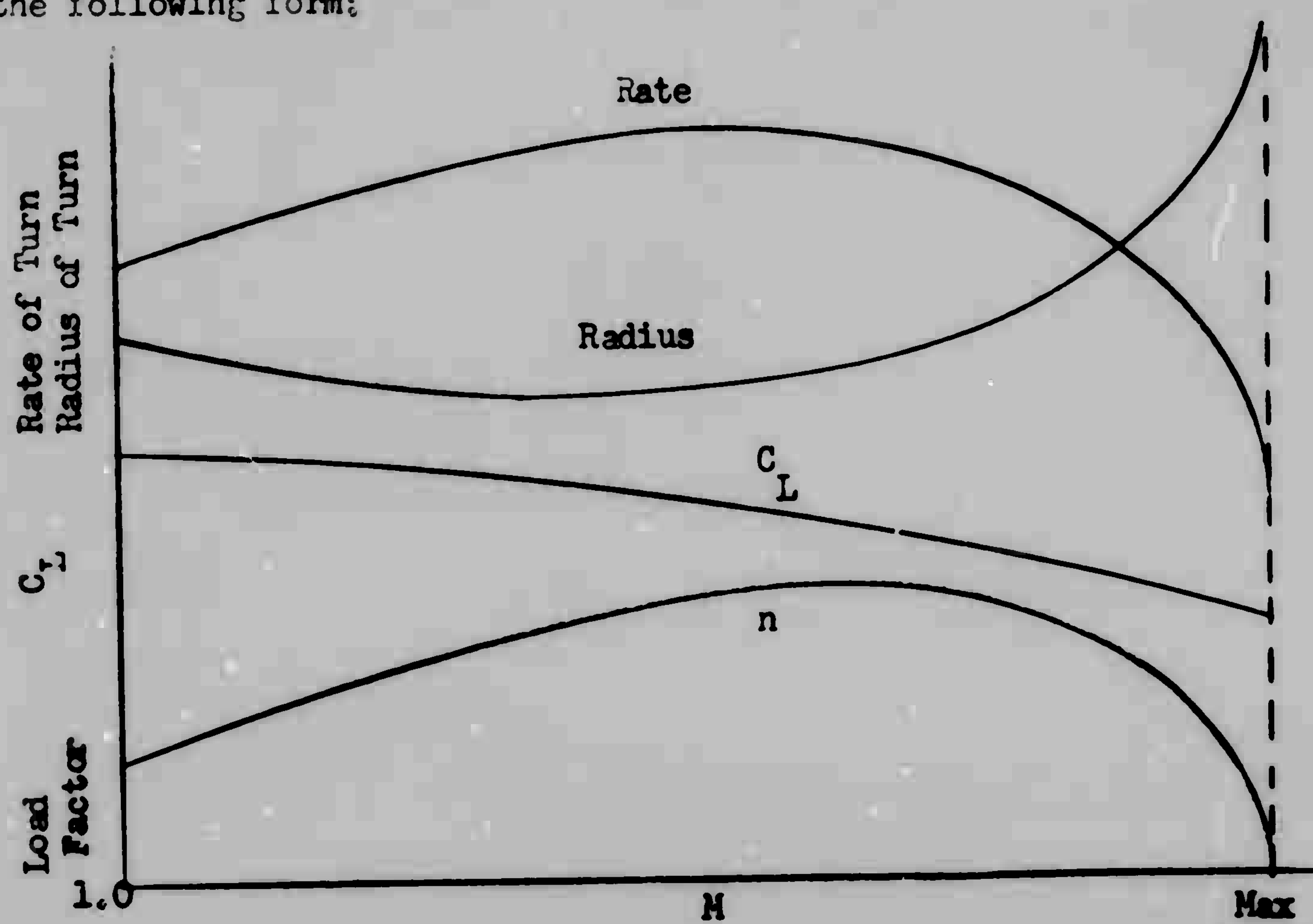


Figure 3.5.7

3.6 SPECIFIC ENERGY ANALYSIS

3.6.1 GENERAL:

The energy analysis of an aircrafts flight envelop was initiated by German engineers during the latter part of World War II. The introduction of the turbojet power plant enabled an aircraft to obtain values of kinetic energy that are of the same magnitude as the value of the potential energy. The use of an aircrafts total energy as the independent variable facilitates time to change from one combination of speed and altitude to another. An aircrafts range capability can also be determined by this analysis.

3.6.2 INTRODUCTION:

The total energy that an aircraft possesses at any given instant is the sum of the potential and kinetic energies.

$$E = W h + \frac{W V^2}{2g} \qquad \text{Equation 3.6.1}$$

$$\frac{E}{W} = h + \frac{V^2}{2g} = h_e \qquad \text{Equation 3.6.2}$$

The term (E/W) or energy per pound of aircraft weight is called specific energy or energy height. (This is the height which the aircraft could theoretically attain if all of its kinetic energy was coverted to potential energy.) It is often given the symbol h_e which has units of feet.

The conservation of energy over a given interval of time can be used to describe the total energy change of the aircraft, that is the energy dissipated against aerodynamic drag, the energy change due to loss of weight and the energy derived from the fuel burned.

$$dE = \eta_o \; H_c \; d\,\overline{FU} - D\,V\,dt - \left(\frac{E}{W}\right) d\,\overline{FU}$$ Equation 3.6.3

where

η_o is the overall efficiency of the power plant

H_c is the heat content of the fuel burned and is given units in this instance of ft-lb/lb. $\overline{FU}$ is the amount of fuel used and is equal to the decrease in gross weight dW.

The total energy equation may be written as

$$\left(\frac{E}{W}\right)\left(\frac{W}{1}\right) = E = Wh + \frac{W\,V^2}{2g} = W\left(h + \frac{V^2}{2g}\right)$$

By differentiation of the left side, this equation becomes

$$dE = \left(\frac{E}{W}\right) dW + W\,d\left(\frac{E}{W}\right)$$

$$dE = -\left(\frac{E}{W}\right) d\,\overline{FU} + W\,d\left(\frac{E}{W}\right)$$

This relationship is valid since $dW = -\,d\,\overline{FU}$. This value of dE is substituted into equation 3.6.3 which becomes

$$W\,d\left(\frac{E}{W}\right) = \eta_o \; H_c \; d\,\overline{FU} - DV\,dt$$ Equation 3.6.4

This equation is divided by W and differentiated with respect to time.

$$\frac{d\,(E/W)}{dt} = \frac{\eta_o\,H_c}{W} \cdot \frac{d\,\overline{FU}}{dt} - \frac{DV}{W}$$

The term $\eta_o \; H_c$ is the useful work output per pound of fuel and can be expressed as thrust output and fuel flow.

$$\eta_o H_c = \frac{\text{work/time}}{\text{\# fuel/time}} = \frac{T \cdot V}{FU/dt}$$

Substituting this value of $\eta_o H_c$ into the equation it becomes

$$\frac{d(E/W)}{dt} = \frac{T V}{W} - \frac{D V}{W}$$

or $$\frac{dhe}{dt} = \frac{(T - D)}{W} V \qquad \text{Equation 3.6.5}$$

Now it can be seen that the rate of change of specific energy or energy height is equal to the product of the excess thrust and velocity per pound of aircraft weight and is the basic unaccelerated rate of climb equation.

Equation 3.6.2 can be differentiated with respect to time and becomes:

$$\frac{dhe}{dt} = \frac{dh}{dt} + \frac{V}{g} \cdot \frac{dv}{dt}$$

This value of dhe/dt is substituted into equation 3.6.5

$$\frac{dh}{dt} + \frac{V}{g} \frac{dv}{dt} = \frac{(T - D)}{W} V$$

$$\left[\frac{dh}{dt} = \frac{(T - D)}{W} V \right] - \frac{V}{g} \frac{dv}{dt} \qquad \text{Equation 3.6.6}$$

This equation is the complete rate of climb equation that includes acceleration in the climb path. Thus it can be seen that the excess power per pound of aircraft weight can be used to give an unaccelerated climb, a level flight acceleration or a combination of the two conditions. Equation 3.6.6 can be rewritten so the acceleration can be corrected to give true rate of climb if all of the excess power was used only to increase the potential energy of the aircraft.

$$\frac{dh}{dt} = \frac{(T - D)\,V}{W}\left[\frac{1}{1 + (V/g)\,(dv/dh)}\right] \qquad \text{Equation 3.6.7}$$

This equation will produce the unaccelerated rate of climb from climb data but is often difficult to use in determining the conditions that will give the optimum rate of climb. Since $\frac{(T - D)\,V}{W}$ is a function of h and V, a functional relationship can be derived for equation 3.6.7

$$\frac{dh}{dt} = f\,(V,\ h,\ dV/dh)$$

The solution of this functional equation to give the optimum climb technique requires the use of calculus of variations which are often complicated. The use of specific energy, equation 3.6.5, give a simplified means of finding the optimum technique and still retains the accuracy of the other climb equations.

3.6.3 SPECIFIC ENERGY SOLUTION FOR CLIMBS

At a given gross weight, the rate of change of energy height becomes a function of only the true airspeed and altitude, which in turn also determines the excess thrust of the aircraft powerplant combination. Thus, Equation 3.6.5 gives a simplified solution and becomes a function of only the two variables or $dhe/dt = f\,(V,\ h)$. The equation does not directly specify the form in which the total energy is changed but gives a relationship for the rate of change of the total energy per pound of aircraft weight. The use of energy height, he, as the independant variable in an aircraft's level flight envelop is illustrated in Figure 3.6.1.

This envelop is constructed for a maximum power setting at all points.

Supersonic Fighter Envelop

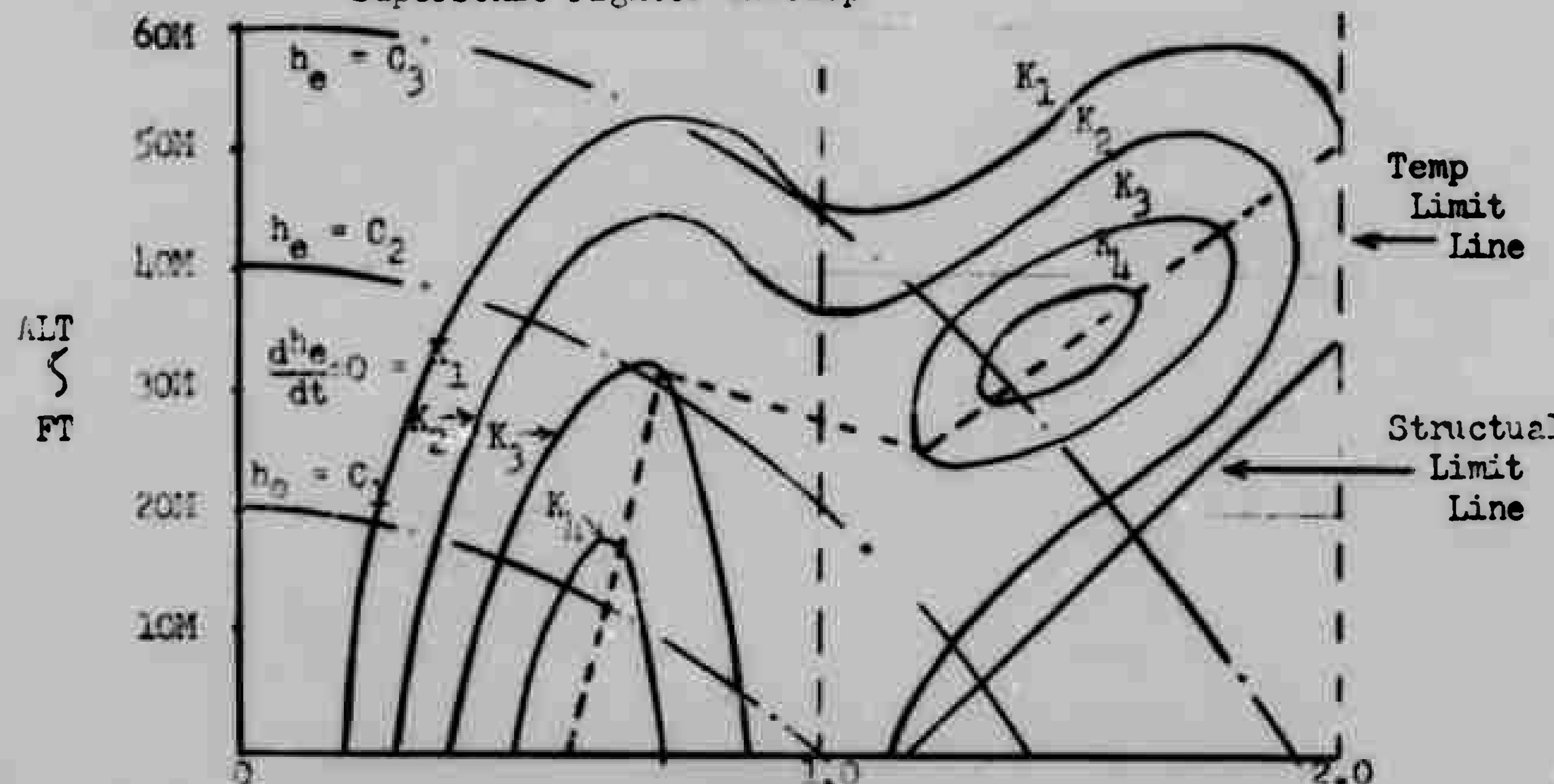


Figure 3.6.1

From this Figure it is seen that the final altitude can be reached at a wide range of speeds or at a wide range of energy heights. Therefore, the problem of a climb schedule may be presented as the problem to minimize the time to go from one level of energy height to a higher level. The use of the concept of specific energy is also useful due to the fact that a rapid estimation can be made of the maximum altitude obtainable in a zoom or the maximum speed in a dive.

The determination of the minimum time to change from one altitude and airspeed combination to another combination is solved as the minimum time to go from one energy level to the desired level. An equation using energy altitude as the independant variable can be written that will solve the minimum time problem.

$$t = \int_{he_1}^{he_2} \frac{1}{dhe/dt} \cdot dhe \qquad \text{Equation 3.6.8}$$

From the previous derivation we have

$$he = h + V^2/2g \qquad \text{Equation 3.6.2}$$

$$\frac{dhe}{dt} = \frac{(T - D)\,V}{W} = f\,(h, V)$$

Since from equation 3.6.2

$$he = f\,(h, V)$$

or

$$h = f\,(he, V)$$

and

$$V = f\,(he, h)$$

it follows that

$$\frac{1}{\left(\frac{dhe}{dt}\right)} = f_1\,(he, V) = f_2\,(he, h)$$

$$t = \int_{he_1}^{he_2} f_1\,(he, V)\,dhe$$

or

$$t = \int_{he_1}^{he_2} f_2\,(he, h)\,dhe$$

The problem reduces to minimizing the above intergals, which in turn minimizes the time to change from one energy level to another. Since two variables are involved the calculus of variations is used and the integral is at a minimum value when

$$\frac{\partial}{\partial V}\left[\frac{dt}{dhe}\right]_{he\,=\,K} = 0$$

or
$$\frac{\partial}{\partial h}\left[\frac{dt}{dhe}\right]_{he\,=\,K} = 0 \qquad \text{Equation 3.6.9}$$

Now if $\frac{\partial}{\partial x}\left[f\,(x\,,\,y)\right] = 0$ it then follows that $\frac{\partial}{\partial X}\left[f^{-1}(x\,,\,y)\right] = 0$

That is to say, if the slope of the function f (x , y) equal zero the slope of $1/f(x\,,\,y)$ is also equal to zero as shown in the following figure.

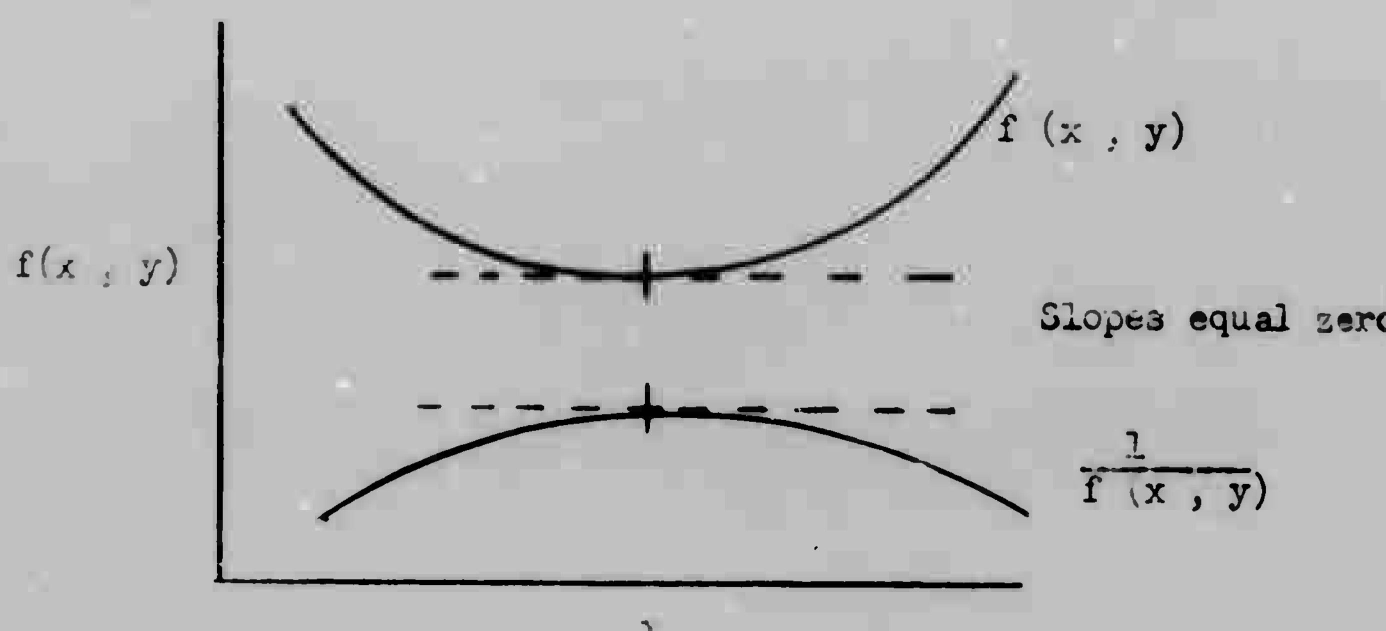


Figure 3.6.2

From this relationship we can write equation 3.6.9 as

$$\frac{\partial}{\partial V}\left[\frac{dhe}{dt}\right]_{he\,=\,K} = 0$$

or
$$\frac{\partial}{\partial h}\left[\frac{dhe}{dt}\right]_{he\,=\,K} = 0 \qquad \text{Equation 3.6.10}$$

Substituting in this equation for dhe/dt we now have:

$$\frac{\partial}{\partial V}\left[\frac{(T - D)\,V}{W}\right]_{he = K} = 0 = \frac{\partial}{\partial h}\left[\frac{(T - D)\,V}{W}\right]_{he = K}$$

Equation 3.6.11

The optimum climb schedule is now the one that satisfies this condition at every point. This path may be difficult to determine by analytical means but can be easily solved using a graphic presentation. The basic rate of climb equation to find the climb speed at each altitude which gives the maximum rate of climb is determined by the equation 3.6.12

$$\frac{\partial}{\partial V}\left[\frac{(T - D)\,V}{W}\right]_{h_e = K} = 0$$

Equation 3.6.12

The two climb schedules, the maximum rate of climb by the conventional method and the optimum by the specific energy method is shown in Figure 3.6.3. Interpreted graphically Equation 3.6.11 requires that at a given value of energy height the speed which results in the maximum excess power per pound of aircraft weight will be the speed which gives a minimum time interval to go from one value of energy height to another. Equation 3.6.12 is used in determining the maximum rate of climb schedule at each altitude.

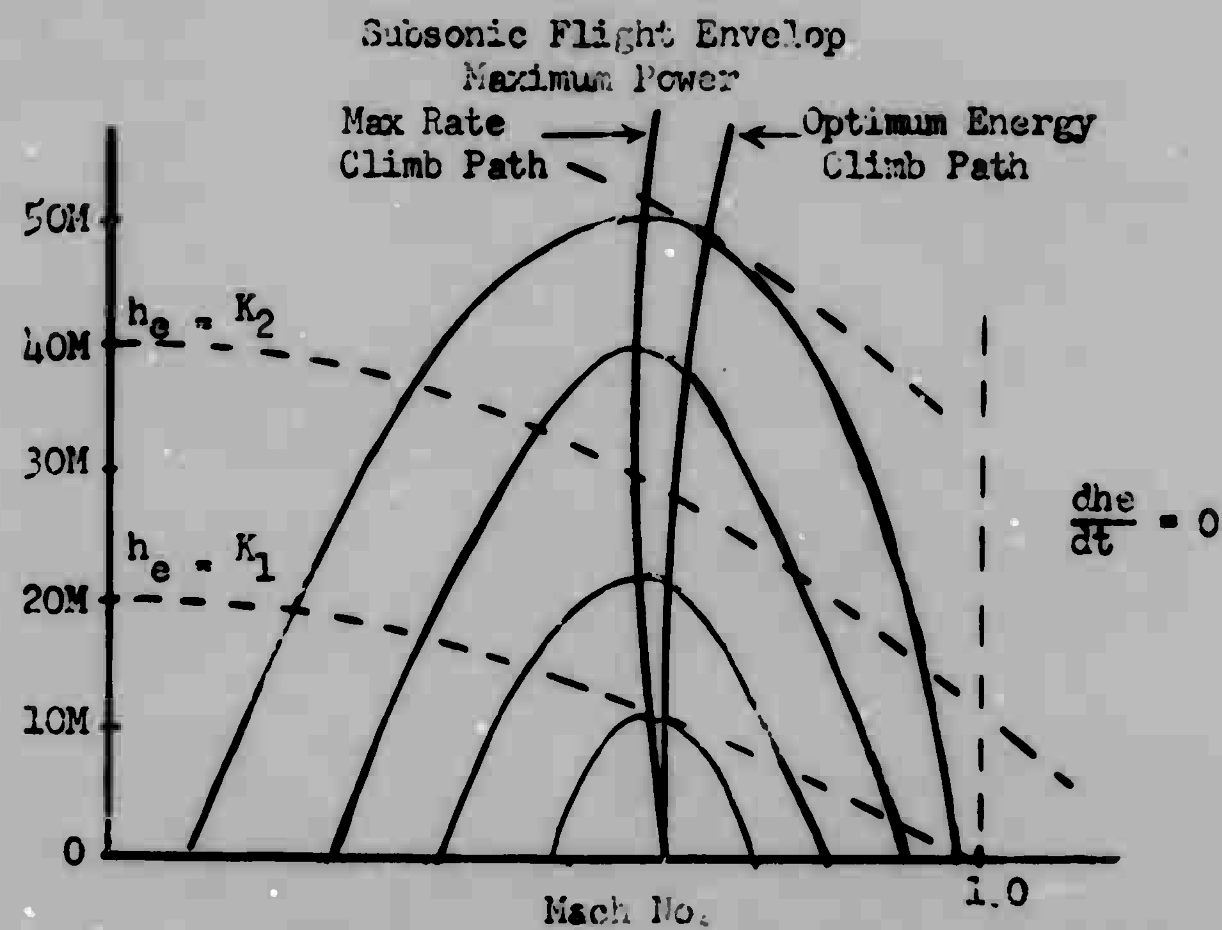


Figure 3.6.3

This Figure shows contours of constant excess power per pound of aircraft weight or dhe/dt superimposed on contours of constant energy height. The two different climb schedules are illustrated on this plot. The plot is for a typical subsonic turbojet fighter and it can be seen that for this case the maximum rate of climb schedule is close to optimum energy climb schedule. As the speed increases above Mach 1.0 the schedules grow farther apart.

The difference between using the two methods as given by equations 3.6.11 and 3.6.12 is that the energy method chooses velocities where the excess power contours are tangent to a line of constant energy height while the conventional method chooses velocities where the contours are tangent to a line of constant altitude. Figure 3.6.4 shows the minimum time to climb for a supersonic fighter of the Mach 2.0 category.

Supersonic Fighter Flight Envelop
Maximum Power

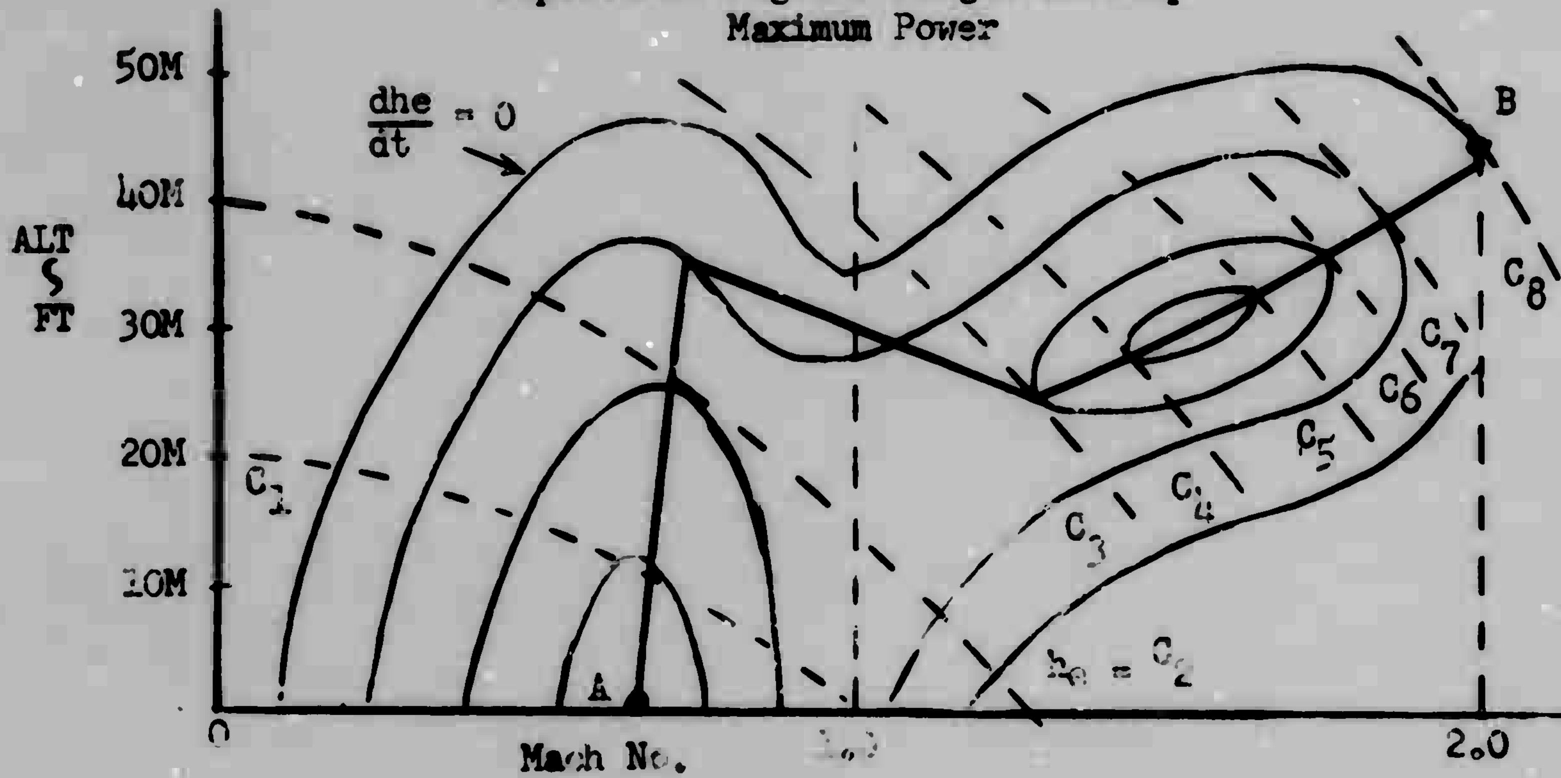


Figure 3.6.4

This diagram of the flight envelope is readily adaptable to a specific energy analysis in order to determine the optimum climb performance for a combination of altitudes and airspeeds. The flexibility of this method is demonstrated in that a climb with acceleration or an acceleration with a dive can be part of the optimum energy path. The maximum altitude in a zoom from any point can also be estimated. The aircraft in Figure 3.6.4 has a certain value of energy height at point B. If the pilot could convert all of this energy into actual altitude he could reach an altitude at zero velocity that would equal the value of the energy height at point B. This is impossible, however due to the fact that when the aircraft is at an altitude above the contour of dhe/dt = 0 it is in a region of negative excess thrust, (T - D) = (-) The value of a negative excess thrust acting over an interval of time will dissipate some of the energy causing a decrease in the total energy height and, therefore, the aircraft will never be able to convert all of the total energy at a given point into potential energy. The same analogy holds for the maximum speed in a dive.

It is interesting to note that for air breathing engines the lines of dhe/dt = K or excess power per pound of aircraft weight decreases as height is increased. This is due to the decrease in the thrust output of the engine as altitude is increased. The rocket engine aircraft combination is different due to the fact that the thrust output of the engine increases slightly with an increase in altitude. The specific energy analysis for this type aircraft is shown in Figure 3.6.5.

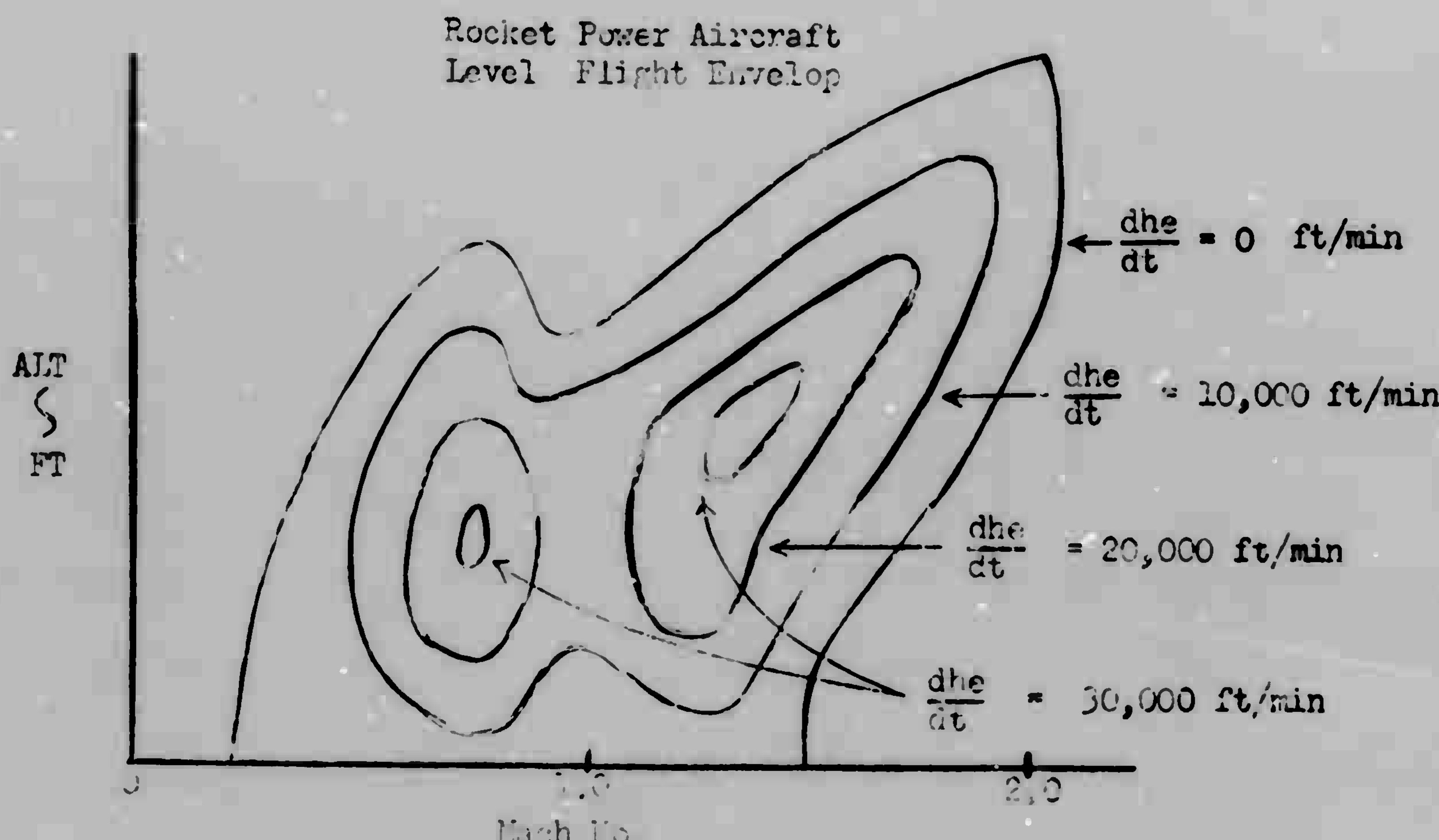


Figure 3.6.5

A three deminsional representation of an aircraft flight envelop using the specific energy analysis can be constructed. This envelop is shown in Figure 3.6.6 The optimum energy climb schedule can be outlined in the h_e and $V_T^2/2g$ plane, Figure 3.6.6a. The maximum rate of climb is outlined by the peaks of the body in the plane of dhe/dt and $V_T^2/2g$, Figure 3.6 6b. The three dimensional analysis is a convenient method for comparison of the two climb schedules.

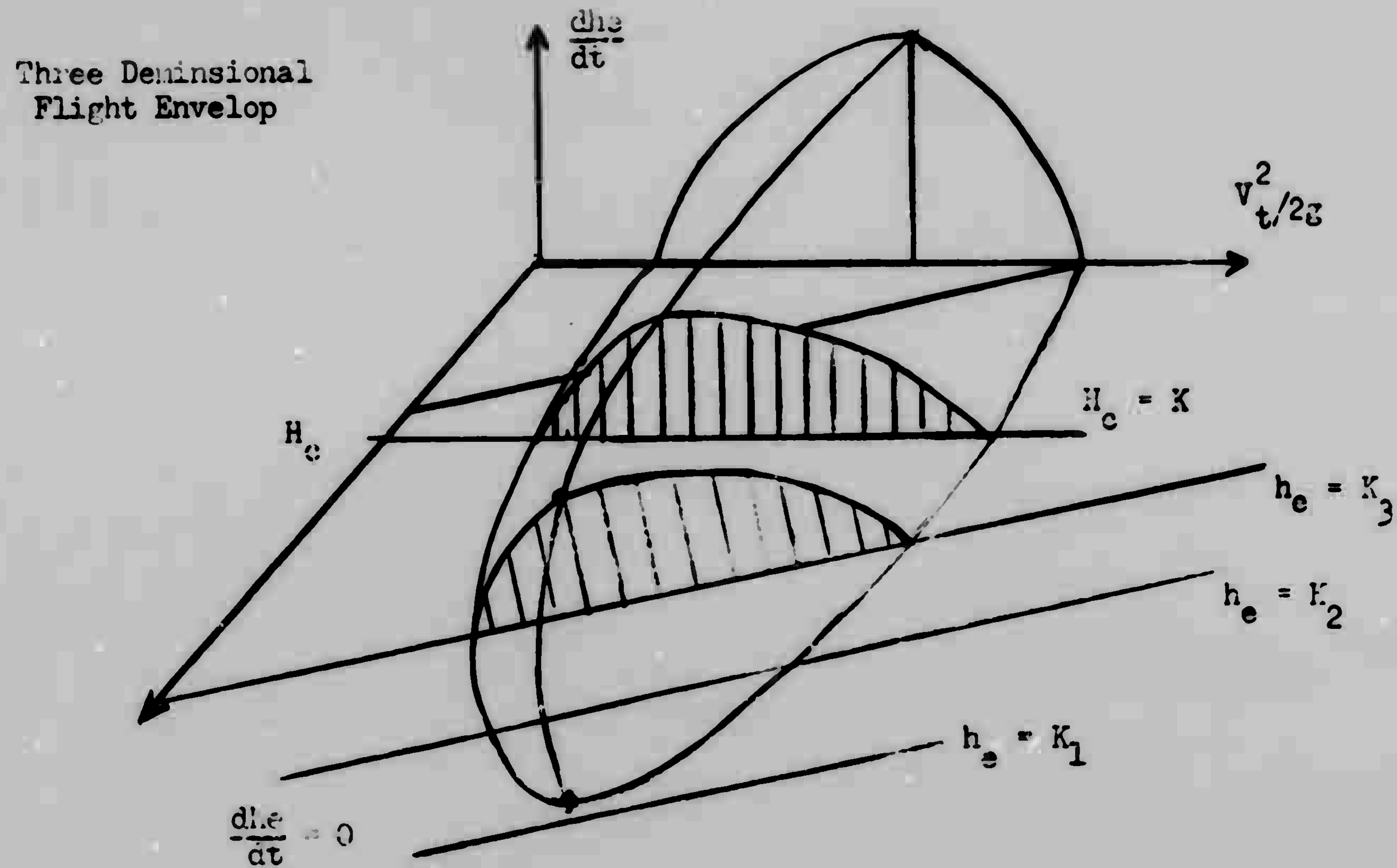


Figure 3.6.6

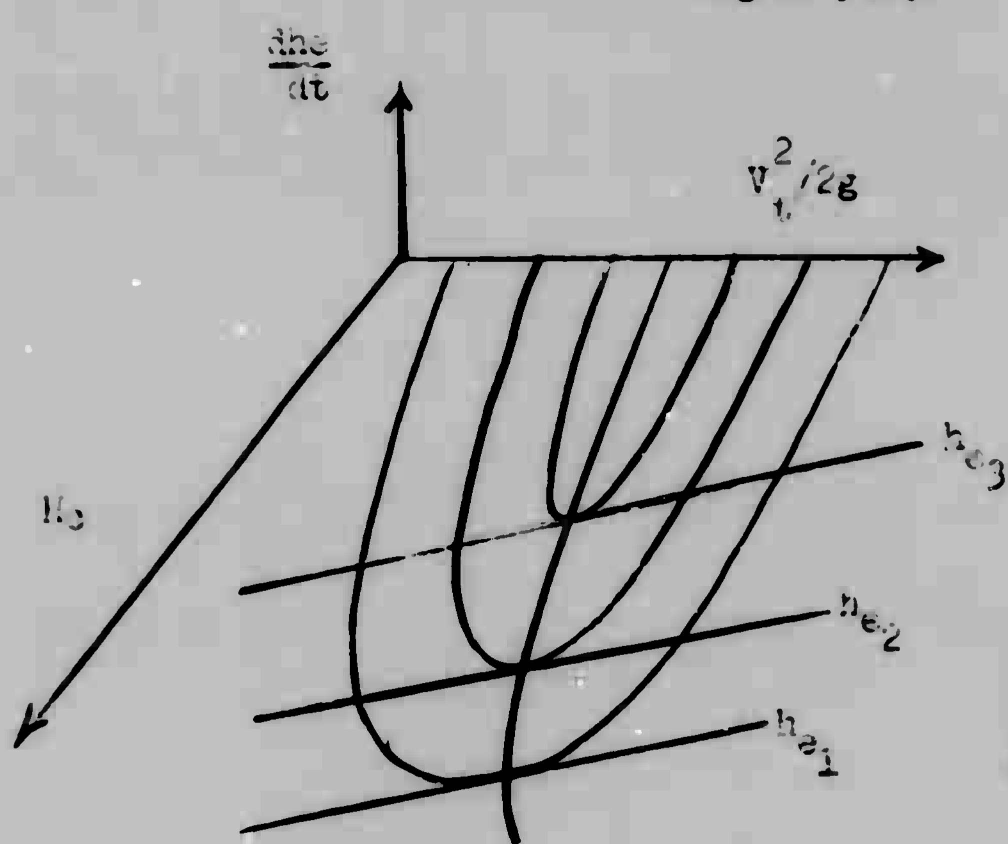


Figure 3.6.6a

Opitimum Energy Climb Schedule

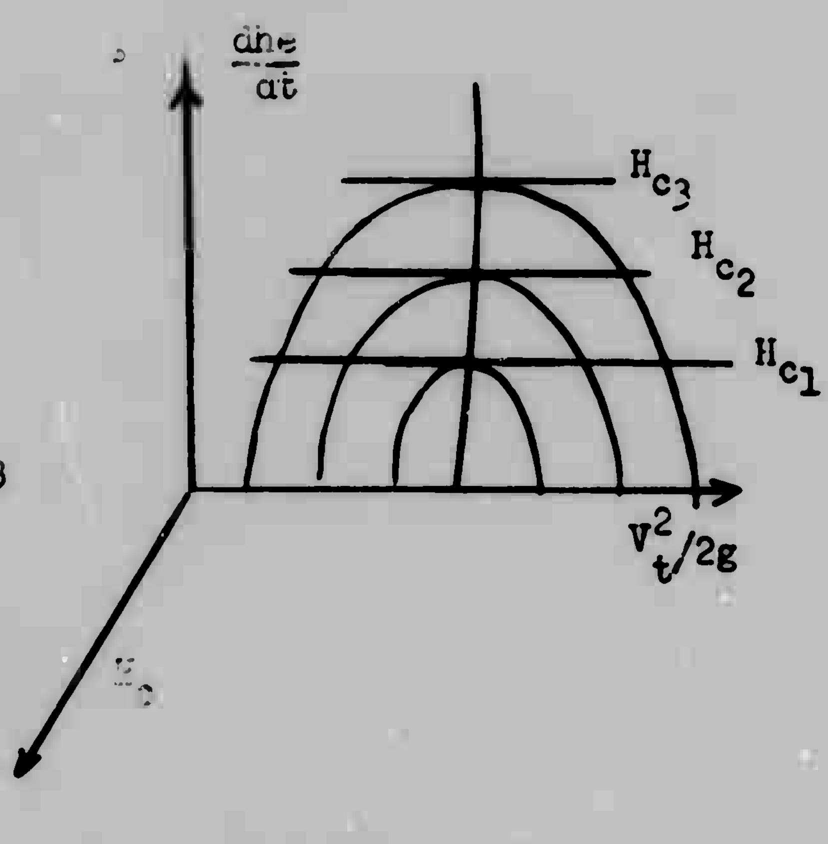


Figure 3.6.6b

Maximum Rate of Climb Schedule

3.6.4 SPECIFIC ENERGY ANALYSIS FOR LEVEL FLIGHT PERFORMANCE

The data for constructing the flight envelop as shown in Figures 3.6.1, 3.6.2 and 3.6.3 may be obtained by continued climbs or level accelerations. The continued climbs are more difficult to fly and they require accurate temperature and wind information for the correction of the data. The level flight acceleration data is not effected by either of the above factors and is easily reduced.

The actual values of excess thrust at various speeds and constant altitudes are useful in obtaining flight test data for turning performance as well as climbs. The value of excess thrust can be determined once the plot of dhe/dt is made.

$$\frac{dhe}{dt} = \frac{(T - D)\,V}{W} = \frac{(T_{ex})\,V}{W}$$

$$\left(\frac{dhe}{dt}\right)\left(\frac{W}{V}\right) = T_{ex} \qquad \text{Equation 3.6.13}$$

Therefore it is seen that at any given velocity during a level acceleration the actual excess thrust may be determined.

The parameter T_{ex}/δ is often required to obtain the final data for acceleration and turning capability. This is accomplished by multiplying both sides of Equation 3.6.13 by $1/\delta$.

$$\frac{dhe}{dt} \cdot \frac{W}{V} \cdot \frac{1}{\delta} = \frac{T_{ex}}{\delta} \qquad \text{Equation 3.6.14}$$

From the level flight acceleration data the positive values of Tex/δ at various airspeeds or Mach numbers can be determined.

A maximum power steady state turn at a constant altitude results in a condition where T = D or excess thrust is zero. A maximum power level decelerating turn at a constant altitude results in a situation where the

drag is greater than the thrust available and the excess thrust becomes a negative value. This will naturally produce a value of dhe/dt that is negative. Plotting the excess thrust data from the accelerations, decelerating turns and stabilized level turns versus the Mach number gives Figure 3.6.7.

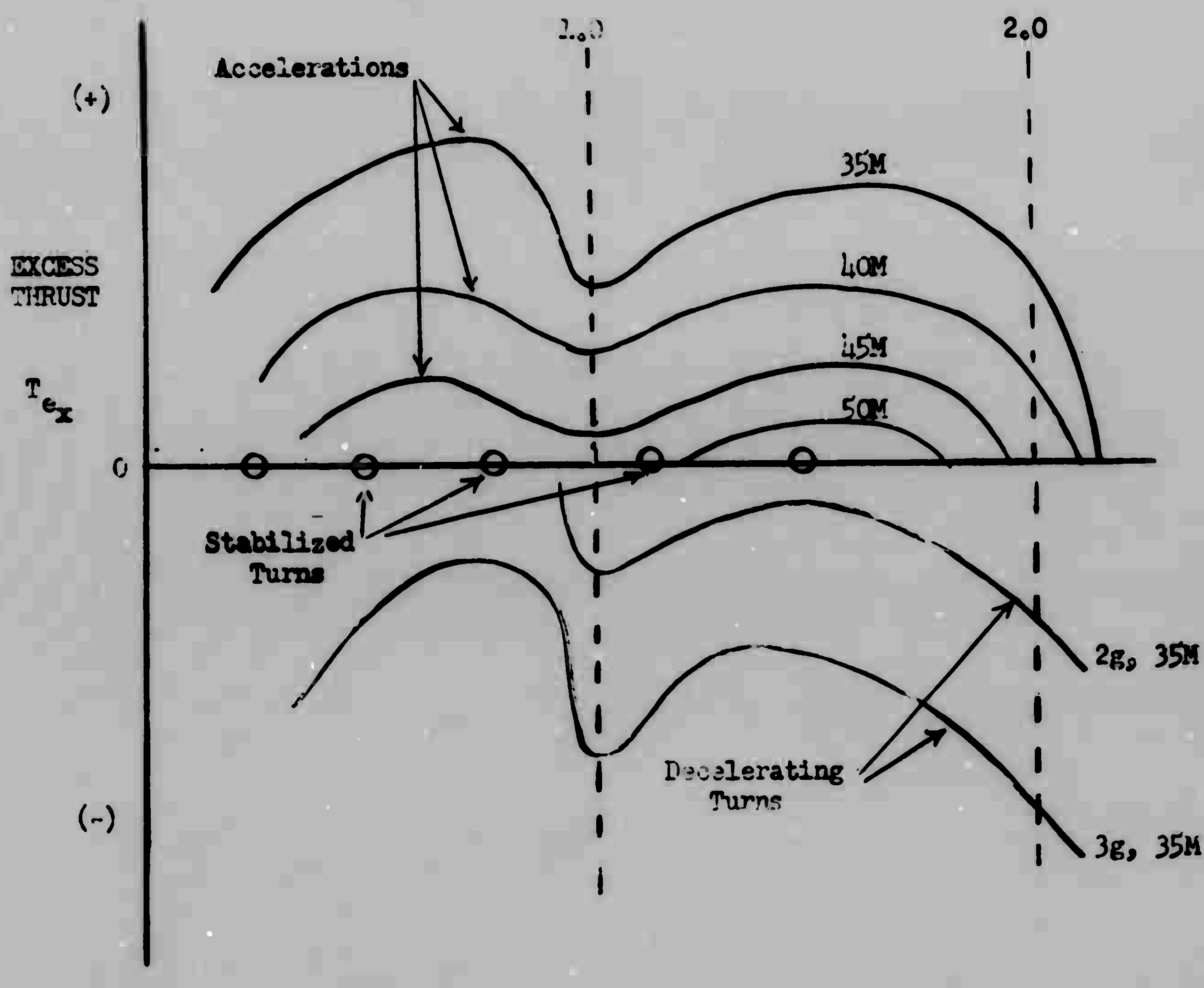


Figure 3.6.7 Excess Thrust Versus Mach Number

This data is then cross plotted at constant Mach number to a more versitile form shown in Figure 3.6.8.

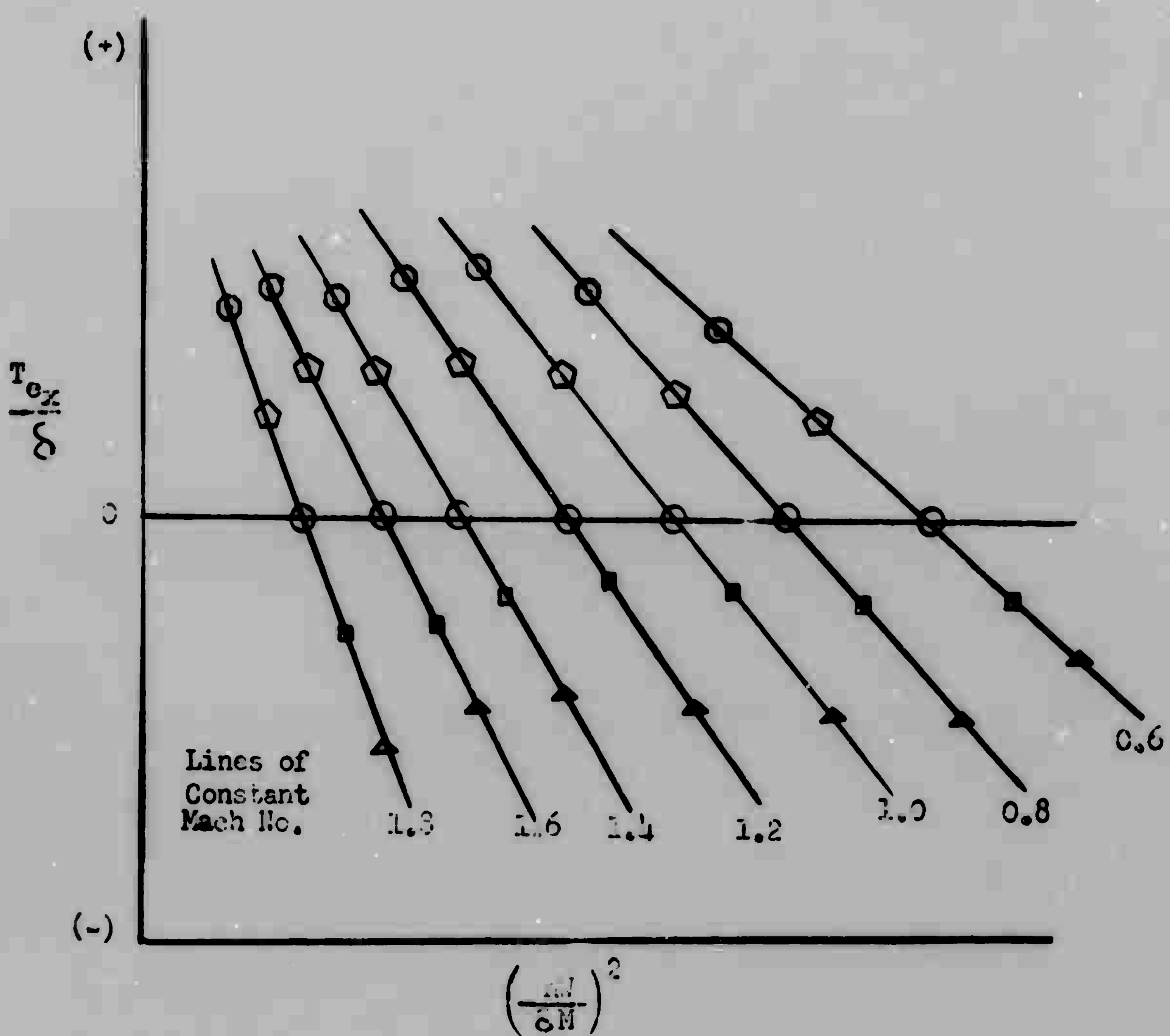


Figure 3.6.8 Specific Energy Summary Plot

The negative values are obtained from the decelerating turn curves, (Figure 3.6.7) the positive values from the 1 g acceleration curves and the zero values from the stabilized turns.

From this plot it is possible to obtain:

1. Acceleration performance - such as time distance and fuel used (requires fuel flow data) to accelerate from one speed to another.

2. Climb performance - both accelerated and unaccelerated rates of climb as well as the climb ceiling at any speed can be obtained.

3. Maximum speed - can be extrapolated for $Tex/\delta = 0$ for any given altitude.

4. Turning performance - both stabilized and decelerating turning performance can be determined.

5. Any combination of the above can be found, such as climbing accelerating turns, descending stabilized turns, etc.

3.6.5 RANGE DETERMINATION FROM SPECIFIC ENERGY ANALYSIS

The determination of an aircrafts range capability can be made using specific energy analysis. Equation 3.6.4 is used as the starting equation for this development.

$$W d\left(\frac{E}{W}\right) = \eta_o \, H_c \, d\,\overline{FU} - D \boxed{V \; dt}$$

The term V dt is equal to dR and can be substituted into the above equation

$$dR = \frac{\eta_o \, H_c \, d\,\overline{FU}}{D} - \frac{W}{D} d\left(\frac{E}{W}\right)$$

In straight and level flight the assumption is made that $L = W$ and $d\,\overline{FU} = -dW$. The equation can be rewritten as

$$dR = -\frac{\eta_o \, H_c \, dw}{D}\left(\frac{L}{W}\right) - \left(\frac{L}{D}\right) d\left(\frac{E}{W}\right)$$

or $$dR = -\eta_o \, H_c \, \frac{dW}{W}\left(\frac{L}{D}\right) - \left(\frac{L}{D}\right) d\left(\frac{E}{W}\right)$$

This equation is now integrated to give total range.

$$R = -\eta_o \quad Hc\left(\frac{L}{D}\right)\ln\left(\frac{W_1}{W_o}\right) - \left(\frac{L}{D}\right)\left[\left(\frac{E}{W}\right)_1 - \left(\frac{E}{W}\right)_o\right] \qquad \text{Equation 3.6.15}$$

where W_o = starting gross weight and

W_1 = final gross weight

As proven previously $\eta_o \quad Hc = \frac{T \cdot V}{dFU/dt} = \frac{T \cdot V}{W_f}$ and $dW_f/dt = Tc_j$ the term can be written as $\eta_o \quad Hc = \frac{V}{c}$. This term is substituted into equation 3.6.15 which becomes

$$R = \left(\frac{V}{c_j}\right)\left(\frac{L}{D}\right) \quad \ln\left(\frac{W_c}{W_1}\right) - \left(\frac{L}{D}\right)\left[\left(\frac{E}{W}\right)_1 - \left(\frac{E}{W}\right)_o\right] \qquad \text{Equation 3.6.16}$$

An examination of the above equation shows that the second term is required only if there is a change in specific energy during the cruising portion of the flight. A turbojet aircraft cruises at a constant angle of attack or constant L/D ratio to obtain the maximum range available. This technique of holding L/D constant will result in the aircraft climbing as fuel is consumed. The increase in altitude will result in a decreased dynamic pressure as the Mach number or V_t is held constant above the tropopause. This increased altitude will increase the value of the specific energy, therefore this change must be considered. The change in the specific energy term is usually less than 1% of the basic term when the aircraft is flown at the tropopause or above and at airspeeds of 400 knots or faster. Thus, the last term of Equation 3.6.16 can be dropped without any appreciable effect on the

accuracy of the equation. The equation now becomes the familiar Berguet Range equation.

$$R = \left(\frac{V}{c_j}\right)\left(\frac{L}{D}\right) \ln \left(\frac{W_o}{W_1}\right)$$ Equation 3.6.17

An examination of the altitude gained during the cruise climb portion of flight can also be made at this time. In straight and level flight

$$nW = L = C_L\left(\frac{\rho_a}{2}\right) V_t^2 \ S$$

Substituting for $\frac{\rho_a}{2} V_t^2$ in terms of M for 1 g level flight

$$W = C_L \ (\gamma/2) \ p \ M^2 \ S$$ Equation 3.6.18

Differentiating:

$$dW = C_L \ (\gamma/2) \ dp \ M^2 \ S$$ Equation 3.6.19

Dividing equation 3.6.19 by equation 3.6.18 gives the following result:

$$\frac{dW}{W} = \frac{dp}{p}$$

Since $dp = -\rho g \, dh$, and $p = \rho g \, RT$

$$\frac{dp}{p} = -\frac{\rho g \, dh}{\rho g \, RT} = -\frac{dh}{RT}$$

$$\therefore \frac{dW}{W} = -\frac{dh}{RT}$$

When the aircraft is flown above the tropopause the ambient temperature will be 216.66 °K on a standard day. Substituting this value for T and 96.06 ft/°K for R the equation becomes

$$\frac{dW}{W} = -\frac{dh}{20{,}940}$$

$$\frac{20{,}940\ dW}{W} = -\ dh \qquad \text{Equation 3.6.20}$$

Equation 3.6.20 gives the altitude gain in feet for a decrease in gross weight during cruise.

3.6.6 ENDURANCE DETERMINATION FROM SPECIFIC ENERGY ANALYSIS

The endurance capability of an aircraft can also be determined using specific energy analysis. Equation 3.6.4 is again used as the starting equation for this development.

$$W d\left(\frac{E}{W}\right) = \eta_o\ H_c\ d\,\overline{FU} - D\ V\ dt \qquad \text{Equation 3.6.4}$$

$$dt = \frac{\eta_o\ H_c\ d\,\overline{FU}}{DV} - \frac{W}{DV}\ d\ \frac{E}{W}$$

For stabilized flight $L = W$ and $d\,\overline{FU} = -\ dW$.

$$dt = -\frac{\eta_o\ H_c\ \ d\,W}{DV}\left(\frac{L}{W}\right) - \frac{L}{DV}\ d\left(\frac{E}{W}\right)$$

$$\text{Endurance} = \int dt = -\frac{\eta_o\ H_c}{V}\left(\frac{L}{D}\right)\ln\left(\frac{W_1}{W_o}\right) - \left(\frac{L}{D}\right)\left(\frac{1}{V}\right)\left[\left(\frac{E}{W}\right)_1 - \left(\frac{E}{W}\right)_o\right]$$

$$\text{Endurance} = \frac{\eta_o\ H_c}{V}\left(\frac{L}{D}\right)\ln\left(\frac{W_o}{W_1}\right) \quad -\left(\frac{L}{D}\right)\left(\frac{1}{V}\right)\ \left(\frac{E}{W}\right)_1 - \left(\frac{E}{W}\right)_o$$

Substituting values for $\eta_o\ H_c$ the equation becomes

$$\text{Endurance} = \frac{1}{c_j}\ \left(\frac{L}{D}\right)\ln\left(\frac{W_o}{W_1}\right) - \left(\frac{L}{D}\right)\left(\frac{1}{V}\right)\left[\left(\frac{E}{W}\right)_1 - \left(\frac{E}{W}\right)_o\right] \qquad \text{Equation 3.6.21}$$

The last term is due to any change in specific energy and may be dropped as indicated in section 3.6.5.

The final equation now becomes

$$\text{Endurance} = \frac{1}{c_j}\left(\frac{L}{D}\right)\ln\left(\frac{W_o}{W_1}\right) \qquad \text{Equation 3.6.22}$$

Thus, it is seen that the concept of specific energy can be an extremely useful means of evaluating an aircrafts performance.

3.7 TAKEOFF AND LANDING

An important part of the testing of any aircraft is the takeoff, landing and operation in close proximity to the ground. Takeoff and landing are greatly dependent on pilot judgement and technique and, therefore, are subject to considerable variation for any given aircraft and set of conditions. Because of this large unpredictable variable, namely the pilot, it is neither possible nor practical to make exact prediction or correction of takeoff and landing performance. It is only possible to estimate the approximate capabilities of an airplane within rather broad limits. For this reason, takeoff and landing performance will be considered from a rather general point of view taking into account only the major variables and making some assumptions concerning the lesser variables.

3.7.1 TAKEOFF PERFORMANCE

Many factors, other than the pilot, effect the takeoff performance. Some are obvious while others are rather subtle. Among the obvious are power, wind, runway slope, rolling friction, weight and aircraft configuration. Some of the less obvious factors are temperature and density effects on aerodynamic lift and drag and on power plant and propeller performance. Still more subtle are the effects of wind shear and ground effect.

Phase of Takeoff and Effects of Pilot Technique

A takeoff may be considered as being made up of three basic phases. These are:

1. Ground roll in which minimum drag and maximum thrust are desired.
2. Transition which consists of rotation to a flight attitude, takeoff and establishing an initial climb angle.

3. Initial climb out to a safe altitude.

Each of these phases are subject to variation in pilot technique and technically speaking require different methods of analysis. For instance, early rotation of the aircraft in the ground phase will increase the ground distance and may even prevent attaining takeoff speed because of the increased drag. Thus, it is of utmost importance to minimize the drag during the ground phase. Transition is another critical period in which the pilot could lift off at so low a speed that he would be unable to accelerate or climb. This is generally known as taking off on the backside of the power curve. On the other hand if the pilot delays rotation too long his ground roll will not be the shortest possible. The pilot also has considerable control over the distance covered in climbing to a given altitude, generally 50 feet. He can either climb slowly and accelerate under constant power or he can pull up sharply, holding speed constant or even decreasing speed slightly in order to gain altitude rapidly. In general, more variation results in the air phase than in the ground roll.

3.7.1.1 FORCES ACTING ON AN AIRCRAFT DURING TAKEOFF

In order to properly orient ourselves to the takeoff problem, let us consider the forces acting on the aircraft during the three phases of takeoff. First let us consider the ground roll. The aircraft is acted upon by thrust, aerodynamic drag and friction as in Figure 3.7.1.

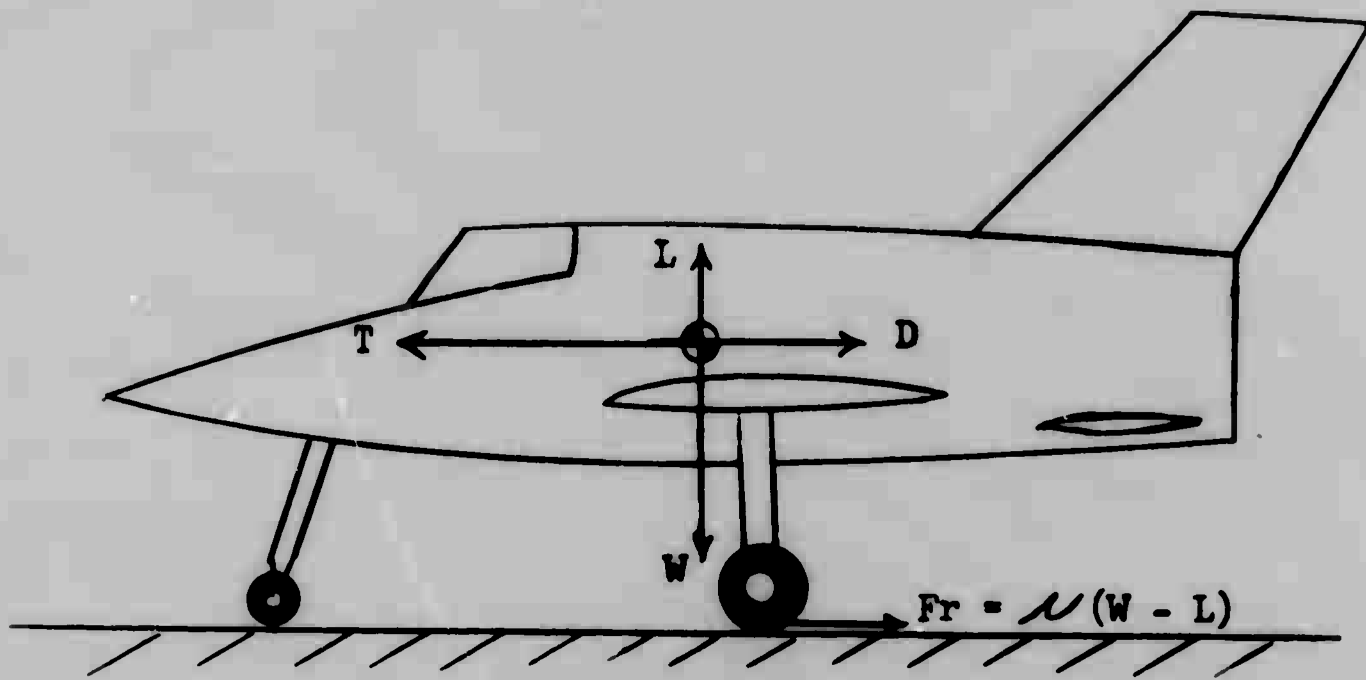


Figure 3.7.1

Summing the horizontal forces

$$T - \left[D + \mu(W - L)\right] = \frac{W}{g} a \qquad \text{Equation 3.7.1}$$

Where

T = thrust of the power plant

D = aerodynamic drag = $C_D\, q\, S$

L = aerodynamic lift = $C_L\, q\, S$

W = weight of the aircraft

Fr = frictional force

μ = rolling coefficient of friction

The term in brackets is called the resistance, R, and must be minimized to obtain the shortest ground roll. For aircraft taking off from soft grass or sod field where the coefficient of friction is high, it is especially important to reduce the weight on the gear by maintaining a higher angle of attack or by lowering flaps. This technique does not produce a significant decrease in takeoff distances when the friction is low, such as on a concrete runway. The optimum angle of attack required for this sort of takeoff is obtained mathematically by minimizing the resistance through the following procedure

$$R = D + \mu(W - L)$$

$$R = (C_{D_p} + C_{D_i})\, q\, S + \mu(W - C_L\, q\, S)$$

$$R = \left(C_{D_p} + \frac{C_L^2}{\pi \mathcal{R} e}\right) q\, S + \quad (W - C_L\, q\, S)$$

$$\frac{dR}{dC_L} = \frac{2\, C_L}{\pi \mathcal{R} e}\, q\, S - \mu(q\, S) = 0$$

Setting the derivative equal to zero gives the C_L required for minimum resistance

$$C_{L_{opt}} = \frac{\pi \mathcal{R} e \mu}{2} \qquad \text{Equation 3.7.2}$$

The angle of attack which will give this C_L is obtained from a C_L versus α curve for the aircraft such as Figure 2.3.9a. Note: Failure to use optimum C_L technique during the ground roll when the friction coefficient is high .01 - .03 can result in doubling the optimum ground roll of some aircraft.

A table for some respresentative rolling, sliding and static ceofficients of friction are presented in Table 3.7.A.

Table 3.7.A
Friction Ceofficients

Type of Surface	Rolling	Static	Sliding (1)
Concrete:			
Dry	.02 to .05	.70 to .85	.15 to .40
Wet	.02 to .05	.20 to .50	.30 to 0
Bituminous Pavements			
Dry	.02 to .05	.75 to .85 (2)	.15 to .40
Wet	.02 to .05	.20 to .70 (2)	.30 to 0
Packed Snow	.02	.25 to .37	.12 to .15
Ice	.02	.10 to .25	.05 to .20
Hard Turf	.04 to .05		
Short Grass	.05		
Long Grass	.10		
Soft Ground	.10 to .30		

1) Except for wet runways the sliding friction coefficient decreases with speed. The value listed first is associated with the higher speed.

2) Note that the average static coefficient is higher for bituminous pavements than for concrete. While this is in fact true it may not be possible to realize these higher braking forces on bituminous surfaces because their nonhomogeneous nature causes alternately high and low coefficients as the tire rolls over the pavement. This generally results in skiding and sliding.

In the transition phase the forces acting on the aircraft are continuously changing. The friction force is reduced as the aircraft lift is increased and of course becomes zero at the lift off or unstick point. The aerodynamic drag is increased during this phase of operation. The induced drag increases because of the increased lift and the total drag increases because of the increased speed. Also, during the transition phase the gear retraction is started and causes increased or decreased drag depending on the gear arrangement and retraction cycle. Naturally, after the gear has been retracted the total drag is decreased but this condition is not generally accomplished until after the initial climb angle has been established. The variation of the takeoff forces with speed are shown in Figure 3.7.2.

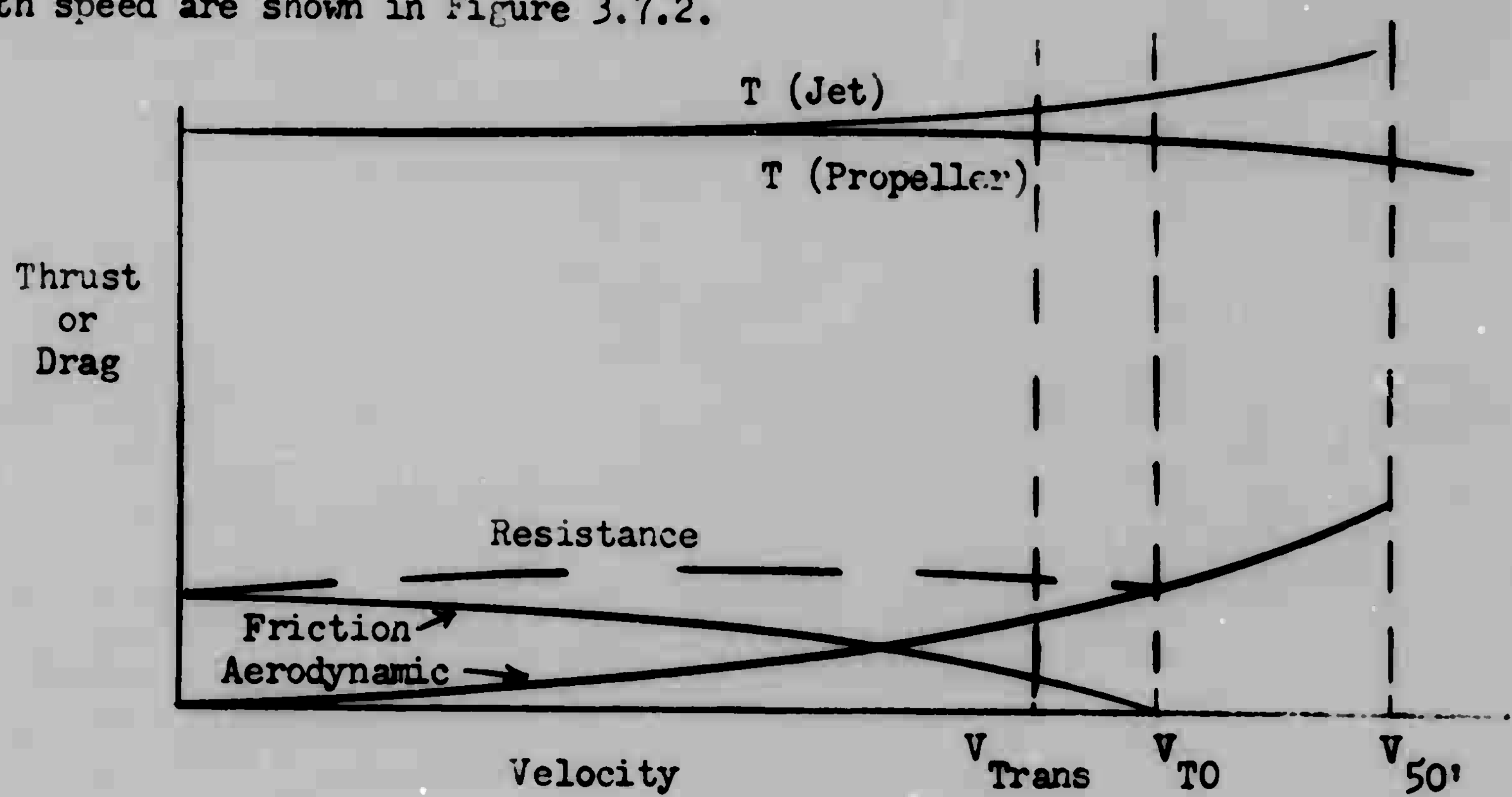


Figure 3.7.2 Force Versus Velocity Plot for Takeoff and Initial Climb

For some high performance aircraft the transition phase is not completed before the aircraft passes through the 50 foot point, in which case there are only two phases of test interest. This characteristic results from the fact that it is physically impossible to rotate the aircraft before it has literally leaped through 50'. It may also be due to a characteristic of the aircraft such as pitch up which prevents rapid rotation to establish the initial climb angle.

The forces acting during the initial climb are the same as those acting during the air phase of transition except that they increase in a more predictable manner. Since the climb is established the drag increases with the square of the velocity. However, as was mentioned previously, the gear retraction cycle is occurring at this time and may cause a variation in the drag; also the induced drag is increasing because the aircraft is climbing out of ground effect.

Since a great deal of variation can occur during takeoff certain assumptions must be made concerning the forces acting during each phase. These will be pointed out at the appropriate time.

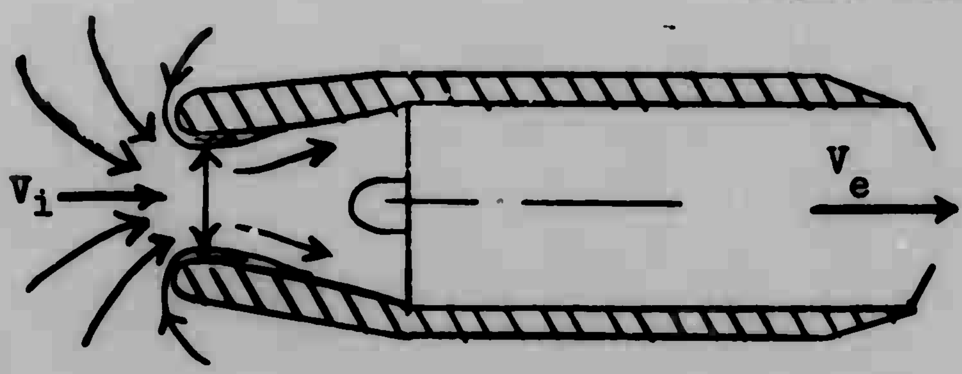
While the previous discussion has been primarily concerned with the variation of frictional forces and aerodynamic drag, they are not the only ones which change during takeoff. Thrust varies depending on the type of powerplant and though it may vary less than 3 or 4 percent during the takeoff roll this change may be more significant than a much larger percentage change in friction and aerodynamic drag.

The thrust of a jet, like that of a rocket would be constant if there were no inlet duct losses. However, the increased momentum of the incoming

air with increasing speed, the increase in duct losses with angle of attack and the requirement for sharp lipped inlets on modern high speed aircraft cause significant changes in net thrust.

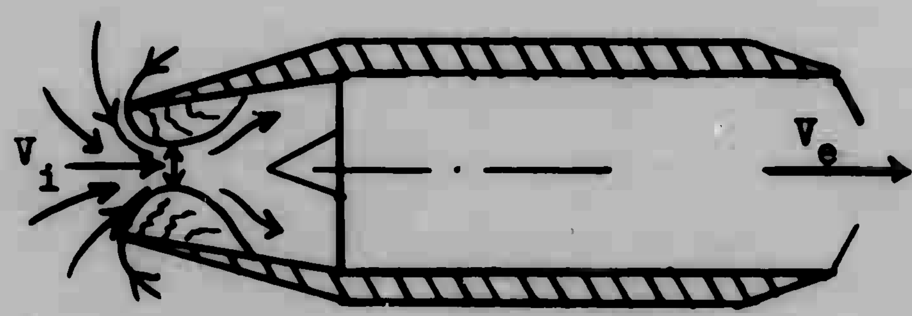
Let us first consider the static thrust of a jet engine mounted in a subsonic blunt lipped duct and a supersonic sharp lipped inlet, Figure 3.7.3a.

Static Condition Duct Flow



Low Speed Duct

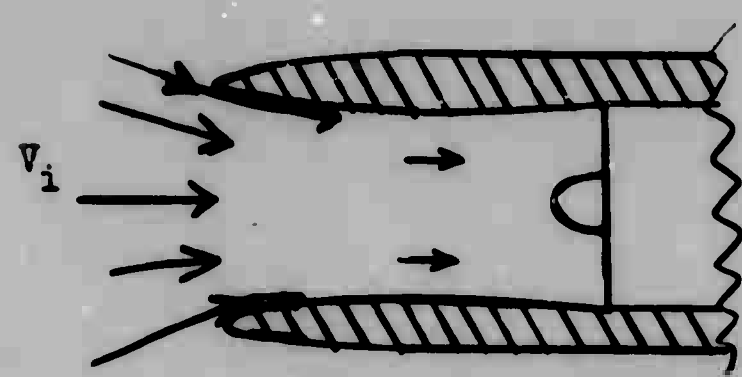
Figure 3.7.3a 1



High Speed Duct

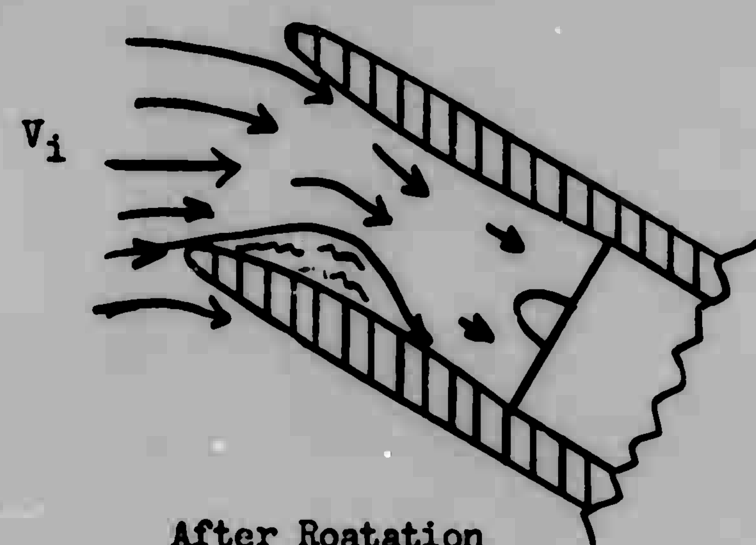
Figure 3.7.3a 2

Duct Flow During Takeoff



Prior to Rotation

Figure 3.7.3b 1



After Roatation

Figure 3.7.3b 2

Since a jet engine derives its thrust from the change in momentum of the air passed through it, $T_{net} = M (V_e - V_i)$, the maximum thrust should result when the velocity, V_i, of the incoming air is zero. This is truly the case if there are no inlet losses as with a bellmouth in a test stand. Engines installed in aircraft, however, do suffer losses because they are designed to give best efficiency at some design forward speed. These losses are caused by the reduction in effective duct area when incoming air approaches the duct from all directions as shown in Figure 3.7.3a. The sharp lipped inlet tends to form a dead air region just inside the lip causing an effective reduction in inlet area which results in a lower mass flow and, therefore, lower static thrust. The low speed inlet does not suffer as much from this effect because of its more favorable streamlined shape for static conditions and, therefore, may nearly develop rated thrust. These losses are reduced rapidly as forward speed increases.

As forward speed is increased the inlet velocity and momentum is proportionately increased causing a reduction in the net thrust, T_{net}. This reduction in net thrust is normally more than offset by the increase in duct efficiency of the high speed inlet resulting in increased thrust during ground roll, Figure 3.7.3b. For the low speed inlet the increase in duct efficiency is not too great so that the increase in inlet momentum may cause the thrust to decrease slightly but generally it is considered constant for a first approximation.

Another effect which is closely akin to the low speed duct loss problem is the duct losses caused by angle of attack during the transition and air phase. The flow pattern is the same except that it occurs on only one side of

the duct as shown in Figure 3.7.3b. This loss will cause a reduction in thrust when the aircraft is rotated to a flight attitude just prior to takeoff and its magnitude will depend on the duct configuration, takeoff speed, etc.

From this it is seen that the thrust of a jet powered aircraft may increase, decrease or remain constant during the takeoff roll depending on the duct configuration.

A propeller driven aircraft is a different story. The thrust of a pure propeller type aircraft (no jet thrust or JATO) almost always decreases during takeoff with a constant horsepower output. This is qualitatively seen for a constant speed propeller from the fact that the thrust horsepower available is defined as

$$THP_a = BHP_a \times \eta_p = \frac{T \times V_t}{550} \qquad \text{Equation 3.7.3}$$

Where

η_p is the propeller efficiency

T is the thrust

V is the true speed

During takeoff the brake horsepower, BHP, is constant at the maximum available and the η_p increases slightly causing the thrust horsepower to increase slightly. However, the increase in velocity is very large, therefore, if the thrust horsepower increases only a small amount the thrust, T, must decrease in some proportion to the velocity, V.

The thrust of a fixed pitch propeller decreases even more than the constant speed prop since the blade angle cannot increase as the forward speed is increased. This reduces the angle of attack of the blade and consequently the thrust. Though not generally the case some fixed pitch propellers give

increased thrust with increased speed for a brief period during takeoff. This is expected for very large blade angles where the blade is partially stalled under static conditions and, therefore, becomes more effective as forward speed is increased because of the lower angle of attack of the blades.

3.7.1.2 TAKEOFF PERFORMANCE EQUATIONS

Now that we have observed the forces acting and how they behave during the takeoff run, let us see more precisely how they effect the takeoff performance. Considering the ground roll, the takeoff distance is given by

$$S_g = \int_0^{S_{TO}} dS \qquad \text{Equation 3.7.4}$$

which can be expressed in terms of velocity and acceleration

$$S_g = \int_0^{V_{TO}} \frac{\frac{dS}{dt}}{\frac{dV}{dt}} dV \qquad \text{Equation 3.7.5}$$

$$S_g = \int_0^{V_{TO}} \frac{VdV}{a}$$

where

V = velocity

a = acceleration

The acceleration is obtained from Newton's Second Law which states

$F = ma$

where

$$F = T - R = \text{excess thrust} = T_{ex}$$

$$R = D + \mu(W - L) = \text{total resistance}$$

Therefore

$$a = \frac{(T - R)}{W/g}$$

Substituting in equation 3.7.5

$$S_g = \frac{W}{g} \int_0^{V_{TO}} \frac{VdV}{(T - R)} \qquad \text{Equation 3.7.6}$$

Thus, if the velocity and the excess thrust as a function of velocity can be determined at each point the mimimum distance to attain any given speed can be determined. Except for preliminary calculations, this is not normally done. For flight test data correction purposes a less exact formulation is required. A glance at Figure 3.7.2 suggests that during the ground roll the excess thrust is nearly constant, that is (T - R) in the above equation does not change greatly. Using the mean value of T - R and integrating gives

$$S_g = \frac{W}{g\ (T - R)_{mean}} \int_0^{V_{TO}} VdV$$

$$S_g = \frac{W}{g\ (T - R)_{mean}} \times \frac{V_{TO}^2}{2} \qquad \text{Equation 3.7.7}$$

Note that rearranging this equation says that the work done during the ground roll is equal to the increase in kinetic energy of the aircraft .

$$(T - R)_{mean} \; S_g = \frac{W}{2g} V_{TO}^2$$

Work done = Kinetic Energy Gain

The air phase of the takeoff can likewise be analysed from the standpoint of energy gained. In fact, this is necessary because the aircraft gains speed as well as altitude during climb out so that the true performance is best evaluated in terms of the energy change.

The distance for the air phase like the ground roll is given by

$$S_a = \int_0^{S_a} dS \qquad \text{Equation 3.7.8}$$

which in terms of the energy gain is given by

$$S_a = \int_{E_{TO}}^{E_{50}} \frac{\frac{dS}{dt}}{\frac{dE}{dt}} \, dE \qquad \text{Equation 3.7.9}$$

where

S_a = air distance

E = total energy of the aircraft relative to the total energy at the brake release point.

V_{TO} and V_{50} are the velocities at takeoff and 50 feet, respectively.

The total energy is given by

$$E = W \left(h + \frac{V^2}{2g}\right) \qquad \text{Equation 3.7.10a}$$

$$\frac{dE}{dt} = W \left[\frac{dh}{dt} + \frac{V}{g}\frac{dV}{dt}\right] \qquad \text{Equation 3.7.10b}$$

where

h = height above lift off point, (normally 50 ft.)

W = aircraft weight

V = aircraft velocity

Note that $\frac{dE}{dt}$ can be written

$$\frac{dE}{dt} = V\,(T - R)$$

since $\frac{W}{g}\frac{dV}{dt}$ is the accelerating force or the excess thrust.

By substitution equation 3.7.9 becomes

$$S_a = \int_{V_{TO}}^{V_{50}} \frac{V\,dE}{(T - R)\,V} \qquad \text{Equation 3.7.11}$$

If, as for the ground roll, we take the mean excess thrust, the above equation becomes simply the change in energy

$$S_a = \frac{1}{(T - R)_{mean}} \int_{V_{TO}}^{V_{50}} dE$$

$$S_a = \frac{W}{(T - R)_{mean}} \int_{V_{TO}}^{V_{50}} d\left(h + \frac{V^2}{2g}\right)$$

$$S_a = \left[\frac{W\left(h + \frac{V^2}{2g}\right)}{(T - R)_{mean}}\right]_{V_{TO}}^{V_{50}} = \frac{W\left(50 + \frac{V_{50}^{\,2} - V_{TO}^{\,2}}{2g}\right)}{(T - R)_{mean}}$$

where the reference height V_{TO} is 50 feet and the quantity $\frac{V_{50}^2 - V_{TO}^2}{2g}$

is defined as h_v .

The equation is then

$$S_a = \frac{W(50 + h_v)}{(T - R)_{mean}} \qquad \text{Equation 3.7.12}$$

While the assumption that the excess thrust is approximately constant is not too good during the airphase the distance traveled and the time duration is small, thus causing only small variations in the total distance required to clear a 50 ft. obstacle.

3.7.1.3 THRUST AUGMENTATION FOR TAKEOFF

It is evident from the previous discussion that the takeoff distance is directly a function of the excess thrust available to accelerate the aircraft and the velocity to which it must be accelerated.

$$F = T - R = T_{ex}$$

This means that there are three approaches to shortening the takeoff roll, (1) Reducing the resistance, (2) Increasing the thrust, and (3) Decreasing the takeoff speed.

Little can be done to reduce the resistance for a given aircraft and runway surface except to select the optimum lift coefficient for minimum resistance either by lowering flaps or by holding a given angle of attack during takeoff.

High lift devices such as slats and flaps are being used to increase the maximum lift coefficient and reduce the stall speed so that the aircraft does

ot have to be accelerated through such a large speed range before takeoff. This approach is being developed to the utmost for V/STOL type applications. Thrust vectoring and boundary layer control are also being considered but as yet are only in the experimental stages. As yet, boundary layer control while showing promise has not been extensively used for takeoff because it requires either a separate power source or the use of engine power to run it. The use of engine power decreases the power available and,therefore,tends to defeat the purpose which it is trying to promote, that is, better takeoff performance. Future development may provide more efficient boundary layer systems which will make it feasible for increased takeoff performance.

The second alternative of increasing the thrust and possibly vectoring it upwards as for a lift wing is a much more promising prospect for significantly increasing the takeoff performance. Increased thrust may be obtained by increasing the size of the primary powerplant or by using some sort of augmentation. Thrust augmentation is generally provided by one of three methods, (1) By auxilliary engine, jet, rocket, JATO, etc. (2) By water injection or otherwise operating the primary engine at greater than maximum rated power for a brief period of time, or (3) For a jet engine by incorporating an afterburner in the tail pipe of the primary engine. If thrust augmentation is possible throughout the takeoff, the theory and analysis are identical to the unaugmented case. If the thrust augmentation is only part time, the takeoff must be analyzed in two parts; that is, augmented and unaugmented.

Part time augmentation such as may be attained with JATO and water injection brings up the problem of determining the most efficient time during the takeoff to utilize the additional thrust. Equation 3.7.7 and 3.7.12 shows that an increase in the excess thrust reduces the ground roll. The question

still remains as to whether the distance is shortened more by using the part time augmentation early or late in the ground roll. It was also previously shown that the energy gained during the ground roll is equal to the work done by the excess thrust. Thus, the most efficient point to use limited augmentation is where it will do the most work; that is, where the thrust horsepower increase is a maximum. To fix this concept, let us assume that the augmentation provides ΔT increase in thrust for a fixed period of time, Δt. The distance traveled during time Δt will be ΔS

$$\Delta S = V \Delta t$$

The work done by ΔT will be

$$\Delta work = \Delta T \times \Delta S = \Delta T \times V \Delta t = \Delta THP \times \Delta t$$

From this it is seen maximum increase in thrust horsepower is attained at the higher speeds. This effect is seen in Figure 3.7.4. From this it is evident that the maximum increase in thrust horsepower is obtained by waiting until the aircraft has gained speed before using the part time augmentation actually it is desireable to time the augmentation so that it terminates just as the aircraft attains a safe speed and altitude.

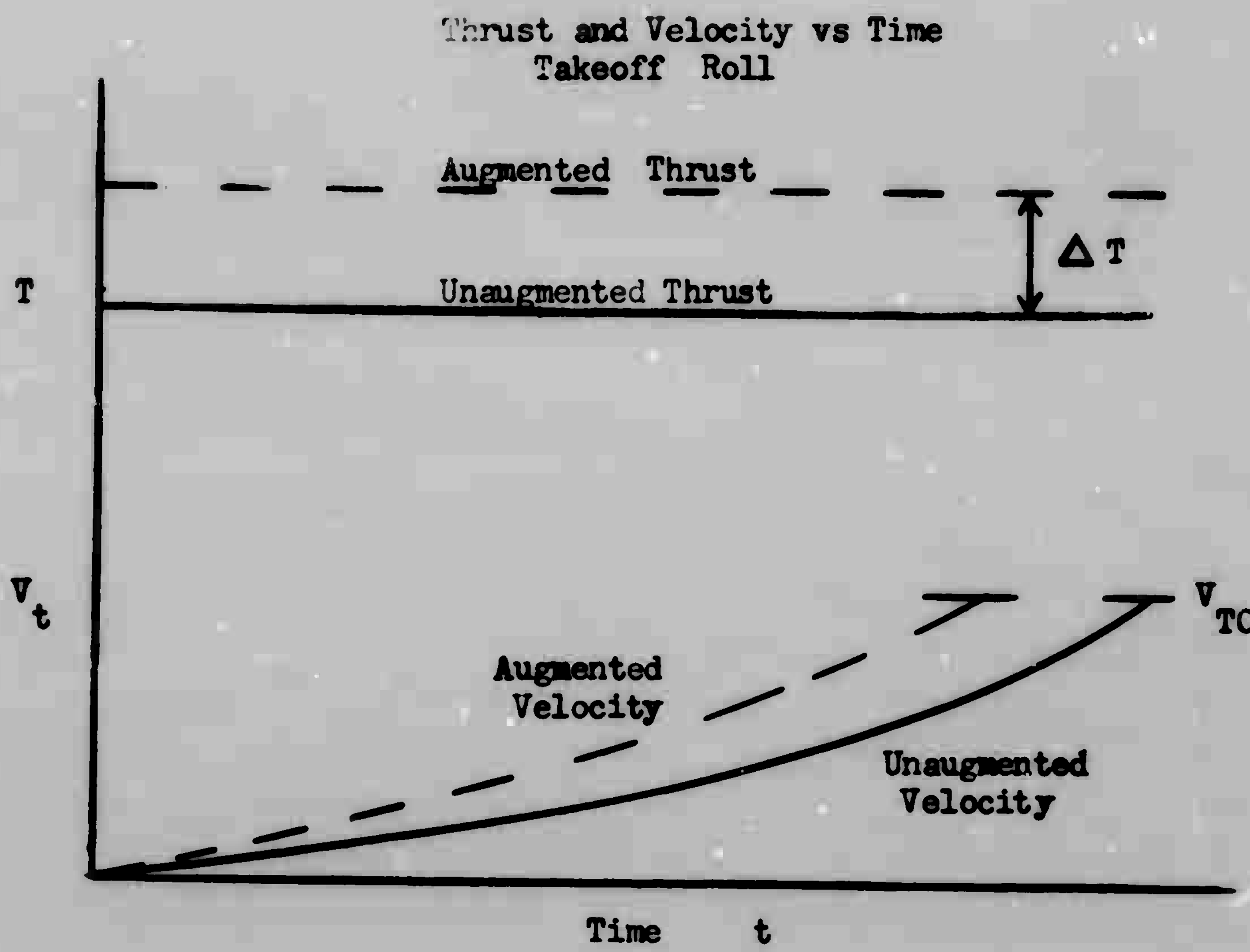


Figure 3.7.4a

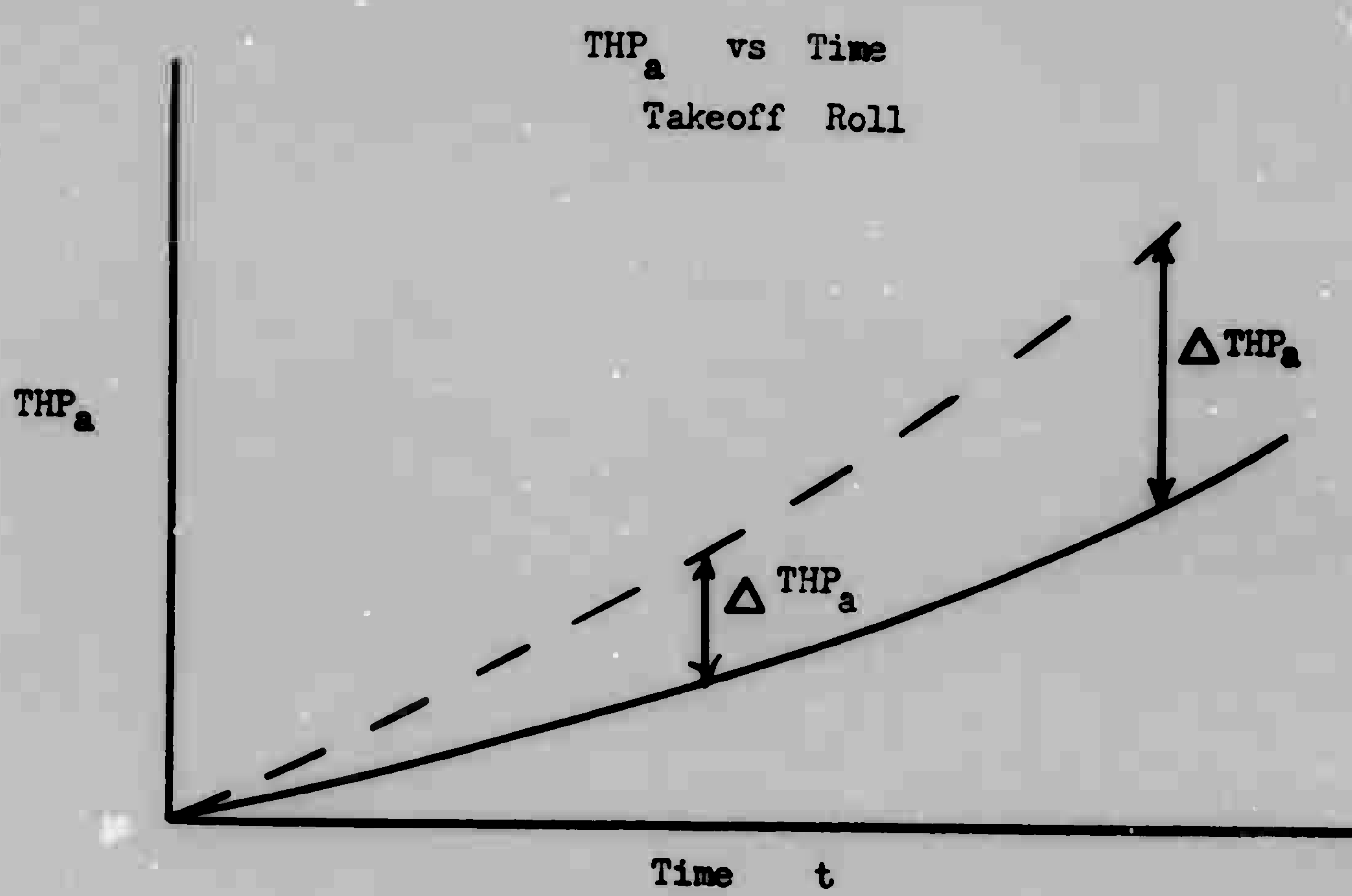


Figure 3.7.4b

3.7.1.4 TEST DATA CORRECTION METHODS

Because zero wind, standard day conditions can not be obtained at will, it is necessary to have methods at our disposal to correct the data to these conditions. Since large deviations from standard are the exception rather than the rule, it is permissible in some cases to use some rather crude methods without serious error. If large corrections are required, however, more precise methods must be used.

3.7.1.4a WIND EFFECTS AND CORRECTIONS

Taking off with headwinds or tailwinds obviously effects the ground roll. The degree to which winds change the takeoff or landing roll may be determined from equation 3.7.7.

Since we are interested in ground distance we shall consider only velocities relative to the ground. That is, when taking off into the wind the ground speed at takeoff is

$$V_{TO_W} = V_{TO} - V_W$$

where V_{TO} = the zero wind takeoff velocity

V_{TO_W} = the takeoff velocity with wind

V_W = the headwind component along the runway

A means of correcting the ground roll for the effects of wind is obtained from equation 3.7.7 which gives the ground roll without wind. Similarly the ground roll with winds is

$$S_W = \frac{W}{g(T - R)_W} \int_0^{V_{TO_W}} V dV = \frac{W}{g\,(T - R)} \quad \frac{V^2_{TO_W}}{2}$$

Substituting $V_{TO_w} + V_w$ for the takeoff velocity, V_{TO}, in equation 3.7.7 and taking the ratio of S/S_w gives

$$\frac{S}{S_w} = \frac{(T - R)_w}{(T - R)} \left[\frac{V_{TO_w} + V_w}{V_{TO_w}}\right]^2$$

$$\frac{S}{S_w} = \frac{(T - R)_w}{(T - R)} \left[1 + \frac{V_w}{V_{TO_w}}\right]^2$$

From this it is seen that the takeoff roll with no wind is

$$S = S_w \left[\frac{a_w}{a}\left(1 + \frac{V_w}{V_{TO_w}}\right)^2\right] \qquad \text{Equation 3.7.13}$$

where a_w = mean acceleration with winds

a = mean acceleration without winds

This, then constitutes a correction equation for reducing the test ground roll, S_w, obtained with the wind blowing to a no wind condition. This correction must be applied before other corrections can be made.

Most aircraft exhibit slightly different mean excess thrust during takeoffs with and without winds, that is

$$\frac{(T - R)_w}{(T - R)} = \frac{a_w}{a} \neq 1.0$$

To account for this factor an empirical relation which is good as long as the winds are less than 10 knots has been developed.

$$S = S_w \left(1 + \frac{V_w}{V_{TO_w}}\right)^{1.85} \qquad \text{Equation 3.7.13a}$$

The wind correction for the air phase is a simple matter. The aircraft moves relative to the air mass with reference to the ground; therefore, it follows the same flight path relative to the air whether the wind is blowing or not. We have only to account for the drift of the air mass relative to the ground during the time the aircraft climbs to 50 feet. If the wind velocity is assumed constant from ground level to 50 feet the ΔS correction is

$$\Delta S = V_w t \qquad \text{Equation 3.7.14}$$

where t is the time from takeoff to 50 feet. The wind actually increases with altitude; however, ignoring this fact does not cause significant error as long as the winds are low. Empirical relations are available to compensate for this effect but they are not generally required.

Adding the above correction (Equation 3.7.14) to the test value (with wind) gives the air distance with no wind

$$S_a = S_{a_w} + \Delta S$$

3.7.1.4b THRUST WEIGHT AND DENSITY CORRECTIONS

The exact relationships within the restrictions of the mean excess thrust assumption are given for the ground roll and air phase in equation 3.7.7 and 3.7.12. From equation 3.7.7 the ground roll correction equation becomes (Ref. 3 Appendix II)

$$\frac{S_{g_s}}{S_{g_t}} = \frac{\dfrac{W_s}{W_t}\dfrac{\sigma_t}{\sigma_s}}{\dfrac{2\,g\,S_{g_t}}{W_t\,V_{TO_t}^2}\left(\dfrac{W_t}{W_s}T_s - T_t\right) + 1} \qquad \text{Equation 3.7.15}$$

where

S_g = ground roll

T = mean net thrust

σ = density ratio, ρ_a / ρ_o

W = aircraft gross weight

Subscripts t and s refer to test and standard conditions.

This equation corrects for nonstandard weight, altitude, and thrust. Its derivation depends on the assumption that the lift, drag and friction coefficients are constant during the ground roll and that lift off occurs at the same lift coefficient on test and standard day. These assumptions are acceptable since the aircraft is at constant angle of attack through most of the ground roll and runway surfaces are uniform, i.e., smooth and made of one type of material. The constant angle of attack assumption is poorest just prior to lift off since the aircraft is rotated to a flight attitude at this point.

These same assumptions are employed in the derivation of the correction equation for the air phase from equation 3.7.12. The derivation is given in Reference 3, Appendix II and the equation is

$$\frac{S_{a_s}}{S_{a_t}} = \frac{\left[\frac{W_s}{W_t}\frac{\sigma_t}{\sigma_s} \quad h_{v_t}\right] + 50}{(h_{v_t} + 50) + S_{a_t}\left(\frac{T_s}{W_s} - \frac{T_t}{W_t}\right)} \qquad \text{Equation 3.7.16}$$

The terms of the equation have been previously defined.

This equation corrects for the same conditions as 3.7.15 and is subject to the same limitations. However, the assumptions made may not be as valid.

Experience has proven them to be sufficiently accurate, however. Equation 3.7.15 and 3.7.16 are referred to as the "exact correction equations" since the assumptions made are acceptable on todays fixed wing aircraft and especially since most normal corrections are less than 5 or 10 percent of the total take-off distances. New or more refined methods may be required for V/STOL, helicopter and other type aircraft.

Another less accurate approach to the correction problem is used because of its simplicity of application. Its validity depends on the assumption that the corrections to be made are small. It is referred to as the differential or exponential correction method since it is derived by differentiating equations 3.7.7 and 3.7.12 and may also be express in exponetial form by integrating the differential forms. The derivation is found in Reference 3, Appendix II and the final equations are as follows:

Differential Form:

$$\frac{\Delta S_g}{S_{g_t}} = \frac{\Delta W}{W}\left[2 + \frac{R}{T - R}\right] - \frac{\Delta \sigma}{\sigma_t} - \frac{\Delta T}{T_t}\left[1 + \frac{R}{T - R}\right]$$

$$\Delta S_a = \frac{\Delta W}{W_t}\left[1 + \frac{D}{T - D} + \frac{h_v}{h_v + 50}\right] - \frac{\Delta T}{T_t}\left[1 + \frac{D}{T - D}\right]$$

$$- \frac{\Delta \sigma}{\sigma}\left[\frac{h_v}{h_v + 50}\right]$$

BLANK PAGE

Exponential Form: Obtained by integrating the differential forms

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{W_s}{W_t}\right)^{\left(2 + \frac{R}{T - R}\right)} \times \frac{\sigma_t}{\sigma_s} \times \left(\frac{T_t}{T_s}\right)^{\left(1 + \frac{R}{T - R}\right)}$$

$$\frac{S_{a_s}}{S_{a_t}} = \left(\frac{W_s}{W_t}\right)^{\left(1 + \frac{D}{T - D} + \frac{h_v}{h_v + 50}\right)} \times \left(\frac{\sigma_t}{\sigma_s}\right)^{\left(\frac{h_v}{h_v + 50}\right)} \times \left(\frac{T_t}{T_s}\right)^{\left(1 + \frac{D}{T - D}\right)}$$

Where T, R and D are the mean values of the thrust, resistance and drag during each phase.

At first glance these equations do not seem to have been simplified at all; the coefficients or exponents are very complicated quantities. Further examination of these exponents,however, reveal that the mean thrust and drag for a given aircraft should remain constant at any given speed during takeoff. Futhermore, it is found that the energy change, h_v, also remains approximately constant. Thus,if these values are determined,the coefficients or exponents are constant for the aircraft concerned. In addition,it has been empirically shown that these exponents remain constant enough for a given class of aircraft to permit respectable corrections to be made to test data providing the corrections are small, say less than five percent.

Recent work on jet and turboprop aircraft has changed the format of the exponential equations to forms more easily applied to particular engine installations. A few of these equations, with representative exponents, are given for current jet and turboprop aircraft.

Jet Aircraft

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{W_s}{W_t}\right)^{2.3} x \left(\frac{\sigma t}{\sigma_s}\right) x \left(\frac{T_t}{T_s}\right)^{1.3}$$

$$\frac{S_{a_s}}{S_{a_t}} = \left(\frac{W_s}{W_t}\right)^{2.3} x \left(\frac{\sigma t}{\sigma_s}\right)^{0.7} x \left(\frac{T_t}{T_s}\right)^{1.6}$$

Turboporp Aircraft - Constant speed propeller

$$\frac{S_{g_s}}{S_{g_t}} = \left(\frac{W_s}{W_t}\right)^{2.6} x \left(\frac{\sigma t}{\sigma_s}\right)^{1.9} x \left(\frac{N_t}{N_s}\right)^{.7} x \left(\frac{P_t}{P_s}\right)^{.5}$$

$$\frac{S_{a_s}}{S_{a_t}} = \left(\frac{W_s}{W_t}\right)^{2.6} x \left(\frac{\sigma t}{\sigma_s}\right)^{1.9} x \left(\frac{N_t}{N_s}\right)^{.8} x \left(\frac{P_t}{P_s}\right)^{.6}$$

where N = RPM

P = ambient pressure

NOTE: The exponents should be checked for each individual aircraft if accurate corrections are to be made. More detailed information for this type correction is found in reference 3 and 4.

3.7.1.4c MISCELLANEOUS CORRECTIONS

Runway slope

Runway slope merely changes the effective thrust of the aircraft. It is seen in Figure 3.7.5 that the effective thrust is increased by an amount proportional to the since of the angle Θ ; $T = W \sin \Theta$ adding this amount to test mean thrust, $(T - R)_m$, gives $(T - R)_m + W \sin \Theta$

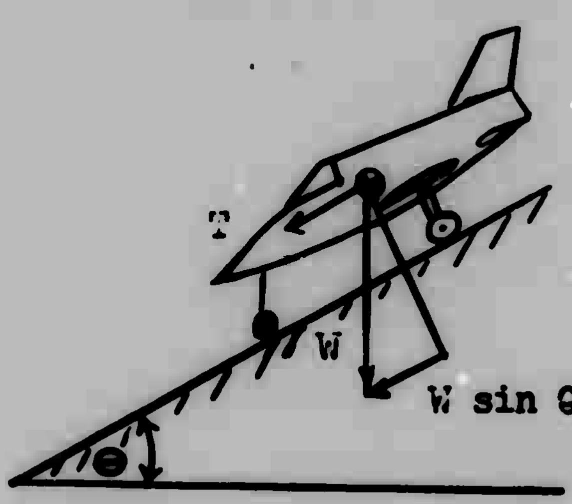


Figure 3.7.5

Writing equation 3.7.7 for test (with slope, subscript t) and standard (zero slope, subscript o)

$$S_{g_o} = \frac{W}{2g} \times \frac{V_{TO}^2}{(T - R)_m} \qquad \text{Equation 3.7.17a}$$

$$S_{g_t} = \frac{W}{2g} \times \frac{V_{TO}^2}{\left[(T - R)_m + W \sin\right]} \qquad \text{Equation 3.7.17b}$$

and taking the ratio of the two gives

$$\frac{S_{g_o}}{S_{g_t}} = \frac{(T - R)_m + W \sin \Theta}{(T - R)_m}$$

This may be reduced to

$$S_{g_o} = S_{g_t} \left[1 + \frac{W}{(T - R)_m} \sin \Theta \right]$$

From equation 3.7.17b

$$(T - R)_m = \frac{W}{2g} \times \frac{V_{TO}^2}{S_{g_t}} - W \sin \Theta$$

Substituting this, the above becomes

$$S_{g_o} = S_{g_t} \left[1 + \frac{W \sin \Theta}{\left(\frac{W}{2g} \frac{V_{TO}^2}{S_{g_t}} - W \sin \Theta \right)} \right]$$

$$S_{g_o} = S_{g_t} \left[\frac{\frac{W}{2g} \frac{V_{TO}^2}{S_{g_t}}}{\frac{W}{2g} \frac{V_{TO}^2}{S_{g_t}} - W \sin\theta} \right]$$

$$S_{g_o} = \left[\frac{S_{g_t}}{1 - \frac{2g\, S_{g_t}}{V_{TO}^2} \sin\theta} \right] \qquad \text{Equation 3.7.18}$$

<u>Lift Coefficient Variation</u>

By varying technique the pilot is capable of changing the lift coefficient at takeoff and at 50 feet. If this variation is large it may be desirable to make corrections to reduce the scatter. This is done by adjusting the takeoff velocity and distance in proportion to the change in C_L. Weight and density are not varied here since they are accounted for in equations 3.7.15 and 3.7.16 which assume C_L constant. If C_L corrections are made they should precede the weight, density and thrust corrections of the forementioned equations.

After the standard liftoff C_L has been selected, the corresponding true speed at liftoff, V_{TO_2}, can be calculated for the test weight and density

$$V_{TO_2} = \sqrt{\frac{2\, W_t}{\rho_t\, S\, C_{L_2}}} \qquad \text{Equation 3.7.19}$$

If the subscripts 1 and 2 are used to denote the test and standard C_L condition the takeoff distance equations can be written as

$$S_1 = \frac{W}{2g} \times \frac{V_{TO_1}^2}{(T - R)_m} = \frac{V_{TO_1}^2}{2\, a_{m_1}}$$

$$S_2 = \frac{W}{2g} \times \frac{V_{TO_2}^2}{(T - R)_m} = \frac{V_{TO_2}^2}{2\, a_{m_2}}$$

dividing the second by the first gives

$$\frac{S_2}{S_1} = \frac{a_{m_1}}{a_{m_2}} \times \frac{V^2_{TO_2}}{V^2_{TO_1}}$$

which reduces to

$$S_2 = S_1 \left(\frac{a_{m1}}{a_{m_2}}\right)\left(\frac{V_{TO_2}}{V_{TO_1}}\right)^2 \qquad \text{Equation 3.7.20}$$

This then is the C_L correction equation for the ground roll. The mean acceleration, a_{m_1}, will generally be different from a_{m_2} because of the change in takeoff speed and will have to be evaluated for each individual aircraft.

<u>The airphase</u> is corrected in a similar manner. The standard C_L is selected for the 50 foot point and the corresponding true speed computed as before. The equations for the test and standard C_L's are written and divided as before.

$$S_{a_1} = \frac{\left(V_{50_1}^{\;2} \quad V_{TO}^{\;2}\right) + 2\,g\,h}{2\,\bar{a}_{m_1}}$$

$$S_{a_2} = \frac{\left(V^2_{50_2} - V^2_{TO_2}\right) + 2\,g\,h}{2\,\bar{a}_{m_2}}$$

where $\bar{a}_{m_1}$ and $\bar{a}_{m_2}$ are the mean accelerations during the airphase.

$$\frac{S_{a_2}}{S_{a_1}} = \frac{\bar{a}_{m_1}\left[\left(V^2_{50_2} - V^2_{TO_2}\right) + 2\,g\,h\right]}{\bar{a}_{m2}\left[\left(V^2_{50_1} - V^2_{TO_1}\right) + 2\,g\,h\right]}$$

For a 50 foot obstacle height, h, the correction equation is,

$$S_{a_2} = S_{a_1} \frac{\bar{a}_{m_1}}{\bar{a}_{m_2}} \left[\frac{V_{50_2}^2 - V_{TO_2}^2 + 3220}{V_{50_1}^2 - V_{TO_1}^2 + 3220} \right] \qquad \text{Equation 3.7.21}$$

For the airphase the ratio $\frac{\bar{a}_{m_1}}{\bar{a}_{m_2}}$ is frequently near 1.0 and, therefore, does not affect the result.

An important consideration in making any corrections is the order in which they are made. For takeoff and landing corrections the order is as follows

1. Wing
2. Runway Slope
3. Lift Coefficient*
4. Weight*, density and thrust*

* Not normally made for landings.

References

1. James H. Dwinnell, Priciples of Aerodynamics
   Mc Graw - Hill, 1949
2. Dommasch, Sherby & Connolly, Airplane Aerodynamics
   Pitman, 1951
3. Kenneth J. Lush, AFFTC TN R-12 Standardization of Takeoff
   Performance Measurements for Airplanes
4. Herrington and Shoemacher AFFTC TR 6273
   Flight Test Engineering Manual

3.7.2 LANDING PERFORMANCE

Equally important if not more important, than getting the aircraft into the air is getting it safely back on the ground. If an aircraft cannot be easily and safely landed it is of little use in accomplishing its mission. Unfortunately landing performance is sometimes woefully neglected in tests because of its great dependence in pilot technique. Sometimes results show virtually no correlation. Not to be thwarted by this past experience, it is the purpose of this section to point out reasons for the lack of correlation, analyze the landing problem and develop theoretical equations which can be used as a basis for making realistic corrections to test data.

3.7.2.1 THE BASIC PROBLEM

The basic problem involved in landing performance is that of dissipating in the shortest time the energy possessed by the aircraft as it clears an obstacle (50 ft) at the end of the runway. This energy can be depleted by any combination of drag, friction, or reversed thrust. The basic force relationship is the same as that used for takeoff (Equation 3.7.1) except that the thrust vector if applicable is reversed.

$$T - D - \mu(W - L) = \frac{W}{g}\, a \qquad \text{Equation 3.7.1}$$

Obviously during the airphase the only force acting to resist the motion of the aircraft is the drag. While theoretically it would be practical to reverse thrust while still in the air it creates rather severe control problems and therefore is not a safe procedure. The drag may be increased in a number of ways. First, the gear is normally down during the approach and creates a large drag. Also, lowering flaps to the full down position creates considerable drag and also adds the additional dividend of lowering the approach and touch-down speeds. Speed brakes or spoilers if the aircraft is so equipped

will add an additional increment of drag when extended. Some aircraft utilize a small approach chute during approach and landing to further increase the drag. It should be recalled that the lift-drag ratio determines the glide slope of the aircraft so that the increased drag shortens the airphase by allowing a steeper glide slope without increasing speed. Hence, high drag and steep glide angle within the limits of the flare capability of the aircraft are desirable during the final approach. In some aircraft, carrying high power and flying on the back side of the power required curve allows a steeper approach to be made with better control and flare capability. The optimum approach for the shortest landing distances must be determined for each model aircraft.

The flare or roundout is an important portion of the landing and is highly dependent on pilot technique. Most aircraft landing gears will not withstand the impact imposed by final approach rates of sink, therefore, it is necessary to reduce the sink rate to within allowable limits by flaring or rounding out. For maximum performance the flare is accomplished at the latest possible time consistant with safety, gear limits and bouncing of the aircraft. If the aircraft bounces after touchdown it looses much valuable time on the ground which could have been used for application of brakes, thrust reversal and deployment of the drag chute. Note: A maximum performance landing is normally rated as a solid to a hard landing with no bounce (never a "grease job"). To prevent bouncing, the flaps are sometimes retracted on impact, thus reducing the tendency to bounce by reducing the lift. (This technique should be approached with caution, however.) This is also accomplished for the "backside" power approach by chopping the throttle.

After touchdown the object is to increase the resistance as much as possible, that is, apply brakes, deploy the drag chute and reverse thrust in a sequence dictated by their relative magnitude and contribution to the total resistance (the most effective being done first). The question arises as to which produces the most resistance, keeping the flaps down thereby maintaining the aerodynamic drag and high lift, or by retracting the flaps, thus reducing the lift and increasing the load on the wheels and braking force. Note, the same problem arises as to keeping a nose high (high C_L) attitude of an aircraft with tricycle gear or lowering the nose (low C_L) to allow maximum braking. Some insight into this problem is gained by plotting the braking force and aerodynamic drag versus velocity for high and low lift and friction coefficients. For an aircraft which is capable of developing high drag as compared to the friction force it is more expedient to maintain a high lift coefficient throughout the ground roll. As shown in Figure 3.7.5a a higher total resistance is attained under these conditions. On the other hand, however, if the aerodynamic drag is low relative to the friction force it does not pay to utilize aerodynamic drag. As shown in Figure 3.7.5b, the total resistance is more at low lift coefficients at high speed but there is little difference at the lower speeds. From this it is seen that the answer to the question is not clear cut but must be decided for each type of aircraft and runway surface.

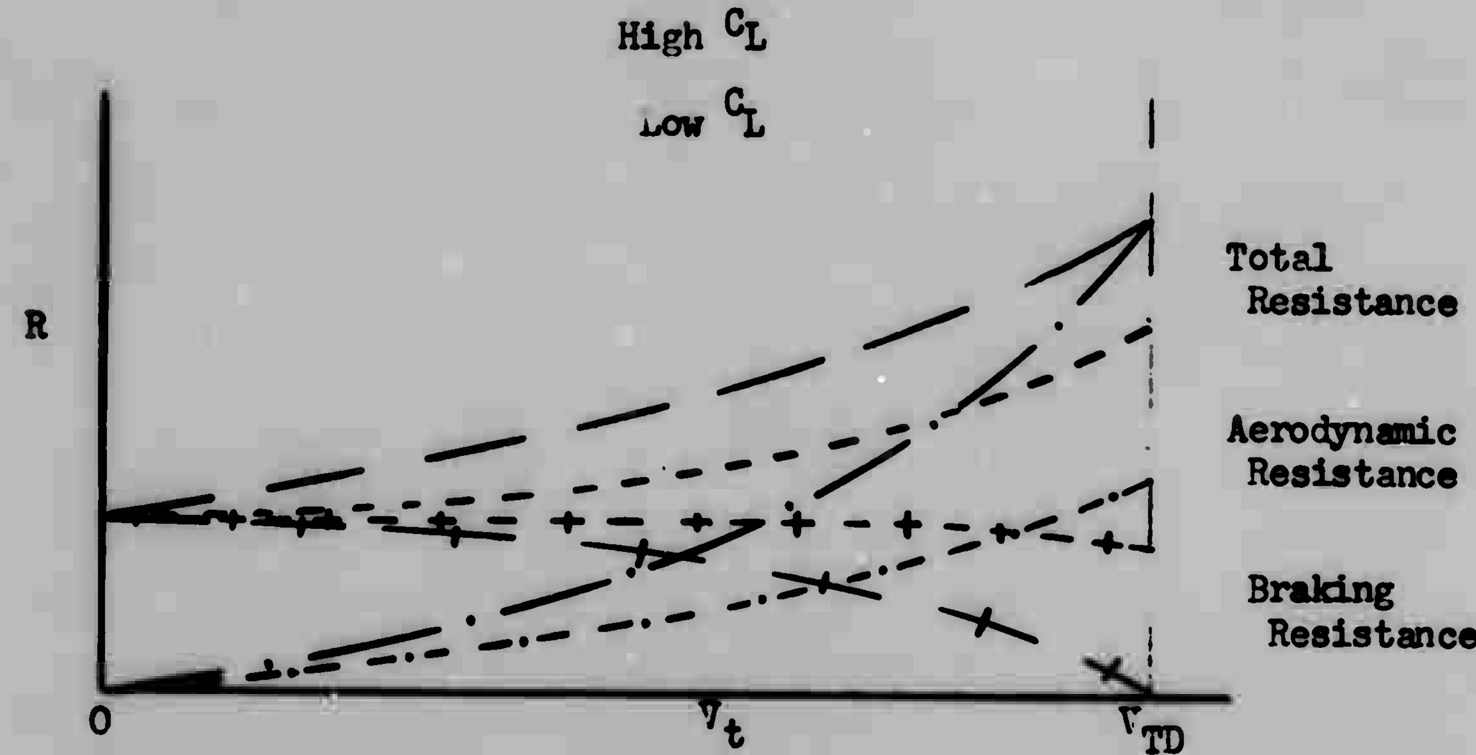


Figure 3.7.5a High Aerodynamic Drag and Low Friction

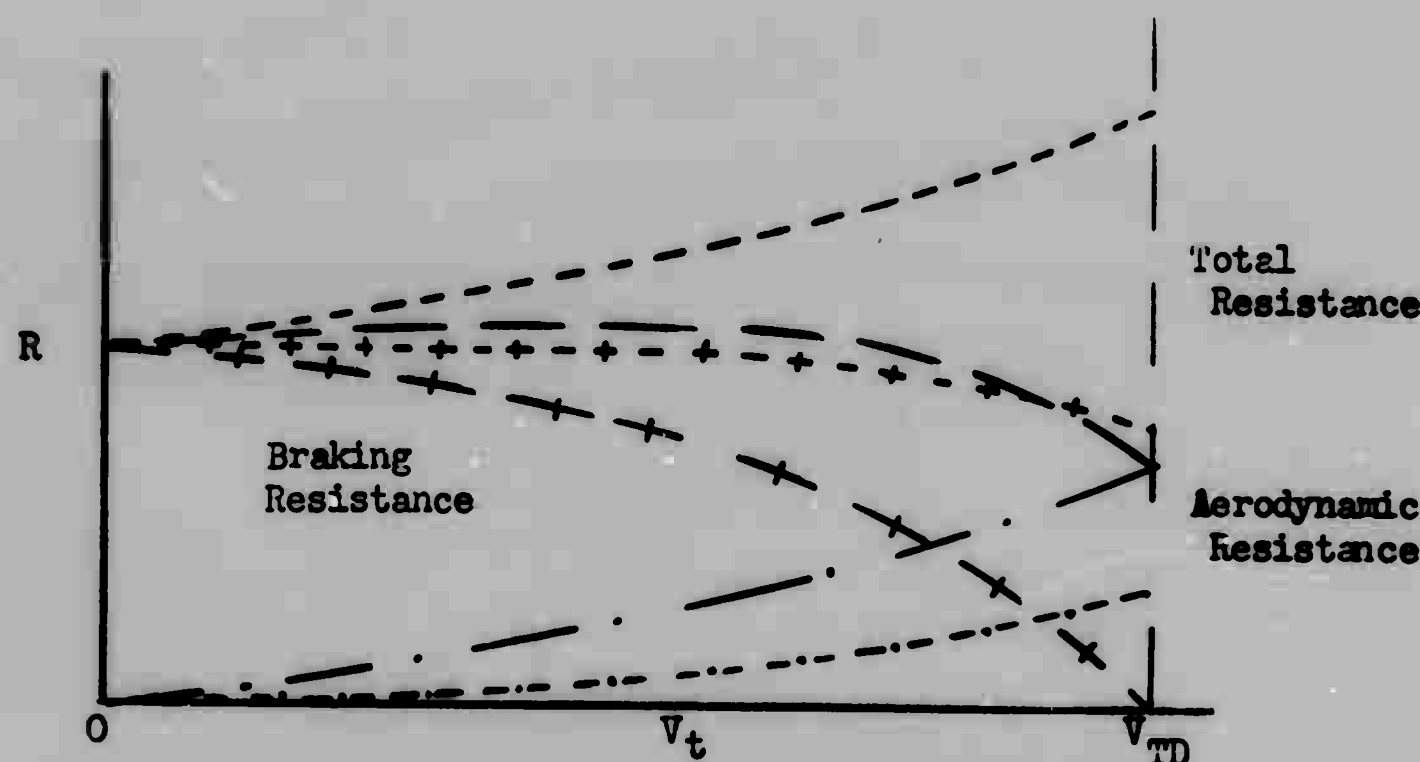


Figure 3.7.5b Low Aerodynamic Drag and High Friction

Figure 3.7.5a,b variation of aerodynamic drag and braking friction with velocity for high and low lift coefficients.

Other techniques can be devised to take advantage of a particular aircraft characteristics and configuration. For instance, holding full back stick during ground roll of an aircraft having tricycle gear not only increases the drag by the amount of the projected frontal area of the control surface but it also increases the down load on the main gear, thus allowing a greater braking force. The improvements in landing performance obtained by some techniques may not be too significant but they should be given due consideration.

To summarize the previous discussion, there is some minimum landing distance which is determined by the maximum approach glide angle consistent with safety; the shortest, most abrupt flare consistent with gear limits and aircraft flying qualities; and the maximum resistance during ground roll. Any deviation from this technique causes increased distance and, therefore, is not optimum. For instance, with a long low approach and a gentle flare it might be possible to attain short ground rolls but it does not improve the total distance to clear an obstacle. On the other hand a steep glide which causes the aircraft to bounce may shorten the air phase but it lengthens the ground roll.

Before leaving the subject of pilot technique it would be well to note that the previous discussion has not been particularly concerned with aircraft handling qualities which quite frequently dictate the permissible technique. This is because control at very low speeds is rather marginal and abrupt flares, etc., may create unsafe flying conditions such as loss of lateral control, stall, etc.

3.7.2.2 LANDING PERFORMANCE

The performance relationships for landing are obtained in the same way as the takeoff equations 3.7.6 and 3.7.11 except that for the ground roll the integration is from the touch down speed to zero velocity and the airphase from V_{50} to V_{TD}. Considering the ground roll first, the equation is

$$S_g = \frac{W}{g}\int_{V_{TD}}^{0} \frac{VdV}{T - R}$$

For landings the thrust is zero or is at least a minimum, therefore, the equation becomes

$$S_g = \frac{W}{g}\int_{V_{TD}}^{0} \frac{VdV}{-R} \qquad \text{Equation 3.7.22a}$$

If as for takeoffs the mean resistance may be considered to act throughout the landing roll the distance is

$$S_g = \frac{W}{2g} \times \frac{V_{TD}^2}{R_{mean}} \qquad \text{Equation 3.7.22b}$$

The airphase equation is obtained in a similar manner except that the thrust term is not neglected since power is sometimes used during the approach.

$$S_a = \int_{V_{50}}^{V_{TD}} \frac{dE}{(T - R)} \qquad \text{Equation 3.7.23a}$$

Using the mean resistance approximation

$$S_a = \frac{1}{(T - R)_{mean}} \int_{V_{50}}^{V_{TD}} dE$$

$$S_a = \frac{W}{(T - R)_{mean}} \int_{V_{50}}^{V_{TD}} d\left(h + \frac{V^2}{2g}\right)$$

$$S_a = \frac{W}{(T - R)_{mean}} \left(\frac{V_{TD}^2}{2g} - 50 - \frac{V_{50}^2}{2g} \right)$$

$$S_a = \frac{-W}{(T - R)_{mean}} \left(\frac{V_{50}^2 - V_{TD}^2}{2g} + 50 \right)$$

or in terms of h_v then

$$S_a = \frac{-W\,(h_v + 50)}{(T - D)_{mean}} \qquad \text{Equation 3.7.23b}$$

Note the similarity between these equations and their takeoff counterparts, equation 3.7.7 and 3.7.12. The negative sign for the landing air distance may appear to be a problem until it is realized that the mean excess thrust, T - R, is also negative causing S_a to be positive as well.

On the surface it would appear that landing performance is equally as predictable as takeoff. This is not necessarily true since landings do not start from the same initial conditions; that is, the same speed over a 50 foot obstacle. Thus, pilot technique is a great variable in setting up the same speed and glide path at the 50 foot point to say nothing of the flare and touchdown or the braking applied. Friction coefficients given in table 3.7A vary widely (50 to 100 percent) for constant braking on a given surface. This alone can account for a large deviation of a seemingly perfect landing from the predicted value. It is for these reasons and others mentioned previously that detailed correction of flight test data is a futile and frustrating process for the engineer. While corrections similar to those made for takeoffs are made for landings, too much cannot be expected. Some of these are outlined in the following section.

Wind and runway slope corrections are the same as those outlined in Section 3.7.1.4a and c.

3.7.2.3 LANDING CORRECTION EQUATION

A correction equation for the ground roll is obtained from equation 3.7.22b

$$S_g = \frac{W}{2g} \times \frac{V_{TO}^2}{R_m} \qquad \text{Equation 3.7.22b}$$

If it may be assumed that the test and standard day lift coefficient are the same at touch down then

$$L = W = \tfrac{1}{2} \rho V_{TD}^2 S C_L$$

or

$$V_{TD} = \frac{W}{\sigma} \times \frac{2}{\rho_o S C_L}$$

Substituting to equation 3.7.22b

$$S_g = \frac{W^2}{\sigma} \times \frac{1}{R_m} \times \frac{1}{g \rho_o S C_L}$$

Writing this equation for test and standard day and taking the ratio gives

$$\frac{S_{g_s}}{S_{g_t}} = \frac{\dfrac{W_s^2}{\sigma_s} \quad \dfrac{1}{R_{m_s}}}{\dfrac{W_t^2}{\sigma_t} \quad \dfrac{1}{R_{m_t}}} = \frac{W_s}{W_t}^2 \times \frac{\sigma_t}{\sigma_s} \times \frac{R_{m_t}}{R_{m_s}} \qquad \text{Equation 3.7.24a}$$

If the resistance is assumed constant on test and standard day the correction equation becomes

$$S_{g_s} = S_{g_t} \left(\frac{W_s}{W_t}\right)^2 \left(\frac{\sigma_t}{\sigma_s}\right) \qquad \text{Equation 3.7.24b}$$

While equation 3.7.14 can be used to correct a given landing to standard conditions (weight and atmosphere), experience has shown the weight correction to be unsatisfactory for correcting a large number of landings to one standard value. This occurs because of the large variation in test data caused by technique. For this reason weight corrections are generally omitted for the reduction of landing data.

BLANK PAGE

SECTION 4

POWER PLANTS

4.1 GENERAL INTRODUCTION

Before we can go on with our analysis of flight test methods for aircraft performance we must learn some fundamentals of the power plants used to propel aircraft. This study is essential toward an understanding of the uses of these engines and of their relationship to the performance of the airplane.

The design of the first successful aircraft engine goes back to 1901 when Charles M. Manly developed a five cylinder water-cooled radial engine. The engine weighed less than three pounds per horsepower. The engine was developed for Professor Langley's experimental airplanes. The Wright brothers engine was the first to fly, but Manly's engine was much more of a engineering achievement.

Pressing needs of WWI provided the impetus for development of the Liberty engine. This engine was unique perhaps because of the speed in which it was designed and built. When we consider that the normal time from the initial conception to the actual running of the engine is several years, the scant 3 months for this work during the summer of 1917 stands out as a remarkable achievement.

By the late 1920's the types and arrangements of engines had pretty well crystallized, and progress for the next decade was largely devoted to refinement of design. In general, all during the thirties and up until the impetus of WWII began to be felt, each year saw a certain percentage increase in reliability and operating life.

Then in the early 1940's Whittle, in England, succeeded in his efforts to demonstrate the worth and possibilities for aircraft use of the turbine jet type of power plant. The idea of jet propulsion was further brought into focus by the German use of the impulse jets in the V-1 and the use of the rocket power plant in the V-2. In England and perhaps elsewhere the athodyd (ramjet) was receiving serious attention. It is interesting to note that the underlying principles for all these types have been known for many years.

The general performance that could be expected from the various types of engines to be discussed can be best shown in the Figure 4.1.1.

Engine Performance Regime

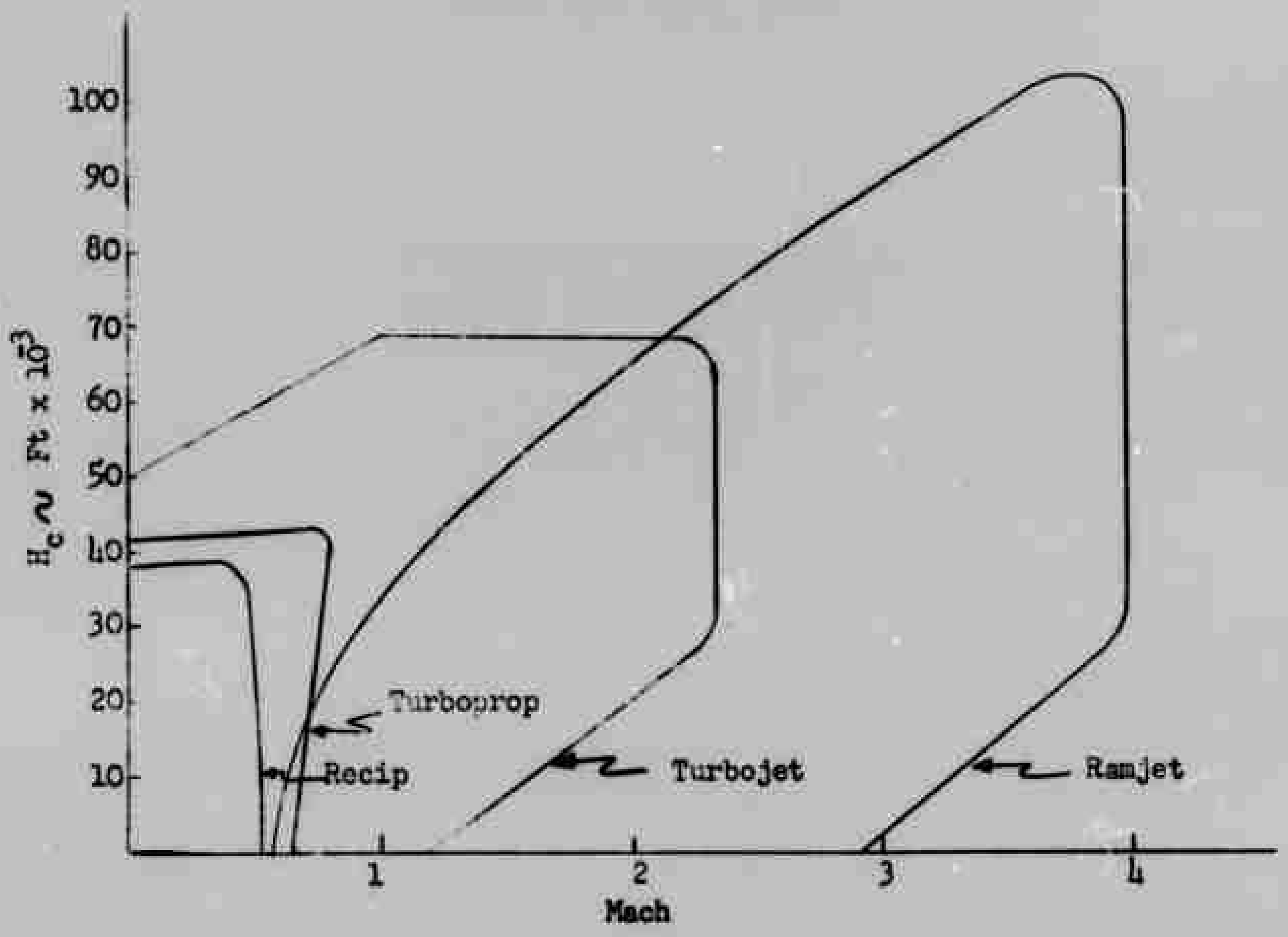


Fig. 4.1.1

4.2 RECIPROCATING ENGINE

We shall begin our study of engines by first considering the internal combustion engine. Neither time or facilities are available here at the Test Pilot School to give a complete course in internal combustion engines, jet engines or rocket engines, nor is the purpose of the school to do so. Such a course would require months of graduate study at an accredited University. It is mandatory, however, that the test pilot have some understanding of the basic fundamentals of these power plants. Much detail has been omitted from the following material, but application of the individual to some outside study will greatly increase his knowledge over that presented here.

4.2.1 Schematic diagrams of the general types of reciprocating engines in current use and listing of their major component parts follows:

a. Supercharged engine:

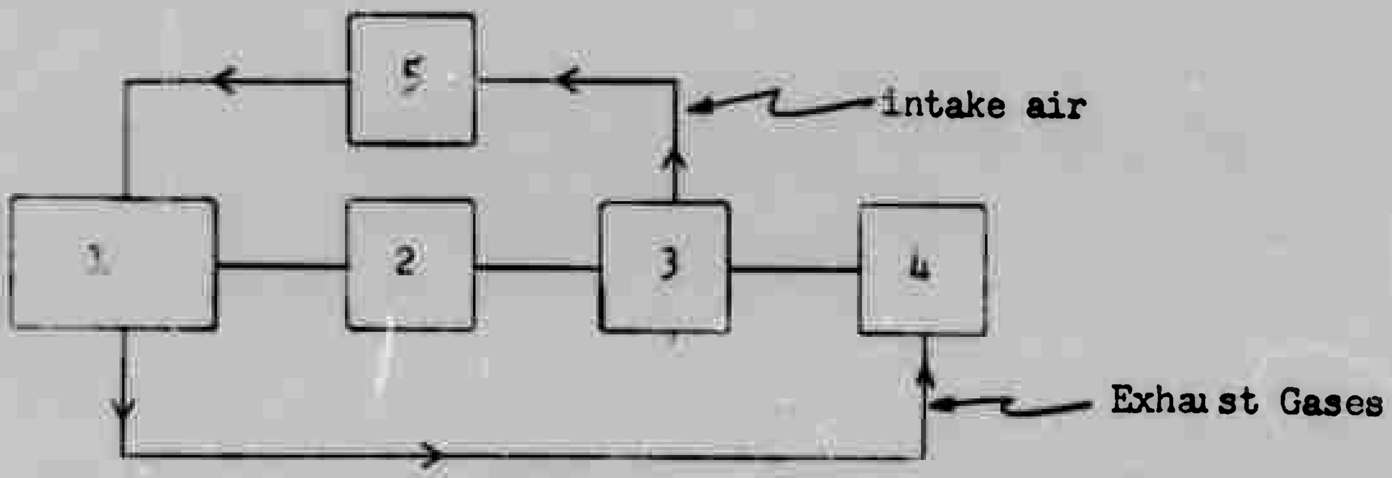


Fig. 4.2.1

1. Engine
2. Gears
3. Supercharger
4. Turbine
5. Aftercooler

b. Compound engine:

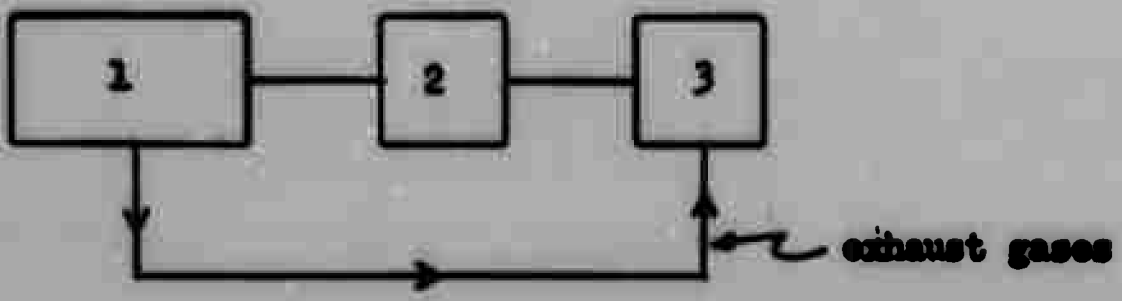


Fig. 4.2.2

1. Engine
2. Gears
3. Turbine

c. Unsupercharged engine. A basic engine without a means for increasing the intake manifold pressure.

4.2.2 Each engine above has the same basic operation with variations in the means for boosting the power output. A piston engine gets its power by burning fuel in a cylinder and moving a piston which is attached to a crankshaft. Normally an engine makes either one revolution (2 cycle) or two revolutions (4 cycles) per power stroke, thus getting its name 2 cycle or 4 cycle. The injection of fuel, ignition, compression and the exhaust is regulated by a camshaft which opens and closes valves. To get a better understanding of the operation of a reciprocating engine let's take a brief look at the strokes of a typical Otto cycle. The name of Otto bears the name of the person who first built the Beau De Rochas engine in 1876. In 1862 Beau De Rochas first put the internal combustion engine on paper as follows:

Strokes of a typical Otto cycle.

a. Intake - Induction of the combustible material (fuel-air mixture). The engine operates as a vacuum pump.

b. Compression - The cylinder volume is decreased, increasing the pressure of the fuel-air mixture.

c. Combustion and expansion or power stroke - Ignition occurs and the chemical reaction increases the pressure of the fuel-air mixture even more and moves the cylinder back.

d. Exhaust - After the mixture has served its useful purpose or done the required work, it has to be removed form the cylinder to make room for the next charge. Following the exhaust the cycle is repeated and since the operation requires four strokes the engine is called a "Four-stroke-cycle".

4.2.3 In speaking of the ideal or theoretical Otto cycle the following assumptions must be made:

Assume no effect on:

a. Fuel air ratio

b. Characteristics of a particular fuel

c. Change in the characteristics of the fuel as it is burned

d. Variations in specific heat

e. Heat transfer losses

f. Dilution of the working medium with exhaust. All the burned gases have been removed and a fresh charge is introduced

g. Leakage or throttling during induction or exhaust

4.2.4 Air is assumed to be the working substance, and the cycle is often referred to as the "Air standard cycle". There is no loss of heat in the system and heat is assumed to be supplied or rejected without heat transfer losses. A pressure-volume (P-V) diagram of an ideal Otto cycle is as follows:

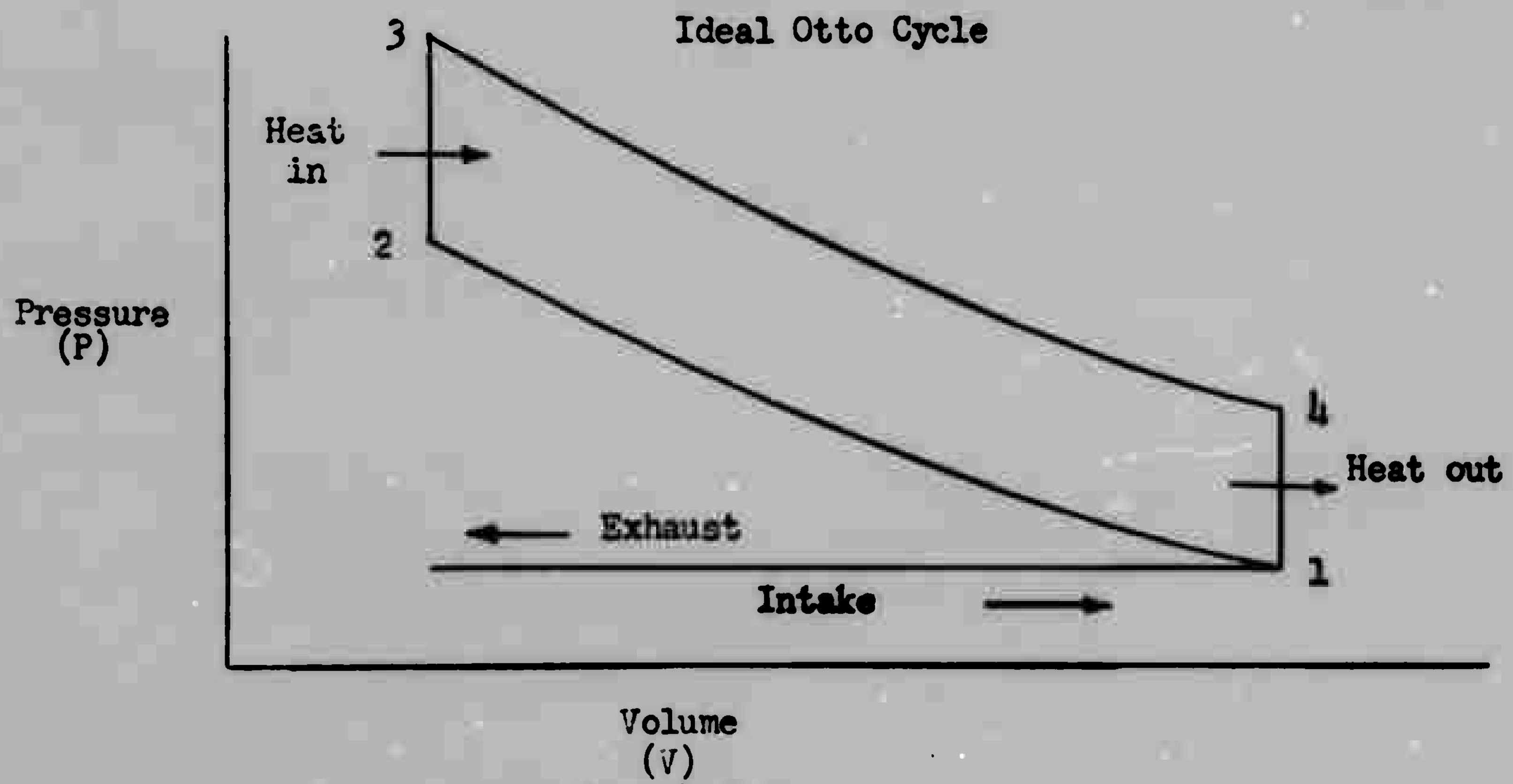


Fig. 4.2.3

1-2 Isentropic compression

2-3 Constant volume heat addition

3-4 Isentropic expansion - power stroke

4-1 Constant volume heat rejection

4.2.5 To determine the net work developed during the cycle mentioned we must determine the area under the curve or, work = $\int P\,dV$. The area under the curve = P V = (lb/ft^2) (ft^3/lb of air) = ft - lb/lb of air = work done = weight of air x B T U /lb of air

where

$$1 \text{ hp} = \frac{33000 \text{ ft-lb/min}}{778 \text{ ft-lb/BTU}} = 42.4 \frac{\text{BTU}}{\text{min}} = 2545 \frac{\text{BTU}}{\text{HR}}$$

4.2.6 A means of determining the P-V diagram of a reciprocating engine is accomplished with a planimeter. As an engine piston reciprocates an attached indicator moves with it. Thus, the indicator pointer moves horizontally in proportion to volume changes in the cylinder. The pressure will be transmitted through the connecting tube to the indicator piston so that it will compress the indicator spring and move the pointer vertically in porportion to pressure changes in the cylinder. The resultant path of the indicator pointer is in a line representing the pressure and volume in the cylinder at all points throughout the cycle. If the planimeter were reading the pressure and volume of an ideal Otto cycle the resultant picture would be as shown in Figure 4.2.3. If we were to consider the actual cycle then all the losses would have to be considered. These losses are:

a. Intake losses - manifold, carburetor, valves

b. The compression stroke is not isentropic

c. Ignition before top dead center, combustion is not 100% efficient, also incomplete and not instantaneous

d. Expansion stroke is not isentropic

e. Exhaust flow losses - exhaust manifold and valves

f. Friction

Considering all the losses mentioned above the actual P-V diagram looks as follows:

Actual P-V Diagram

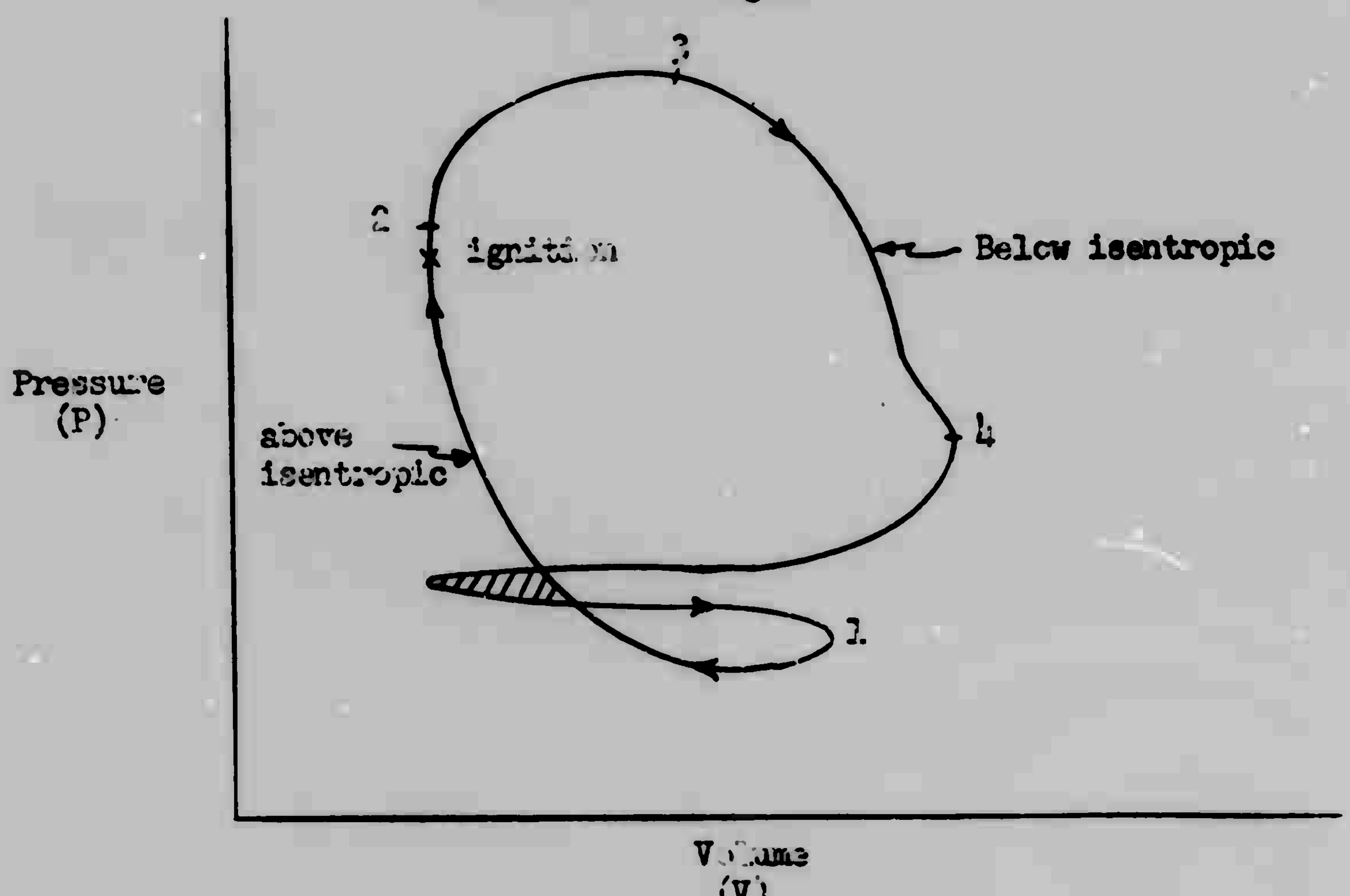


Fig. 4.2.3

In considering the work done during an actual cycle, one must consider the number of cylinders, engine RPM and air density for a given fuel air ratio. It should be noted that the shaded area in Figure 4.2.4 is negative work due to pumping losses.

4.2.7 The average pressure on a piston during a cycle is called the indicated mean effect pressure (imep or P). Due to friction losses between moving parts of the engine, not all of this pressure is available to drive the propeller and the equivalent at the shaft is called brake mean effective pressure. The average net pressure, imep, times the piston area is the average net gas force on the piston, and since

work = force x distance

the work in foot-pounds per cycle is

work = imep x A x L

where imep is in p s i

A = area of piston in square inches

L = length of stroke in feet

Work per unit time is power; therefore the indicated or internal horsepower in a four cycle engine is:

$$IHP = \frac{(\text{net work/cyl/cycle}) \times (\text{cycles/min}) \times \text{no. cyl.}}{\text{horsepower constant}}$$

or more conveniently expressed

$$IHP = \frac{P\ L\ A\ N\ K}{33000}$$ **Equation 4.2.1**

where P = imep

N = RPM/2 for four cycle engine

K = number of cylinders

As mentioned earlier the difference between internal horsepower and bhp is the mechanical efficiency or

$$bhp = ihp \times \eta_{me} \qquad \text{Equation 4.2.2}$$

$$bhp = \frac{P \times \eta_{me} \times L \times A \times N \times K}{33{,}000 \times 2}$$

$$bhp = \frac{bmep \times L \times A \times N \times K}{33{,}000 \times 2} \qquad \text{Equation 4.2.3}$$

If we consider piston displacement then

$$D = \text{displacement} = 12\ L\ A\ K = \text{cubic inches}$$

then

$$bhp = \frac{bmep \times D \times N}{12 \times 33000 \times 2} = \frac{bmep \times D \times N}{792{,}000} \qquad \text{Equation 4.2.4}$$

4.2.5 The engine efficiency is energy out/energy in or power out/power in. The following are the efficiencies related to engines:

a. overall eff $= \eta_o = \dfrac{T \times V}{\text{Fuel Flow} \times J \times H} = \eta_t \times \eta_{me} \times \eta_p$ Equation 4.2.5

where fuel flow = #/sec

J = 778 ft #/BTU

H = Heating value of fuel

b. Thermal efficiency $= \eta_t$

$$= \frac{\text{indicated work produced by the cycle}}{\text{energy added by fuel plus oxidizer}}$$

$$= \frac{\text{IHP} \times 550}{\text{Fuel flow} \times J \times H}$$ Equation 4.2.6

c. Mechanical efficiency $= \eta_{me} = \frac{\text{BHP}}{\text{IHP}}$ Equation 4.2.7

d. Propulsive efficiency $= \eta_p$

$$= \frac{\text{useful work done on an airplane}}{\text{mechanical work done by the cycle}}$$

$$= \frac{T \times V}{\text{BHP} \times 550} = \frac{\text{THP}}{\text{BHP}}$$ Equation 4.2.8

4.2.9 Supercharging

a. The supercharger simply raises the pressure at which the charge is supplied to the cylinder. Basically, any kind of an air compressor having sufficient capacity and capable of developing the desired manifold pressure could be used as a supercharger. This pressure rise can be shown on an ideal Otto cycle as below:

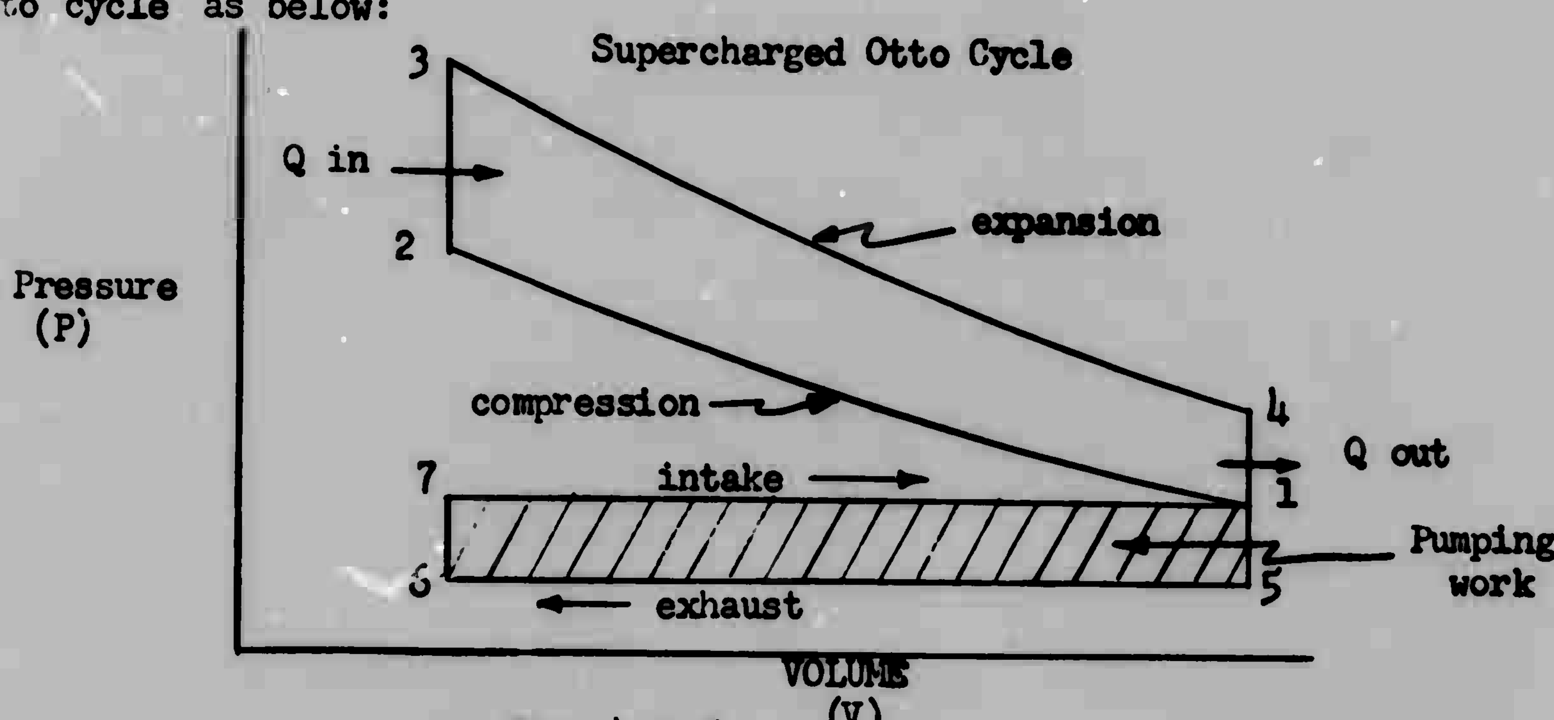


Fig. 4.2.5

(4-5) beginning of exhaust - goes to Pa as in unsupercharged cycle.

(5-6) exhaust stroke

(6) intake valve opens and exhaust valves close, and the clearance volume is filled with air to $P_{manifold}$ by the supercharger.

(7-1) intake stroke fills entire cylinder volume at constant $P_{manifold}$.

(1-2) Compression stroke, etc, around cycle as described in previous discussion of the Otto cycle.

b. In order to supply the engine cylinders with a charge at $P_{manifold}$ greater than P_a a compressor is used. Consider the Otto cycle in greater detail than given in Figure 4.2.5.

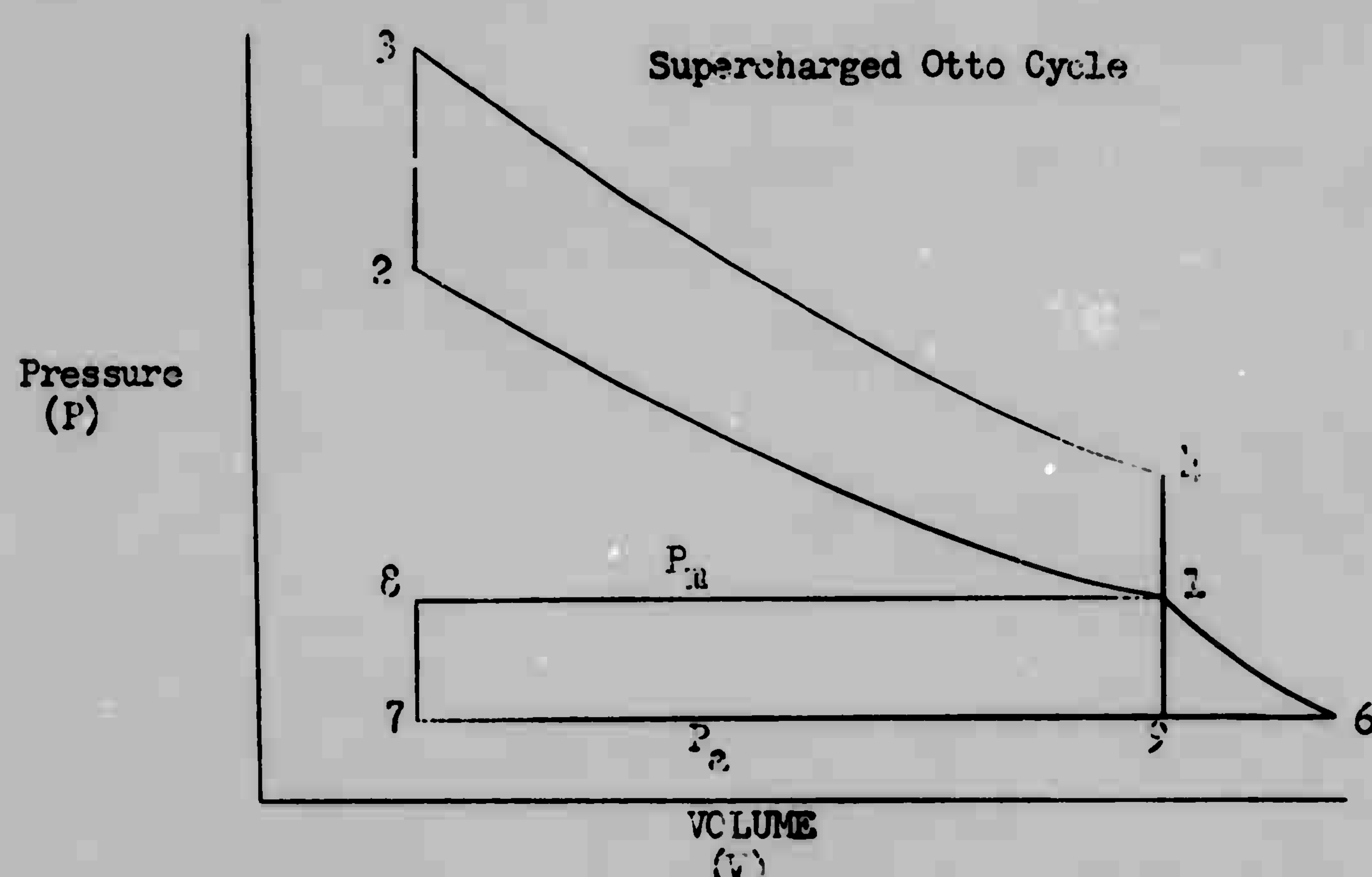


Fig. 4.2.6

(1) Assume that the volume of air is drawn into the compressor at P_a. This is point 6.

(2) The air is compressed isentropically to P_m represented by the area 7-6-1-8 as the work done during compression. This is pumping work. The line 9-6 represents the volume of the compressor.

(3) Intake stroke at P_m is 8-1

(4) Compression is 1-2

(5) Combustion is 2-3

(6) Power stroke is 3-4

(7) Expansion or exhaust is 4-9-7

c. The area 1-2-3-4 is the resultant work done by the compression and power stroke. The area 7-8-1-9 is the resultant work done by the exhaust cycle. The area 7-8-1-6 is the work required to compress the air up to P_m which is accomplished by the compressor. Therefore, area 1-6-9 is the difference in work between exhaust and the intake cycle which results in a negative work required. This is the work required to boost the P_a to a manifold pressure P_m. The net output of the supercharged cycle is area (1-2-3-4) + (7-8-1-9) - (7-6-1-8) or area (1-2-3-4) - (1-6-9).

This cycle exhausts to condition 9. Some of this exhaust could be used to expand isentropically from condition 1 to help drive the compressor. This method is utilized by the turbo-supercharger system.

4.2.10 COMPOUNDING

a. In addition to supercharging, there is a process called compounding which can be used to increase the net output of an engine. This method uses exhaust gases to drive a turbine which is connected to the engine. This is shown in Figure 4.2.2. Lets consider a P-V diagram for a compounded engine.

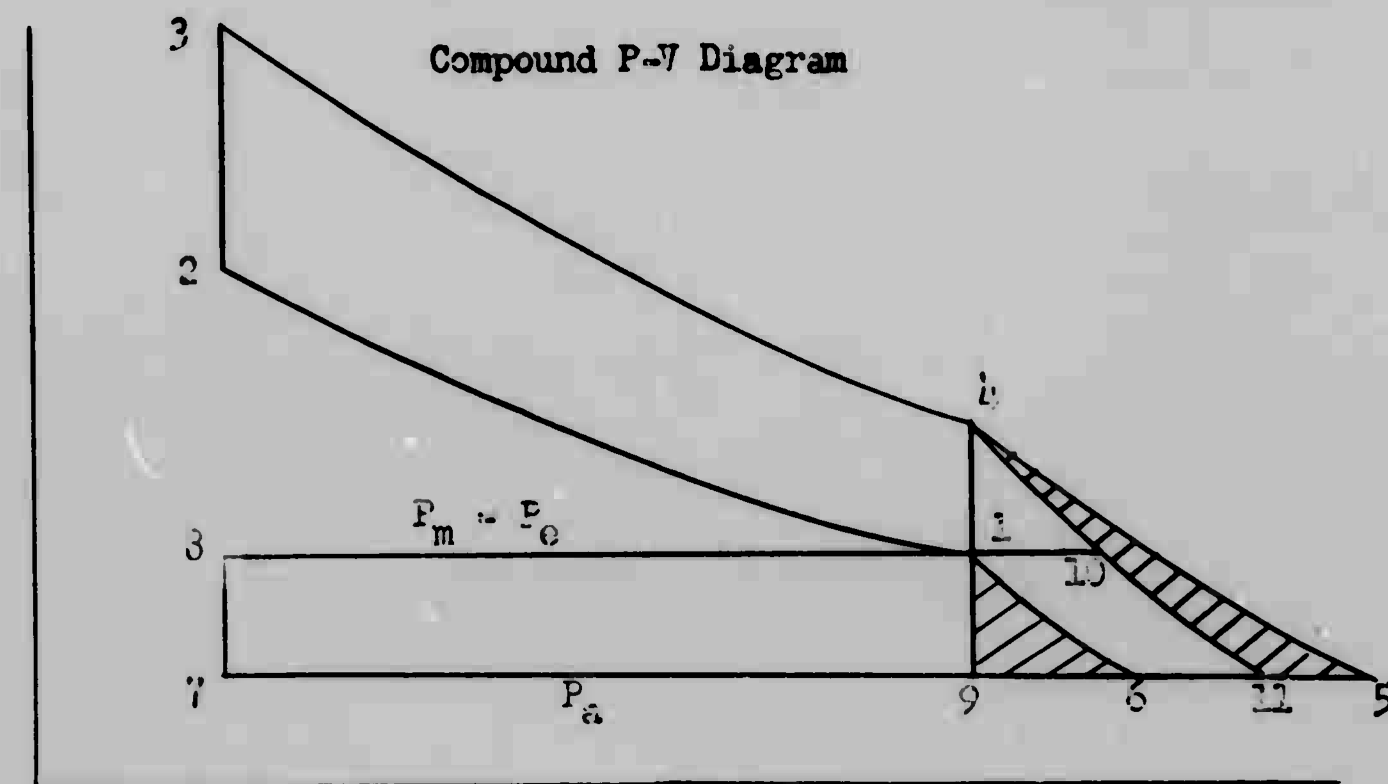


Fig. 4.2.7

(1) It can be seen that this diagram is the same as the super-charged cycle in Figure 4.2.6 with the addition of area 1-4-5-6. If it were possible to continue isentropic expansion along the line 4-5 more work could be delivered by the engine. This has never been accomplished; however, gains have been made. The area 4-5-9 would be available for additional work if 4-5 were isentropic. This is not possible, therefore, area 4-11-9 is practically available and area 4-5-11 represents the losses. However, the pressures in the exhaust manifold cannot equal P_4 since it is necessary to discharge

the exhaust gas rapidly from the cylinder; therefore, exhaust from P_4 to some lower exhaust pressure, P_e, must be accomplished. At sea level the exhaust pressure, P_e, is approximately equal to ambient, P_a. As altitude increases P_e remains approximately sea level pressure while P_a decreases; therefore, a pressure drop P_e to P_a is available between exhaust manifold and the atmosphere to be used to operate a turbine placed in the exhaust system. The exhaust gas expands without doing useful work from P_4 to P_{10} and exits at state 10 to 11. The work available, therefore, is area 8-10-11-7 which exceeds the pumping work 8-1-6-7 required to drive the supercharger. The resultant increase in available work is the area 1-6-11-10. This is used to compound the engine.

(2) Consider a diagram of a supercharged and compounded engine as shown below.

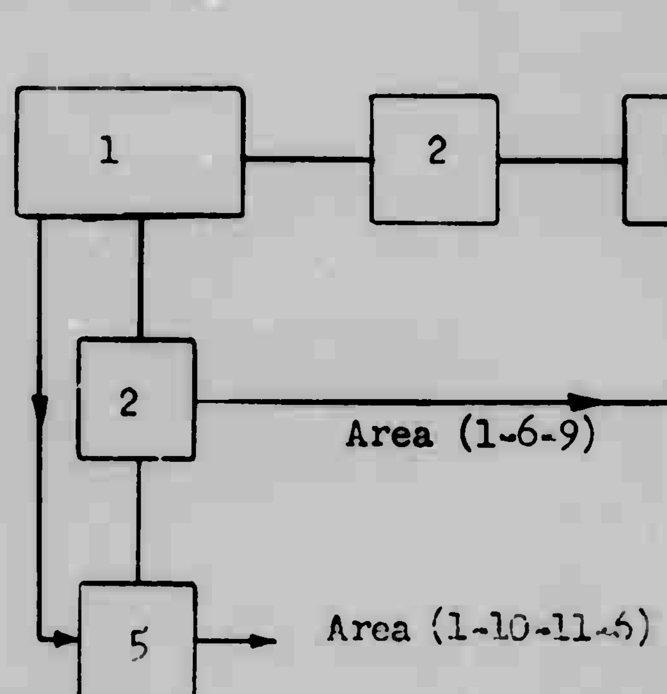


Fig. 4.2.8

1 - engine

2 - gears

3 - supercharger

4 - turbine for supercharging

5 - turbine for compounding

The above diagram shows that area (1-6-9) is required to drive the supercharger which is negative work. Area 1-10-11-9 is the work output of the compounding and the resultant work output of both supercharging and compounding is area (1-10-11-6).

4.2.11 ENGINE LOSSES

a. The approximate heat losses in the actual reciprocating engine are as follows:

(1)	In the combustion process, of total heat added	- - - - - 0.3%
(2)	Combustion process due to heat transfer	- - - - - 2.0%
(3)	Incomplete combustion	- - - - - 2.0%
(4)	Expansion of power stroke	- - - - - 4.0%
(5)	Heat loss to oil	- - - - - 0.3%
(6)	Intake and exhaust	- - - - - 2.6%
	Total energy loss	11.2

This indicates that η_{ms} of the reciprocating engine is about 88.8 percent of the thermal efficiency. At 50,000 feet the density is approximately 12 percent of the density at sea level; therefore, at 50,000 feet the power output of the engine will just about make up for the losses and there is no useful work available from the engine. Some slight differences between engine may be expected, but for a first approximation this simple relationship is

very useful. This situation requires that the engine is at wide open throttle, giving its maximum horsepower. There are a couple of equations which may be used to approximate the horsepower at altitude. The first is:

$$BHP_{alt} = BHP_{SL} \left(\sigma - \frac{1-\sigma}{7.55}\right) \qquad \text{Equation 4.2.9}$$

Consider BHP at 20,000 feet where σ is .5327, therefore,

$$BHP_{20M} = 1\left(.5327 - \frac{1-.5327}{7.55}\right) = .4708$$

This states that the BHP at 20,000 feet is equal to approximately 47 percent of the horsepower at sea level.

The second equation is:

$$BHP_{alt} = BHP_{SL}\left(\frac{\sigma - .1}{.9}\right) \qquad \text{Equation 4.2.10}$$

therefore

$$BHP_{20M} = 1\left(\frac{.5327 - .1}{.9}\right) = \frac{.4327}{.9} = .48$$

4.3 TURBO-JET ENGINES

4.3.1 HISTORY AND DEVELOPMENT

As far back as 1680 Sir Isaac Newton had built a model horseless carriage using the jet propulsion principle to demonstrate his third law of motion: "for each action there is an equal and opposite reaction". There are no records showing that this model was operational. The theory was that the jet of steam escaping from the nozzle at the rear of the boiler would propel the carriage, but it is very doubtful that enough thrust was produced to move the carriage. The records show that the first gas turbine in the United States was begun in 1902 under the supervision of Stanford A. Moss at Cornell University. The information he received through this project was the basis for his doctor's thesis.

The Heinkel Aircraft Company of Germany developed a research aircraft, designated the He 178, with a jet engine for power This is the first aircraft to fly using the principle of jet propulsion and made its first flight 27 August 1939. The power plant was a Heinkel turbojet, the He S3B, with a thrust of 880 to 1,100 pounds.

In 1928 Sir Frank Whittle, a cadet at RAF College in England, prepared a thesis describing the possibility of using jet propulsion to power an aircraft and in January 1930 he applied for his first patent. Because of lack of funds and a sponsor he was not able to develop the engine until 1936 at which time he formed the company called Power Jets, Limited. The Air Ministry

supplied the financial backing, thus making it possible to proceed with the development of the engine. The first successful flight of this engine and the first jet flight in England was on 14 May 1941 using a Gloster E 28/39 aircraft. The pilot was Flight Lieutenant P. E. G. Sayer.

The United States and England had an agreement during World War II whereby there was a constant interchange of ideas so that the development and production could be expedited. The first Whittle engine under this agreement arrived in this country on 1 October 1941. The General Electric Company was called on to build the engine to American standards and Bell Aircraft Corp. was to build the airframe. One year was allotted for the project and the actual time was one year and three weeks to the first flight date. The first pilot to fly the P-59A, the first USAF jet aircraft, was R. M. Stanley. Colonel Laurence C. Craigie was the first United States military person to fly the P-59A which was the day after the flight by R. M. Stanley. These flights were conducted at Edwards AFB, California.

This is a very quick look at the history of the gas turbines and considering the progress made we may safely conclude the future of jet type propulsion cannot be predicted.

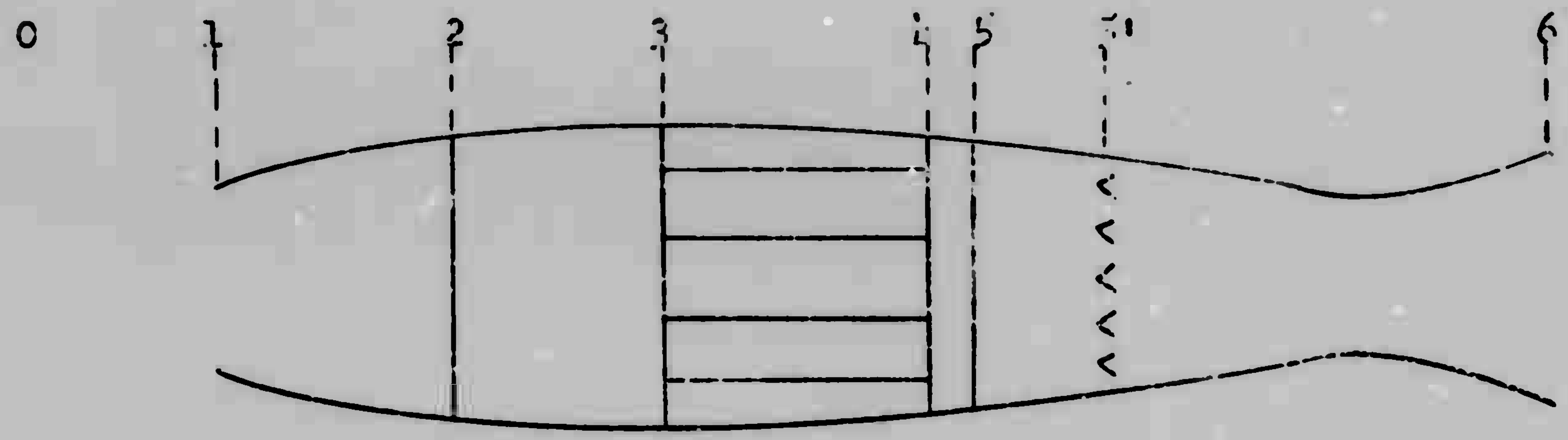
4.3.2 GENERAL:

The basic principles of the turbo-jet are quite similar to those of the reciprocating, except that it employs the Brayton cycle which is not so familiar to most Air Force people. Most present day reciprocating engines employ the Otto cycle. The fact that the turbo-jet deals with large changes in temperature, pressure and velocity with greater volumes

of air over a longer period than does the reciprocating engine requires that a far greater detailed and more complicated analysis be conducted of each phase of the system. The following material presented involves many assumptions and is extremely condensed; however, it should give the student a more familiar working knowledge of the component parts of the jet engine and of their relationships to each other.

4.3.3 BRAYTON CYCLE:

A schematic diagram of the engine and its corresponding P-V diagram is shown below in Figures 4.3.1 and 4.3.2.



TURBOJET ENGINE

Figure 4.3.1

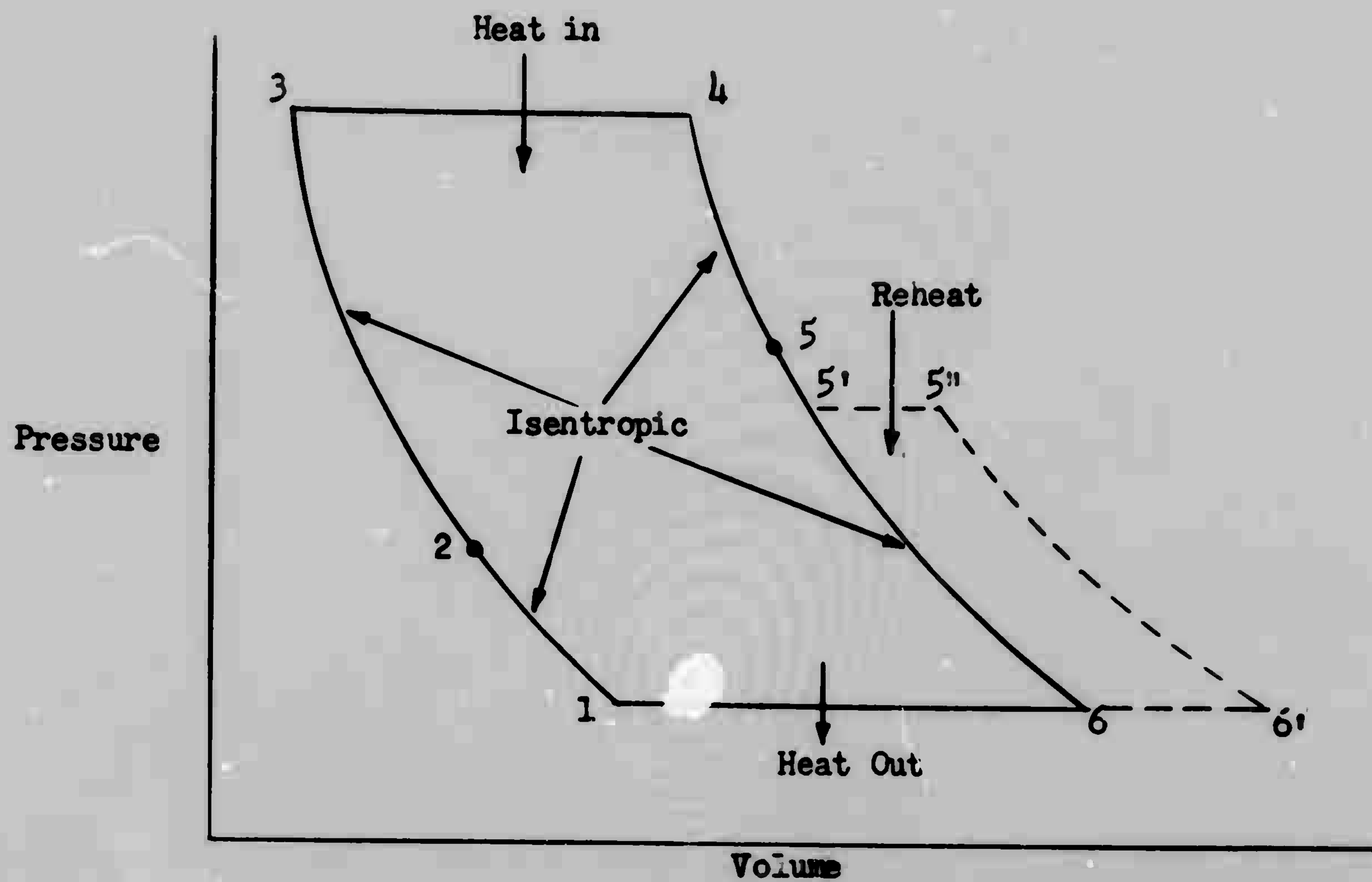


Figure 4.3.2

(a) The schematic of the turbo-jet shows the stations as follows:

Station 0	- - - - -	Ahead of aircraft
Station 1	- - - - -	Lip of intake or diffuser
Station 1-2	- - - - -	Diffuser
Station 2-3	- - - - -	Compressor
Station 3-4	- - - - -	Burner
Station 4-5	- - - - -	Turbine
Station 5-6	- - - - -	Nozzle
Station 6	- - - - -	End of Nozzle (atmospheric)

Station 5' to 6' (Dashed Area) - - - Afterburner

(b) Consider the ideal P-V diagram. The diffuser, station 1-2, slows the air down allowing the pressure to rise. The compressor, station 2-3, increases the pressure more. From station 3-4 fuel is added to the air and ignited. This burning takes place at constant pressure. This heated air is ejected through a turbine allowing expansion. At point 5 the air is completely through the turbine and is entering the nozzle. For a non-afterburning engine the air is expanded to point 6 which is the atmosphere. An afterburner, which is usually part way down the nozzle at point 5', will change the P-V diagram as shown. The ideal Brayton cycle adds heat at constant pressure, thus increasing volume to point 5". The gas is then expanded further to point 6' which is again atmospheric pressure. The afterburner section can be considered a ramjet and the P-V diagram for a ramjet is a Brayton cycle. Therefore, the afterburner will add a section to the cycle which looks like a Brayton cycle.

(c) DIFFUSER:

Air entering the engine is first diffused to convert its kinetic energy to pressure. In an ideal condition this compression takes place reversibly and adiabatically, that is, isentropically. In an actual cycle the process may be considered essentially adiabatic but it cannot be considered as reversible since fluid friction is present. The most widely used efficiency of the inlet diffuser, normally called ram efficiency, is based upon the actual pressure rise compared to the pressure rise which would take place if the process were isentropic. Therefore, ram efficiency is;

$$\eta_R = \frac{P_{t2\,\text{actual}} - P_o}{P_{t2} - P_o} \qquad \text{Equation 4.3.1}$$

If we assume isentropic compression through the diffuser then η_R would be 100 percent and in this case $\frac{T_{t_2}}{T_{t_1}} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + .2M^2$

In the analysis above we assume that the velocity of the air is zero at station 2, face of the compressor.

(d) COMPRESSOR AND TURBINE

The work of the turbine must equal the work of the compressor, because the turbine drives the compressor.

Work of Compressor $= W_C = C_{Pc} (T_3 - T_2)$ Equation 4.3.2

Work of turbine $= W_t = C_{Pt} (T_4 - T_5)$ Equation 4.3.3

There are some losses through the turbine and the compressor and these are taken into account through their respective efficiencies. Although the compressor work may be considered to take place abiabatically, it cannot be considered frictionless. The conventional method of expressing compressor efficiency is the ratio of the frictionless, or reversible, abiabatic compressor work that would be necessary, between the limits of the compressor pressures, to the actual work required for compression.

The present day trend is to present the compressor performance using the parameter $W_a \sqrt{\theta_{t_2}} / \delta_{t_2}$ since it is more pertinent to over all engine performance. The plot most commonly used for this presentation is P_{t3}/P_{t2} versus $W_a \sqrt{\theta_{t_2}} / \delta_{t_2}$ with line of constant $N/\sqrt{\theta}$. Line of constant compressor efficiencies are shown also. The operating line for the compressor is shown

when it is matched with a gas turbine and are operating at a constant ram-pressure ratio. This operating line will shift slightly with a change in ram-pressure ratio.

The surge line shown represents the limit of stable compressor operation. Blade stall will occur during a condition left of the line accompanied by a sharp local drop in pressure and air flow. This causes a flow reversal and results in a pulsating flow and changing load requirements causing rapid cyclic variations in the rotational speed. The surge line position will vary with the type and design of the compressor. The turbine selected must have characteristics such that when matched with the compressor, the operating line is to the right of the surge line for all expected flight conditions, rotational speeds and ram-pressure ratios. A typical axial flow compressor performance map or performance chart is shown in Figure 4.3.3

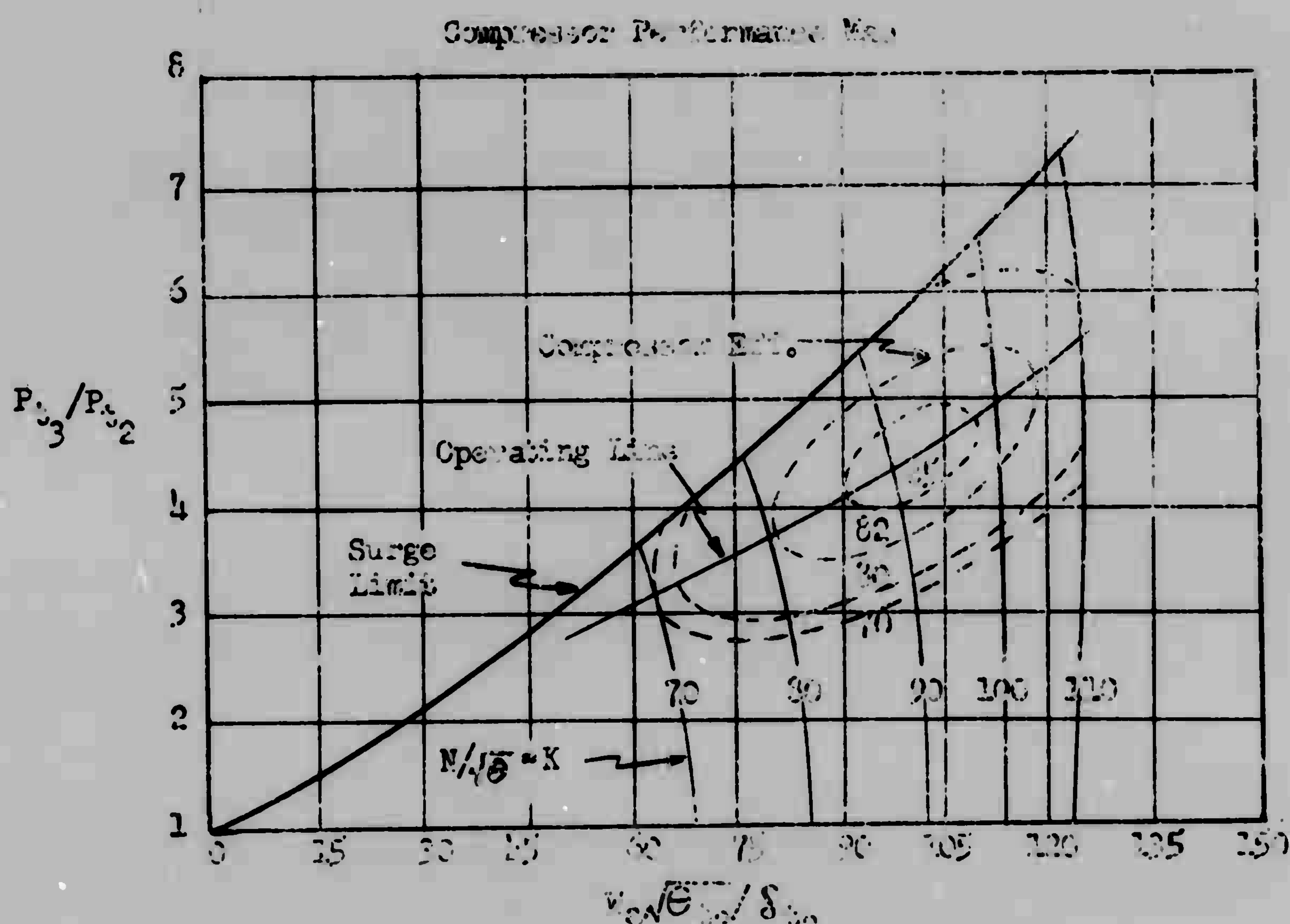


Figure 4.3.1

The effects of compressor efficiency is to vary the horsepower required to drive the compressor when delivering a given pressure at a given airflow. A lower efficiency would result in a higher temperature rise and a higher efficiency would result in a lower temperature rise, considering pressure ratio being unchanged. A lower compressor efficiency means that a greater turbine pressure ratio must be produced to develop the required horsepower. The result is a lower pressure ratio across the jet and lower tailpipe temperature, both of which mean a lower thrust output.

The reason for presenting compressor performance in terms of the previously mentioned parameters is that, by running one series of tests, performance can be predicted at all other operating conditions. It has been found that Reynolds' number has no effects on full scale compressors except in a region of very low speeds and flows, and for this reason it is not even considered. There are indications that Reynolds' number does have significant effects at high altitude operations. The selection of the type of compressor, centrifugal or axial, to be used in designing an engine normally are as follows:

Selection of a centrifugal compressor

(1) Higher pressure boost from a single compressor, thus lower cost.

(2) Much shorter engine.

Slection of an axial compressor.

(1) High efficiency at high pressure ratio.

(2) High capacity for small diameter.

(3) Better adaptability to operation at pressure ratios in excess of four.

(e) BURNERS

Normally there are three types of burners used in jet engines: the counterflow can-type burner, the straight flow can-type burner, and the angular burner.

The counterflow can-type burner requires the primary air to travel the full length of the cans before mixing with the fuel and the secondary air flow reverses its flow as it enters the burner cans or inner chamber. This was first used to take advantage of very high turbulance which is conducive to high combustion efficiencies. The disadvantage of this setup is two fold. First, the high friction losses far exceed the gains in combustion efficiencies. Second, the physical arrangement is such that it results in a very large engine diameter.

The straight flow can-type burner is similar to the counterflow type, but the diameter of the engine can be made much smaller. This setup has a very low friction loss, provides even air distribution and is relatively easy to manufacture. The system is the most used in the aircraft industry.

The angular burner is a straight flow type which offers low friction losses, small diameter and high flow rates. This is a single burner which completely surrounds the engine between the compressor and turbine and contains several fuel nozzles spaced evenly around the inlet to the burner.

All burners are normally designed to produce constant pressure burning; therefore, there must be low velocity in the area of initial combustion to keep the flame from blowing out. A high degree of turbulence is conducive to complete combustion but is also attended by high friction losses and,

therefore, high undesirable pressure drop. The length of the burner must be so that all burning is completed prior to the turbine and low velocity will aid in accomplishing this. The pressure losses mentioned are caused by two factors: first, the pressure loss due to fluid friction; second, pressure loss due to the acceleration of the gases by the addition of heat. In most of the present day jet engines fluid friction is accountable for a greater part of the pressure drop than is the acceleration resulting from heat addition. The design of the perforated inner lining and other obstructions cause high friction losses and the low fuel air ratio does not release enough heat to cause extremely high velocities anywhere in the chamber.

From this we can consider the burner only a place where the temperature of the gas is raised with a slight effect on the pressure of the cycle. The gas pressure entering the turbine stations is slightly lower pressure than compressor discharge. This is due to the friction losses mentioned above.

(f) NOZZLE:

For a compressible flow a nozzle may be defined as a channel operating between a low pressure and a high pressure whose purpose is to change enthalpy into kinetic energy. Nozzle flow is associated with an expansion from a region of one pressure to another pressure which is usually lower.

Despite the simplicity of the nozzle their efficiencies, characteristics and functioning are not completely understood. The nozzle, like the turbine, is a flow device which is concerned directly with the total and static pressure ratio across the nozzle. Like the turbine, the nozzle has what is known as critical flow; therefore, it has measurable effects on turbine performance since it is in series with the turbine.

Nozzle performance curves are normally presented as in Figure 4.3.4.

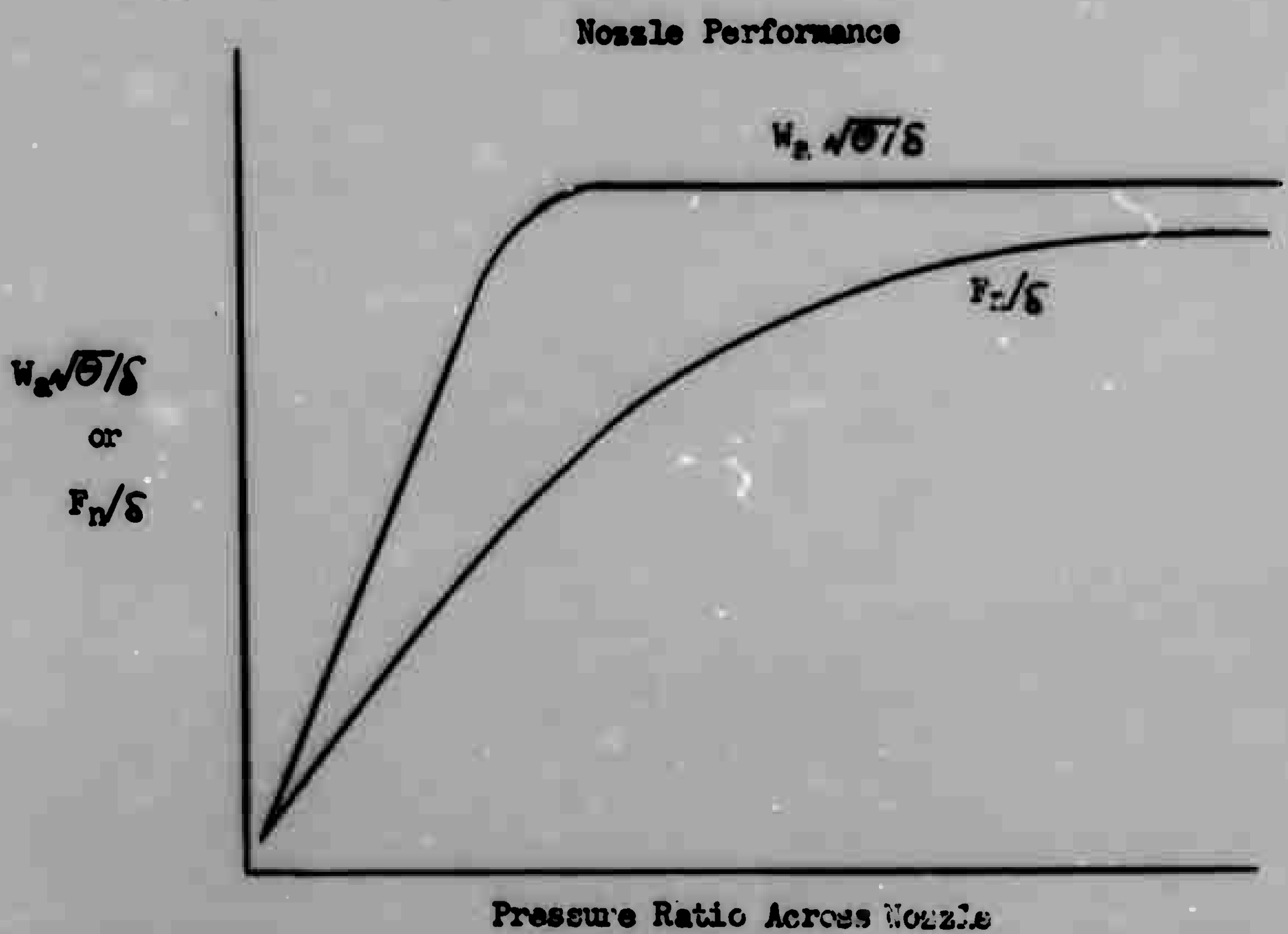


Figure 4.3.4

It can be seen that the thrust continues to increase as pressure ratio increases despite the fact that the airflow has reached a maximum at a low pressure ratio.

An afterburner may be used inside the nozzle much like a ramjet engine. An afterburner, or reheater, is a device to augment thrust by burning fuel in the unused oxygen of the turbojet exhaust gases. The afterburner has a very high rate of fuel consumption which may run about two or three times more than the basic engine, but its take-off thrust augmentation can reach 50%

and at high speed as much as 100%. For short range interceptor type aircraft the disadvantages of the additional weight of the burner, the lowered performance of unaugmented operation, and the large fuel flow are more than offset by the large thrust increase.

The theory and the design of the nozzles are discussed in another section of this handbook and will not be covered here.

4.3.4 EFFICIENCY

For an air breathing engine the propulsive efficiency may be written as:

$$\eta_P = \frac{T \times V_o}{\text{Kinetic Energy of the system}}$$

Consider mechanical work as a change of kinetic energy which is $\frac{1}{2} M (V_W^2 - V_o^2)$

V_W = velocity of wake

V_o = velocity

M = Mass = W/g

then:

$$\eta_P = \frac{T \times V_o}{\frac{1}{2} M (V_W^2 - V_o^2)} \qquad \text{Equation 4.3.1}$$

and $\qquad T = M (V_W - V_o)$

$$\eta_P = \frac{M (V_W - V_o)\ V_o}{\frac{1}{2} M (V_W - V_o)(V_W + V_o)} = \frac{2 V_o}{V_W + V_o} \qquad \text{Equation 4.3.2}$$

$$= \frac{2 \frac{V_o}{V_W}}{1 + \frac{V_o}{V_W}}$$

The following can be deducted:

When:

$V_o = V_W$, η_P = 100% and T = 0

$V_W < V_o$, there is drag

$V_W > V_o$, there is thrust

A plot of η_P versus $\frac{V_W}{V_o}$ is shown below. (see section on Rocket Engines also).

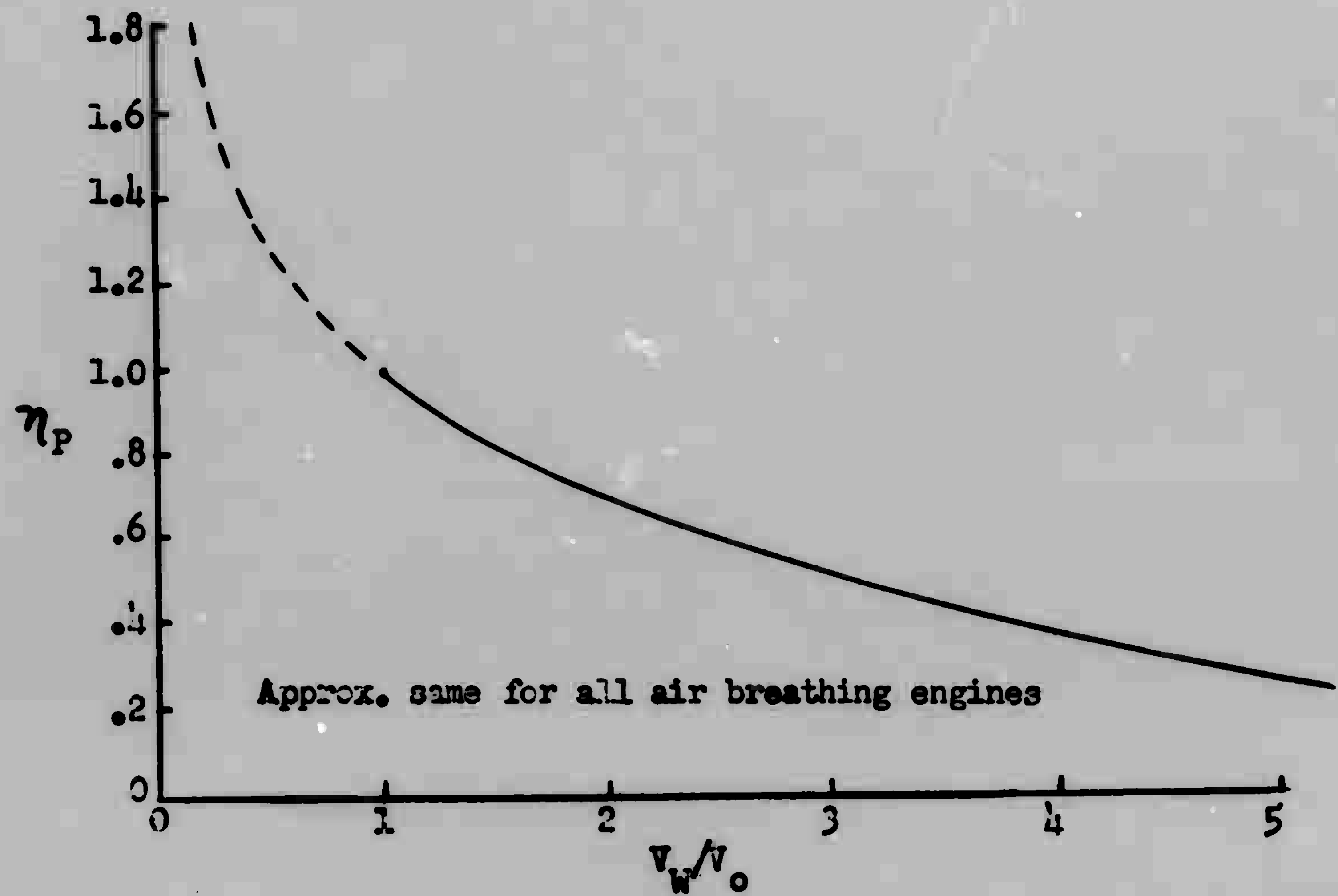


Figure 4.3.4

4.4 PROPELLERS

4.4.1 INTRODUCTION

Propeller theory may seem a rather obsolete subject; however, rather recent developments such as the turboprop, ducted fan designs, STOL aircraft and helicopters, to name a few, should convince the reader that propeller theory is indeed a live subject and should be for some time to come. This section covering propeller theory is very basic and will present two methods used extensively to develop the propeller momentum theory and one basic development of the propeller blade element theory.

4.4.2 PROPELLER MOMENTUM THEORY

The momentum theory for propellers considers the propeller as a devise which changes the momentum of the air as the air passes through it. The change in momentum of the air by the propeller creates a thrust. For continuity of air flow through the propeller, i.e., uniform air flow in front of and behind the propeller, the propeller is considered to have an infinite number of blades and is usually called an actuator disk. This is only one of the assumptions used in the momentum theory that obviously causes the results to be a little less than perfect. Other losses not considered in the development of this theory are:

1. Energy lost in the slipstream rotation
2. Propeller profile drag losses
3. Losses due to non-uniform thrust loading
4. Blade interference losses
5. Losses due to increased drag in the compressible range, i.e., tip losses

It is interesting to note, however, that even though the above losses are not taken into consideration, the ideal propulsive efficiency arrived at using the momentum theory is still less than 100%.

There are several analytical methods of developing the momentum theory. For simplification only two methods will be discussed in this section. The first method to be considered is developed as follows:

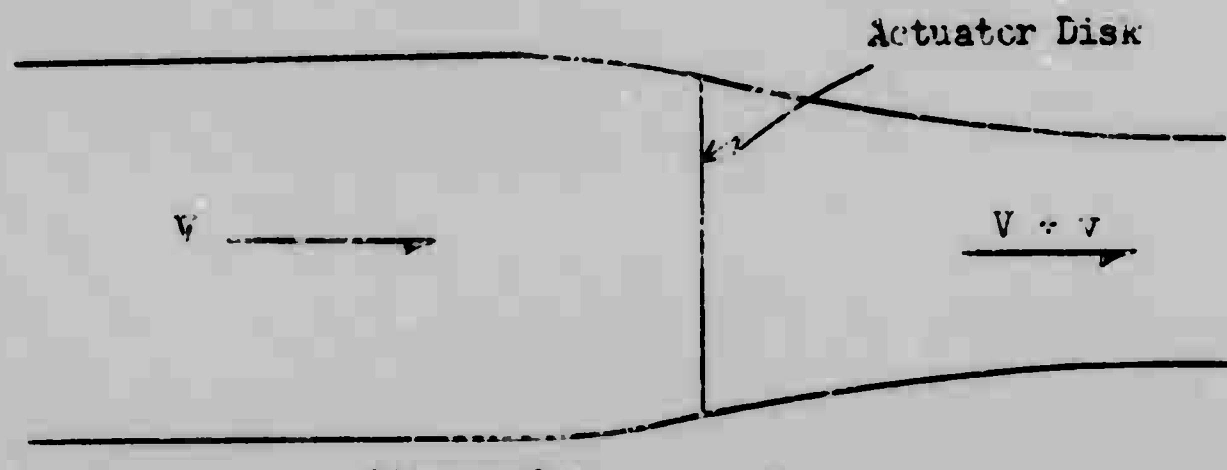


Figure 4.4.7

First, assume the actuator disk is stationary in an airstream as shown above. The air moves toward the disk with a velocity (V), and its velocity is increased by an amount (v) after passing through the disk. The thrust produced is equal to the rate of change of momentum where momentum is equal to the mass of air being accelerated through the disk times its velocity. If we let $\dot{m}$ equal the mass of air that goes through the disk in unit time, then the rate of change of momentum would be equal to $\dot{m}\, v$ which as stated previously is the thrust produced. Now, if we consider the actuator disk as having a forward velocity equal to V, the rate at which work is expended is equal to the increase in kinetic energy of the air through which it is passing, i.e.,

$$\frac{1}{2}\,\dot{m}\left[(V+v)^2 - V^2\right] \text{ or } \dot{m}\, v\left(V + \frac{1}{2} v\right). \qquad \text{Equation 4.4.1}$$

Now, since efficiency is defined as the ratio of the power output to the power input, and since power is equal to the product of velocity and thrust, the power

output is equal to $\dot{m}\, v\, V$. We then get the following equation:

$$\eta_p = \frac{\dot{m}\, v\, V}{\dot{m}\, v\, (V + \frac{1}{2} v)} = \frac{1}{1 + \frac{v}{2V}} \qquad \text{Equation 4.4.2}$$

From the above, it is readily apparent that the propulsive efficiency will always be less than 100% since the term $v/2V$ in the denominator will always have some finite positive value. Another interesting item that can be noted from the above equation is that to approach 100% propulsive efficiency it is necessary to decrease v or increase V. It follows, therefore, that the most efficient case would be to have a large forward velocity (V) and a large actuator disk that produces a small increase in the velocity of a large amount of air passing through it. This also defines why the propulsive efficiency of a turbojet is quite low in comparison to a propeller, since the turbojet derives its thrust from a comparatively large increase in the velocity of the small amount of air that passes through it.

Another method of explaining momentum theory for propellers is as follows: The propeller is again considered to be replaced by an actuator disk which is normal to the air flow direction and has an area of a circle with a diameter equal to the diameter of the propeller. The air is considered to be incompressible so that Bernoulli's incompressible equation may be used between points of constant energy.

This development does not take into consideration the same losses as previously discussed in the first part of this section.

Consider the actuator disk and the properties of the air passing through the stream tube as shown below.

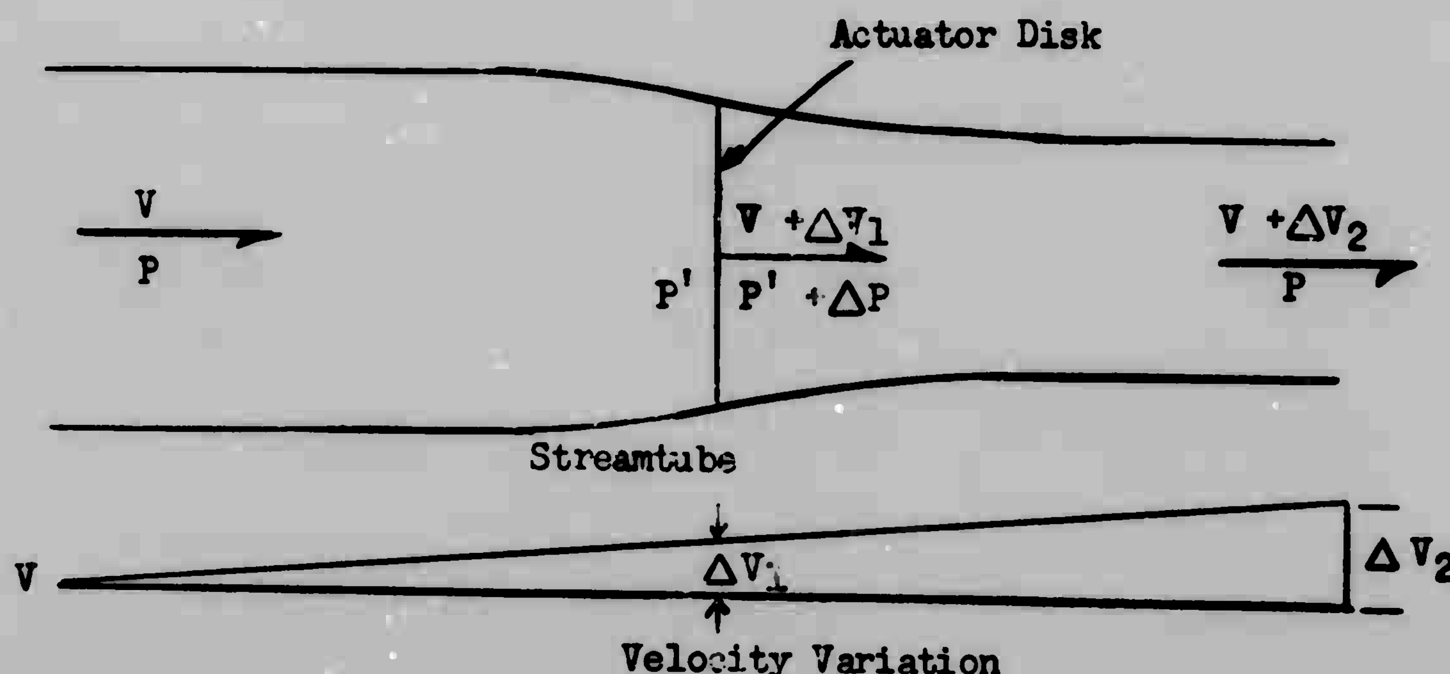


Figure 4.4.2

The velocity (V) is the undisturbed free stream velocity and (P) is the ambient pressure. The increase in velocity up to the disk is denoted ΔV_1 and the total velocity increase is denoted as ΔV_2. The pressure rise through the disk is ΔP.

From Bernoulli's equation the total energy in front of the disk (H_1) equals:

$$P + \frac{\rho V^2}{2} = P' + \frac{\rho(V + \Delta V_1)^2}{2} = H_1 \qquad \text{Equation 4.4.3}$$

The energy behind the disk (H_2) equals:

$$P' + \Delta P + \frac{\rho(V + \Delta V_1)^2}{2} = P + \frac{\rho(V + \Delta V_2)^2}{2} = H_2 \qquad \text{Equation 4.4.4}$$

From these equations it is easily seen that by subtracting H_1 from H_2 the change in pressure between the front and rear of the actuator disk can be found.

$$H_2 - H_1 = \Delta P = \rho\frac{(V + \Delta V_2)^2}{2} - \rho\frac{V^2}{2} \qquad \text{Equation 4.4.5}$$

or $$\Delta P = \rho V \Delta V_2 + \frac{\rho \Delta V_2{}^2}{2}$$

and $$\Delta P = \rho \Delta V_2 \left(V + \frac{\Delta V_2}{2}\right) \qquad \text{Equation 4.4.6}$$

The thrust can be considered as the actuator disk area times the difference in pressure between the front and rear surfaces of the disk, thus:

Thrust (T) $= A \;\Delta P = A \;\rho\; \Delta V_2 \;V + \frac{\Delta V_2}{2}$ which is equal to the change in momentum across the disk. Equation 4.4.7

The Mass Flow ($\dot{m}$) $= A \rho \;(V + \Delta V_1)$ Equation 4.4.8

or $T = \dot{m}\;(V_w - V_o)$ where V_w = wake velocity and V_o = free stream velocity.

or $V_w - V_o = \Delta V_2$

$$\therefore \quad T = A \rho (V + \Delta V_1)\; \Delta V_2 \qquad \text{Equation 4.4.9}$$

Equating these two equations for thrust we have:

$$A \rho \Delta V_2 \left(V + \frac{\Delta V_2}{2}\right) = A \rho \;(V + \Delta V_1)\; \Delta V_2$$

$$V + \frac{\Delta V_2}{2} = V + \Delta V_1$$

or $$\Delta V_2 = 2 \;\Delta V_1 \qquad \text{Equation 4.4.10}$$

Therefore,half of the velocity increase occurs in front of the disk and half occurs behind the disk.

The propulsive efficiency is obtained in conventional fashion from the ratio of output to input power.

$$\eta_p = \frac{\text{output power}}{\text{input power}}$$

The output power, or useful work per second, is the product of the remote velocity and the thrust, whereas the input power is the product of the thrust and the disk velocity: hence

$$\eta_p = \frac{V A \rho \Delta V_2 \left(V + \frac{\Delta V_2}{2}\right)}{(V + \Delta V_1)\left[A \rho \Delta V_2 \left(V + \frac{\Delta V_2}{2}\right)\right]} = \frac{V}{V + \Delta V_1}$$

or

$$\eta_p = \frac{1}{1 + \frac{\Delta V_1}{V}}$$

or in terms of ΔV_2

$$\eta_p = \frac{1}{1 + \frac{\Delta V_2}{2V}} \qquad \text{Equation 4.4.11}$$

which is consistent with the kinetic energy method of arriving at η_p.

4.4.3 PROPELLER BLADE ELEMENT THEORY

The simple blade element theory treats the propeller as an airfoil and attempts to arrive at a value for thrust by analyzing the forces acting on an infinite number of elements that have a span dr. (See Figure 4.4.3)

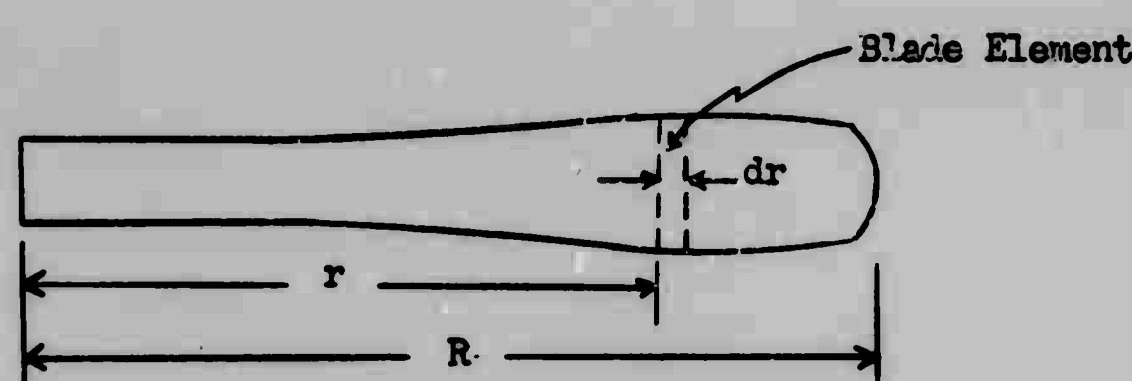
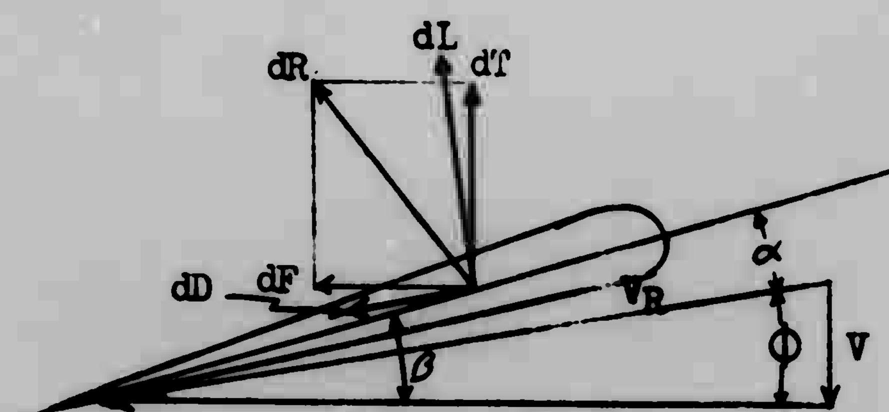


Figure 4.4.3

The angle of attack (α) that each element has is dependent upon the remote velocity of the airplane and the rotational velocity of the particular element. The angle of attack is described as the angle between the relative velocity and the chord of the element just as on an airfoil. The relative velocity (V_R) is the resultant velocity between the airplanes remote velocity and the elements rotational velocity. Figure 4.4.4 shows an element in cross section.



$2\pi r n$ = Rotational velocity where n = RPS

Blade Element Angle & Force Relationships

Figure 4.4.4

Where,

- β is the angle between the plane of rotation and the chord line, degrees
- α is angle of attack, degrees
- Φ is $\beta - \alpha$, propeller helix angle, degrees
- $2\pi r n$ is rotational velocity where n is in RPS.

The differential drag (dD) and lift (dL) vectors produce a differential resultant (dR) which is then resolved into two vectors, differential thrust (dT) and differential torque force (dF). The differential thrust force acts

parallel and opposite to the aircraft's remote velocity vector.

The total thrust is found by integrating the differential thrust force:

$$T = B\int_0^R dT \quad \text{where } B = \text{number of blades} \qquad \text{Equation 4.4.12}$$

Likewise torque (Q) is found by integrating the product of the differential torque force (dF) and the radius (r) at which it acts:

$$Q = B\int_0^R r\,dF \qquad \text{Equation 4.4.13}$$

Non-dimensional propeller coefficients are developed similarly to wing coefficients, with thrust coefficients, and torque and power coefficients corresponding to the drag coefficient; thus

$$T = T_c \rho V^2 D^2 \qquad \text{Equation 4.4.14}$$

where T is thrust, lb

T_c is thrust coefficient, dimensionless

ρ is density slugs/ft^3

V is velocity in ft/sec

D is propeller diameter, ft.

The propeller diameter squared is used to make the T_c dimensionless. The $\frac{1}{2}$ is deleted since dynamic pressure has less significance in propeller analysis than in airfoil calculations.

The thrust coefficient (T_c) could be plotted versus angle of attack as in presenting airfoil characteristics; however, the angle of attack is variable along the blade since it is a function of β, V, r, and n. A more convenient variable is ϕ, because for a praticular β, ϕ defines α. Therefore,

$$\tan \phi = \frac{V}{\pi n D} \quad \text{and since } \phi = (\beta - \alpha)$$

$$\alpha = \beta - \arctan \frac{V}{\pi n D}$$

Therefore,

$$\alpha = f\left(\beta, \frac{V}{n D}\right)$$

The parameter $\frac{V}{n D}$ is called J or advance ratio. J is then usually used as the variable against which propeller coefficients and efficiency is plotted. But, since angle of attack is also a function of β, a β at the three-quarter radius is usually chosen as a reference. Therefore, instead of plotting C_L versus α as is commonly done when presenting airfoil characteristics, C_T which is merely a T_c that is independent of velocity but dependent on RPM is plotted versus J. By defining C_T as equal to $T_c \left(\frac{V}{nD}\right)^2$ then

$$T = C_T \rho V^2 D^2 \left(\frac{n D}{V}\right)^2$$

$$T = C_T \rho n^2 D^4 \qquad \text{Equation 4.4.15}$$

The above equation is the equation normally used to define propeller thrust. By the same reasoning, then

$$F = C_F \rho n^2 D^4 \qquad \text{Equation 4.4.16}$$

and, since torque (Q) is equal to the product of force and distance, then

$$Q = C_F \rho n^2 D^4 R$$

or since $R = \frac{D}{2}$ then

$$Q = \frac{C_F}{2} \rho n^2 D^5$$

By letting $\frac{C_F}{2} = C_Q$ then

$$Q = C_Q \rho n^2 D^5 \qquad \text{Equation 4.4.17}$$

Power coefficients may then be developed in a manner similar to that used for thrust coefficients. Since power is equal to torque per unit time, then.

$$P = Q / \frac{1}{2\pi n} \quad \text{where } 2\pi n = \text{rad/sec}$$

$$P = Q\, 2\pi n$$

or $$P = C_Q \rho n^2 D^5\, 2\pi n$$

$$P = 2\pi C_Q \rho n^3 D^5 \qquad \textbf{Equation 4.4.18}$$

By letting $2\pi C_Q = C_P$ then

$$P = C_P \rho n^3 D^5 \qquad \textbf{Equation 4.4.19}$$

which is the power actually supplied by the engine.

Since propulsive efficiency is the ratio of the power output to the input. Then

$$\eta_p = \frac{TV}{P} \qquad \textbf{Equation 4.4.20}$$

$$\eta_p = \frac{(C_T \rho n^2 D^4) V}{C_P \rho n^3 D^5}$$

or $$\eta_p = \frac{C_T}{C_P} \cdot J \qquad \textbf{Equation 4.4.21}$$

From the above it is obvious that propeller efficiency is a function of C_T, C_P and J.

To show the similarity between airfoil and propeller characteristics, note the following plots depicting common propeller curves and their airfoil counterparts.

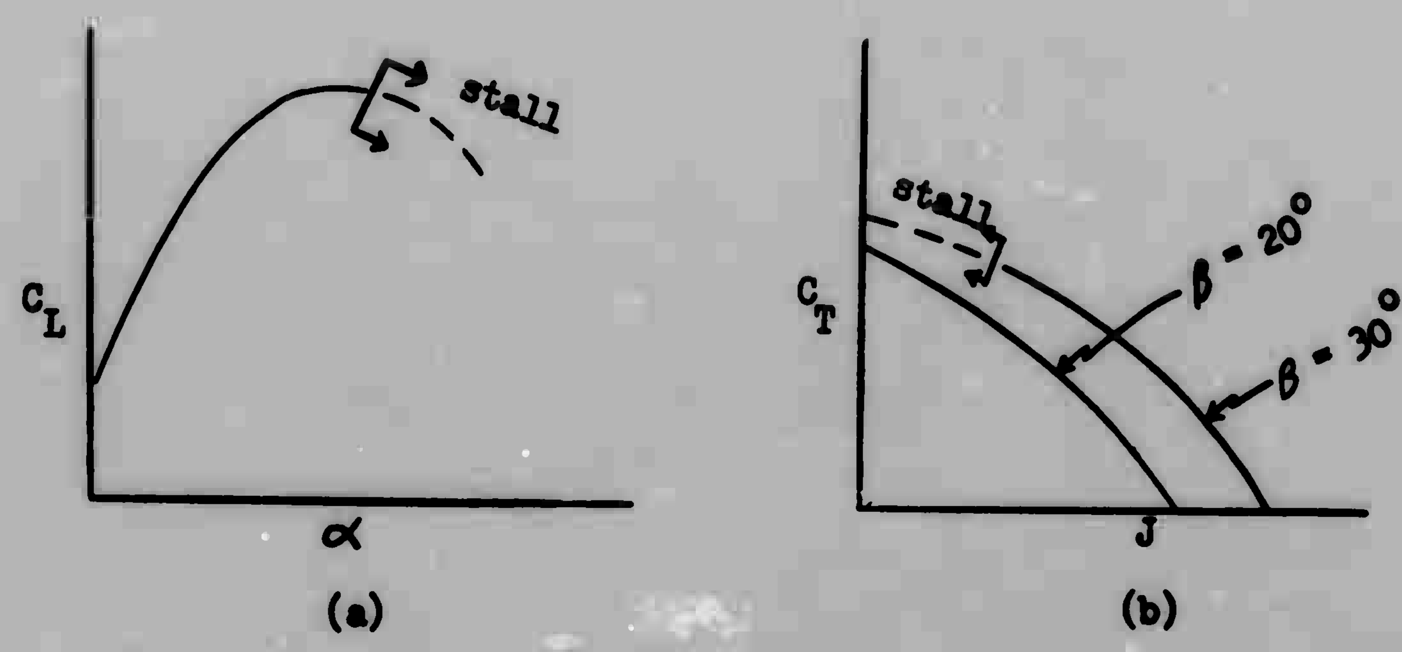


Figure 4.4.5

From the above plots it can be noted that the C_T curves are essentially C_L curves plotted backwards, also when $C_T \rightarrow 0$, $C_L \rightarrow 0$ and when $C_T \longrightarrow C_{T_{max}}$, $C_L \longrightarrow C_{L_{max}}$.

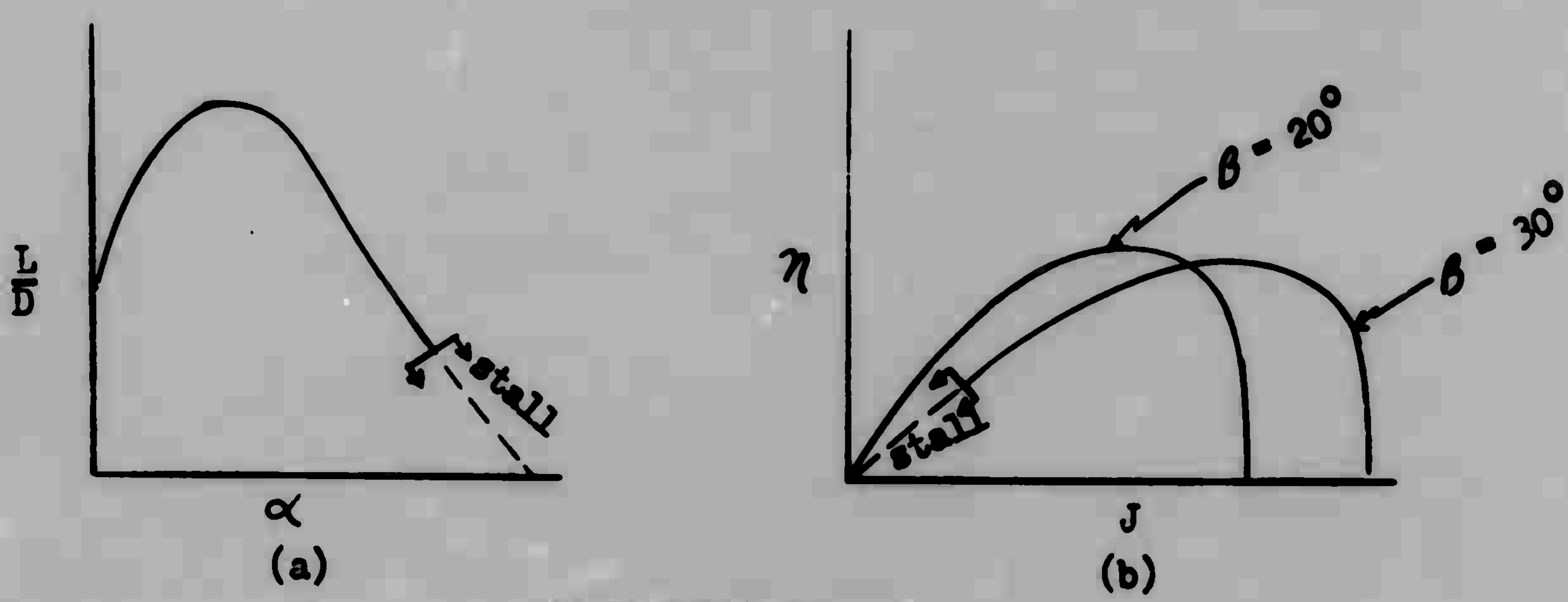


Figure 4.4.6

From the above plots it can be noted that the efficiency curves are similar to curves of L/D that have been plotted backwards like the C_T curves

and when $\eta \longrightarrow 0$ at high J, $L/D \longrightarrow 0$ at low α , and when $\eta \longrightarrow 0$ at low J, $L/D \longrightarrow 0$ at high α ; furthermore when $\eta \rightarrow \eta_{max}$, $L/D \rightarrow (L/D)_{max}$.

Comparative Coefficients and Parameters for Airfoils and Propellers.

Propeller Characteristic	Airfoil Characteristic
J and ϕ	α
η	L/D
C_T	C_L
C_P	C_D

4.5 TURBOPROP ENGINES

4.5.1 GENERAL

The turboprop power plant can be considered as a turbojet engine combined with a propeller. The difference between the two is the propeller and turbine sections. The turbine of a turboprop is designed so as to absorb enough additional energy from the high temperature gases to drive the propeller shaft through the reduction gears on the extension of the shaft through the front of the compressor. The air passing through the turbine is then expanded through the nozzle to near atmospheric. Some jet thrust may be obtained through the exhaust to help power the aircraft.

4.5.2 TURBOPROP ADVANTAGES:

a. Simplicity of design and construction .

The turboprop has much less moving parts than the reciprocating engine; therefore, less parts to wear out and need replacing.

b. Low specific weight. The specific weight of the turboprop may run as low as .5 while a reciprocating engine may run as high as 1.00 to 1.75.

c. Low fuel consumption. The specific fuel consumption on a turboprop runs as low as .4 and further development could run it even less.

d. Low drag installation. The parasite drag of the turboprop is very low due to the small frontal area.

e. Operational flexibility. The turboprop operates well at take-off, high altitude (30,000 to 35,000 feet maximum) and speeds up to 400 to 500 knots.

f. Power response to throttle movement is more rapid than a turbojet.

4.5.3 TURBOPROP DISADVANTAGES

a. High Speed. Being equipped with a propeller, the turboprop is limited by high speeds in the same way as the reciprocating engine.

b. Vibration. The structural vibration with a turboprop installation is usually excessively high. This should be expected when one considers the high horsepower output of turboprops that are being installed on present day aircraft. For example, fuselage vibrations are primarily influenced by the propeller blade tip to fuselage clearance. For a satisfactory vibration level, this tip clearance can be expressed as follows:

$$L = f\ (HP,\ RPM,\ N,\ D.)$$

where

HP = Horsepower output of engine

RPM = Propeller speed

N = Number of propeller blades

D = Propeller deck diameter

The aircraft laboratories at Wright-Patterson AFB developed an empirical equation for this relationship that proved acceptable for all aircraft up to and including World War II types. When this relationship was applied against the C-133, it was found that the propeller tip clearance should have been in excess of five feet. The C-133 tip clearance is actually about 15 inches. When this equation was applied to the XC-132, which was never flown, it was found that the inboard engine would have to be installed beyond the wing tips.

These clearances of course are not realistic. The only answer appears to accept the high vibration levels, and use every means possible to reduce them. At the same time, all effected aircraft structure must be designed to absorb the energy of vibration without fatiguing the structure to the failure point.

4.5.4 TYPES OF TURBOPROP ENGINES

There are many types and configurations of turboprop engines but only a few will be discussed here. To start with there is the simplest of all turboprops which is nothing but a single stage turbojet with a propeller installation. The turbine section is enlarged to allow enough power to be delivered to run the propeller. This system is sometimes called a "direct drive" turbine. Figure 4.5.1 shows this example.

Direct Drive Turbine

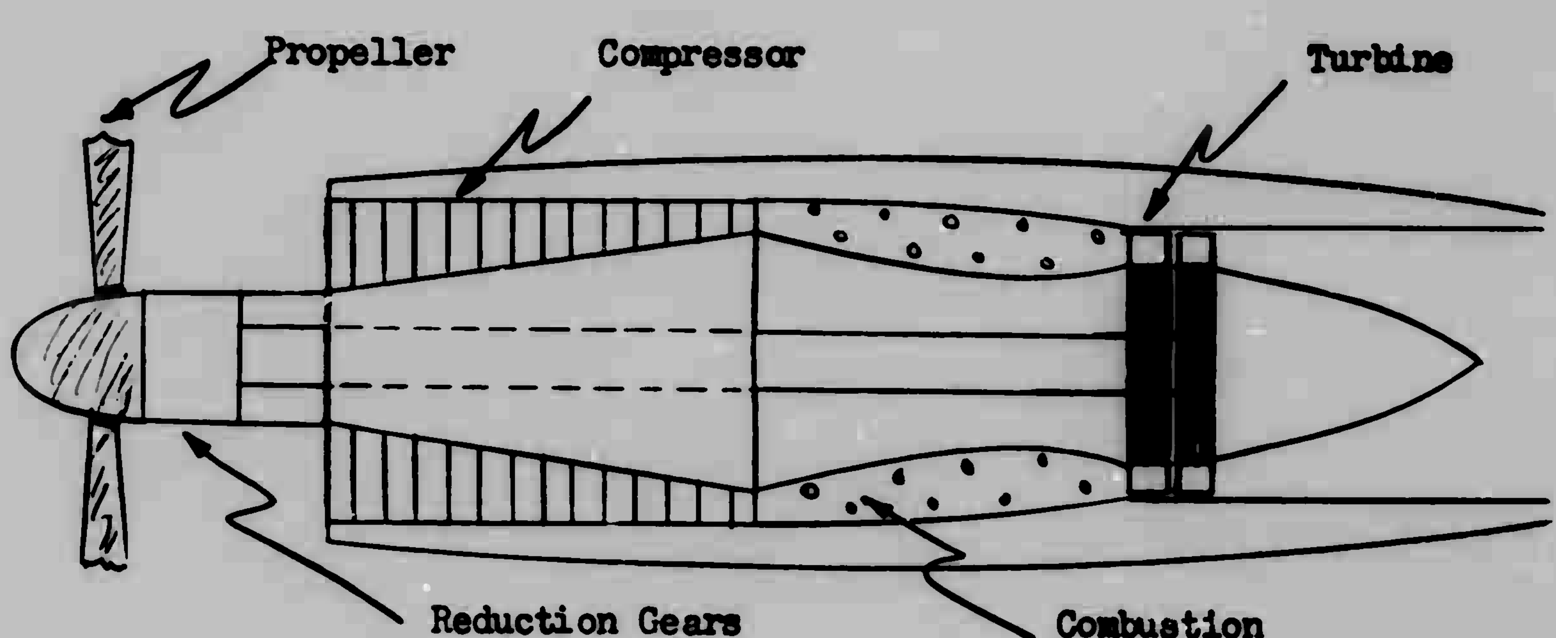


Figure 4.5.1

Another version which is similar to the example in Figure 4.5.1 is the "free turbine" or "seperate drive" turboprop. This engine has one stage of the turbine driving the compressor directly by a hollow shaft. Another stage of the turbine drives the propeller by a shaft within the hollow shaft. This type engine is used in the HU-1 helicopter. Figure 4.5.2 shows a simplified version of this engine

Free or Seperate Drive Turbine

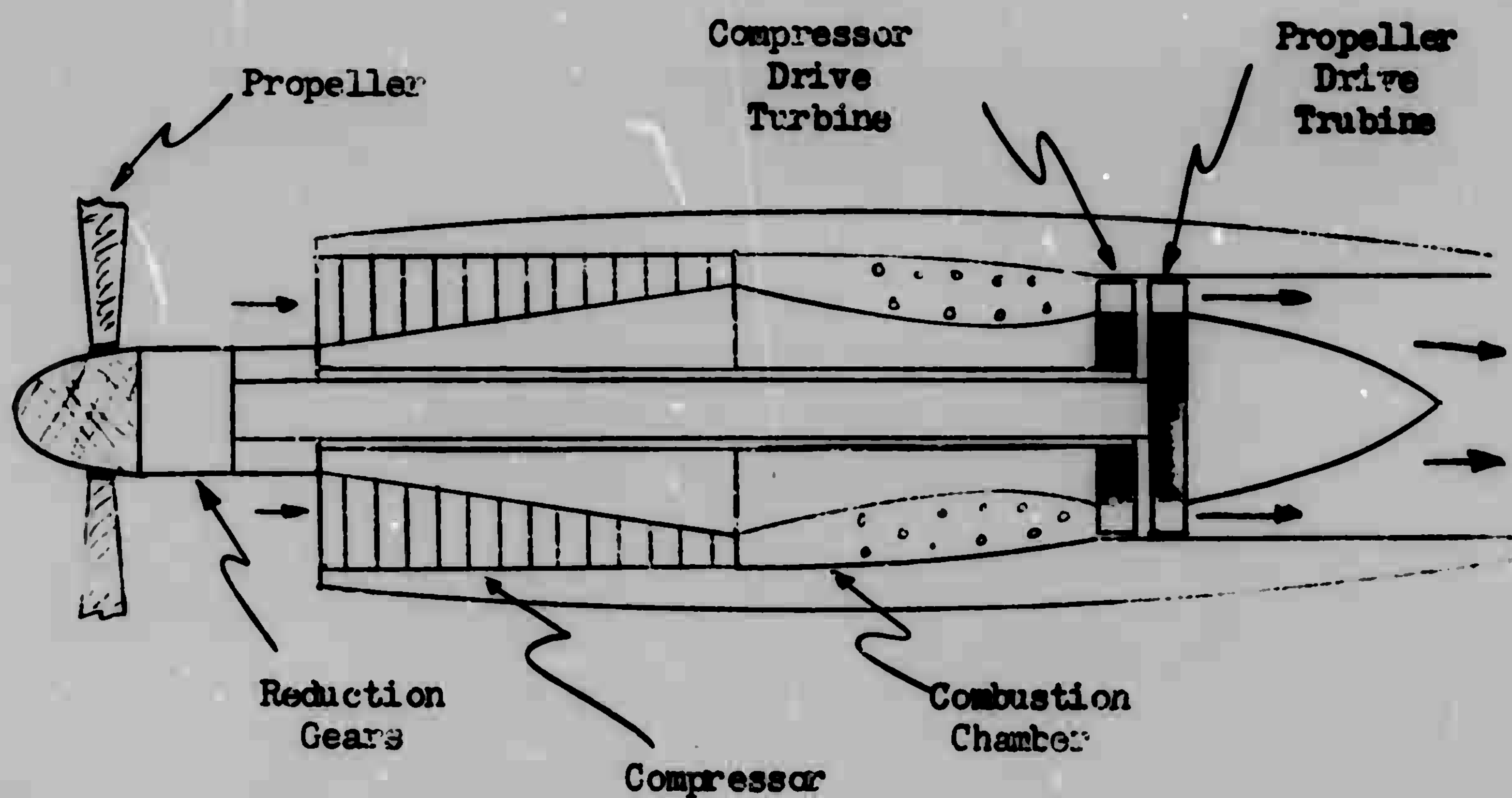


Figure 4.5.2

The T-57 engine is typical of the tandem compounded turboprop. This engine has a low pressure and a high pressure compressor with a separate shaft and turbine for each. As in a twin spool turbojet engine these compressors are not connected mechanically. This engine is shown in Figure 4.5.3.

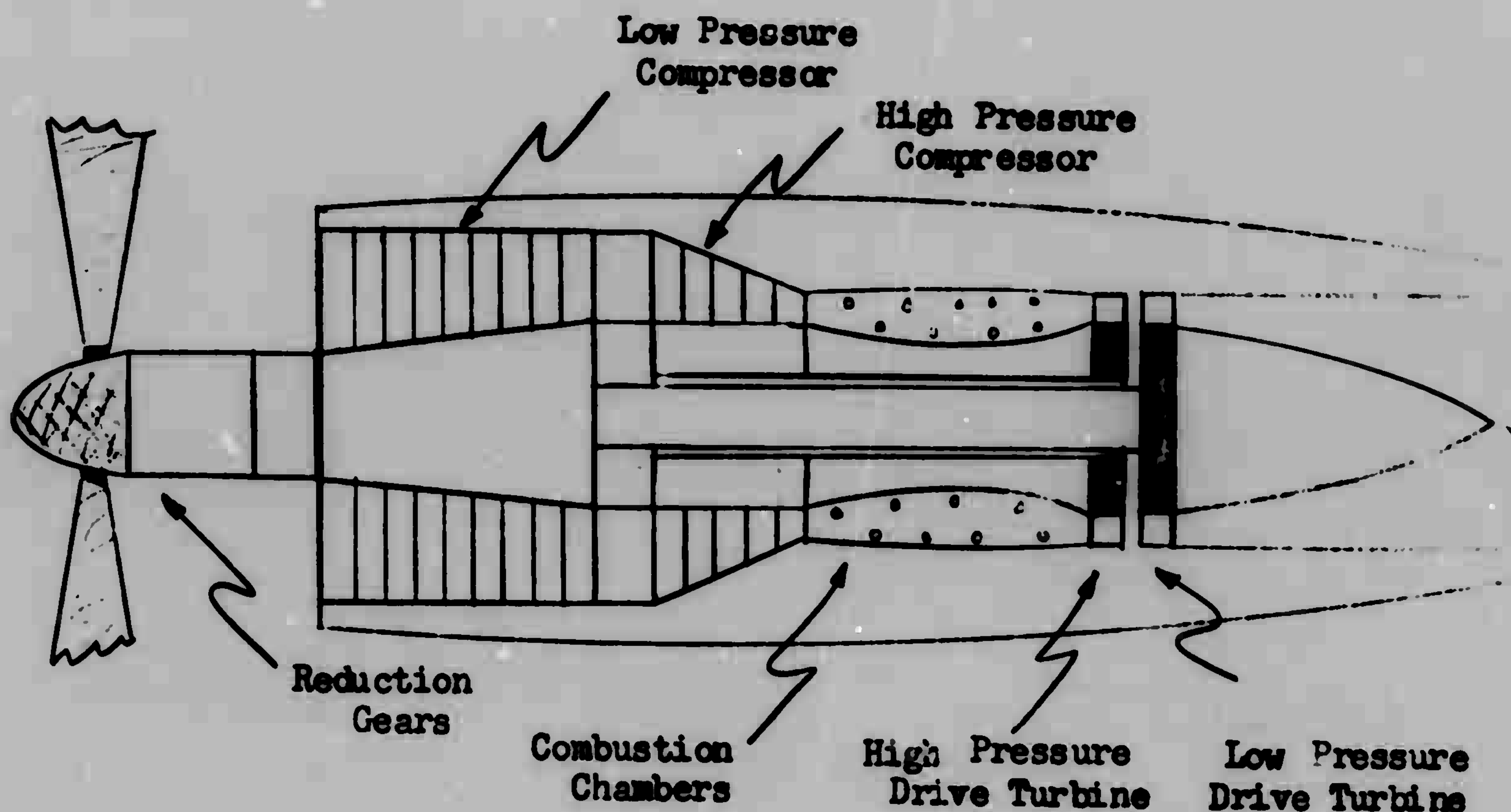


Figure 4.5.3

The engine which is being used more and more by the jet airlines is called the tubo-fan engine. This is nothing but a turboprop with the propellers being housed inside the compressor casing. This system uses the first few stages of the compressor, or maybe some additional stages, as the propeller. These blades extend out into an angular duct that bypasses some of the intake air around the engine. This bypassed air is discharged through a nozzle arrangement the same as the hot jet gases. The propulsive efficiency of a turbofan engine is usually higher than the turbojet because cold air reduces the temperature and increases the mass flow of the jet exhaust. Figure 4.5.4 shows the turbo fan principle.

Ducted Fan Engine

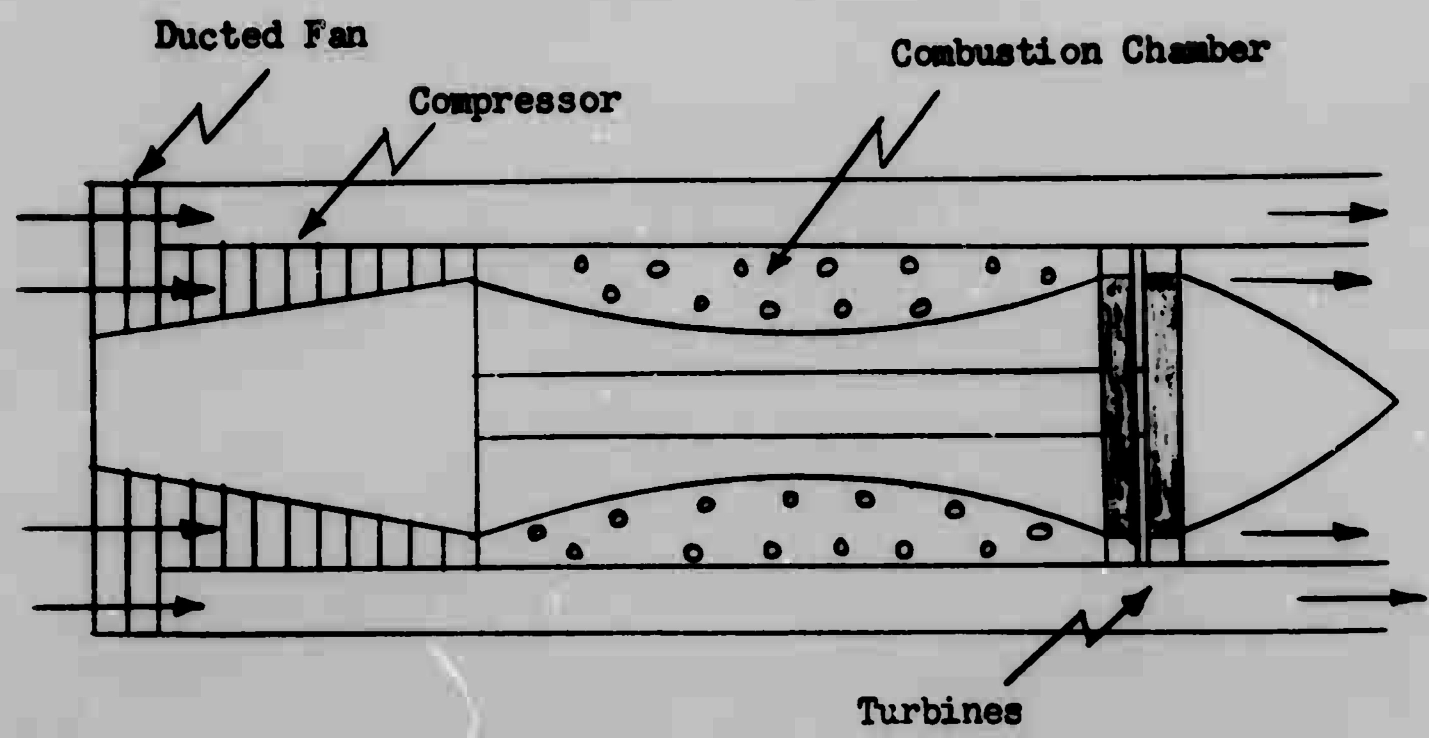


Figure 4.5.4

Another example of the turbofan arrangement is the General Electric CJ-805-23 aft fan engine. This engine, forward of the turbine section, is the same as the CJ-805 turbojet. From here on back to the end of the tailpipe is where the two differ. The CJ-805-23 aft fan engine has an aft fan attached to the turbine stator casing rear flange. It consists of a single stage free floating rotor which is part compressor and part turbine; and a front and rear frame incorporating concentric angular flow passages.

Gases flowing aft from the turbine section flow into the inner passage of the fan front frame and through the partitions of the fan turbine nozzle. The nozzle directs the gases, at the correct angle of attack, to the turbine airfoil of the rotor blades. The gases impart energy to the blades to drive the fan rotor and then flow through the inner passage of the fan rear frame and into the exhaust section.

Secondary air flows through the outer passage to the outer section of the fan. The fan compresses the air at a ratio of 1.6 to 1.0. The air then flows through a row of guide vanes which straightens the direction of flow and then is exhausted along with the primary flow to provide additional thrust. Secondary airflow through the fan is about 1.5 time that of the primary airflow. Below is a simplified drawing of an aft fan type engine.

Aft Fan Engine

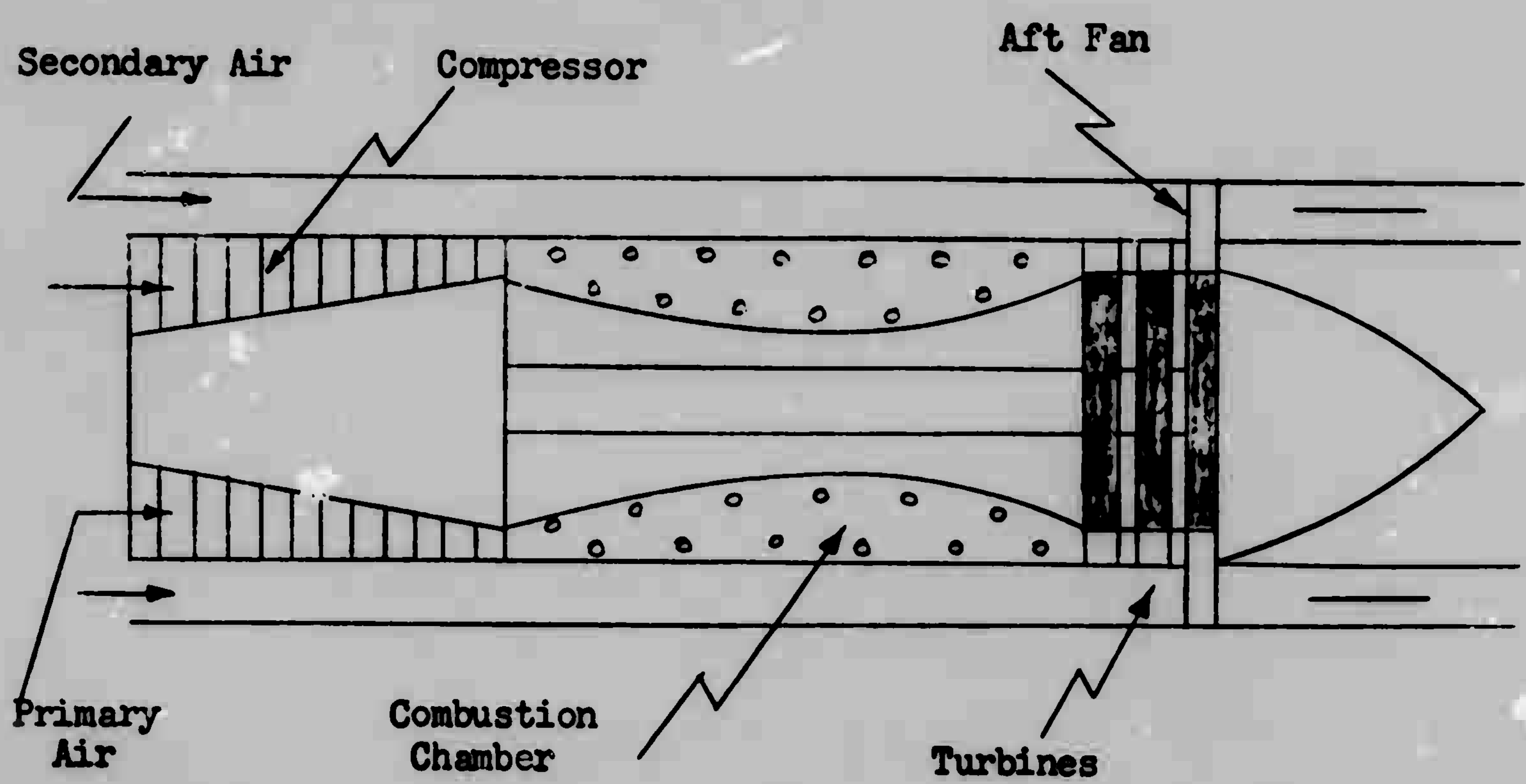


Figure 4.5.5

4.5.5 REDUCTION GEARS

Since the operating RPM range of most gas turbine engines is 8,000 to 12,000 and more, a reduction gear between the propeller and the engine must be used so lower RPM may be obtained. The reduction gears used are similar to those used on a piston engine; however, because of the greater reduction, epicyclic gearing, or a combination thereof and spur gearing are used. On engines which use a counter-rotating propeller, spur gears are used to transfer the driving torque to the rear and front propeller, with some reduction being obtained in this way.

4.5.6 CONTROLLING POWER IN A TURBOPROP ENGINE

The turbojet power output is determined and controlled by rate of fuel flow into the burners. The turboprops power output may be varied by altering the propeller pitch and hence the amount of power it can absorb at a given RPM. Generally the propeller is operated at a constant speed during all flight power settings. Both the fuel flow and the propeller pitch are controlled by a single lever. In cruising flight the engine runs at a constant RPM, usually maximum continuous, with the constant speed unit governing the propeller at constant RPM by changing pitch to adjust to the torque from the engine. When the throttle is opened, the fuel flow is increased and at the same time the pitch is increased to absorb the higher power. Closing the throttle reduces the fuel flow and the pitch is decreased to maintain RPM.

When the throttle is opened past the maximum cruising condition to take-off power, the fuel flow is increased and the constant speed unit is altered to allow the engine RPM to increase to take-off RPM. This is normally obtained by decreasing the propeller pitch, thus allowing the engine to accelerate to the

BLANK PAGE

desired RPM. There is a tendency for the engine RPM to overshoot the take-off RPM but this is taken up by the engine overspeed governor which limits the RPM by reducing the fuel flow,and thus the power. To prevent this loss in power at a time when the throttle is being opened, an <u>anticipator mechanism</u> is put between the fuel control and the governor. The anticipator delays any change to the basic constant speed unit setting when the throttle is opened rapidly. This ensures that the RPM selected by the constant speed unit is slightly lower than the indicated RPM. This will ensure a steady increase in propeller pitch until at full throttle the correct pitch is obtained to absorb the take-off power.

4.5.7 ENGINE FAILURE

When a failure occurs on a reciprocating engine, the constant speed unit moves the propeller RPM to full decrease or about a 25° angle. The propeller will windmill until feathered. On the turboprop the same thing will happen but the propeller will reduce pitch to about 8 to 12 degrees. This would present a maximum frontal area and could result in an extreme hazard. To prevent this very undesirable state of affairs, a reverse torque switch in the reduction gear operates whenever torque reverses, i.e., when the propeller tends to drive the engine. The reverse torque switch overrides the constant speed unit and causes the feathering motor to reduce RPM, thereby reducing the high windmilling drag. On occasions a momentary period of reverse torque may be encountered, however, when the pitch adjusts itself after a moment, feathering action is halted and normal operation is resumed. The engine may be feathered by the pilot at anytime; this usually entails closing the throttle and energizing the feathering motor.

4.5.8 POWER OUTPUT OF A TURBOPROP

The average power-weight ratio of a turboprop engine is about 50% better than that of the reciprocating engine at sea level. Up to its rated height (critical height) the power of a reciprocating engine remains virtually constant (actually increases slightly) while the gas turbine power decreases linearly. This greater power per engine pound allows a greater payload for the turbine engine of the same weight as the reciprocating engine.

The power developed by a turboprop engine is normally given in terms of equivalent shaft horsepower (eshp). Inside the gas turbine the nozzle guide vanes lead the gas flow to the turbine at a set direction to the plane of the turbine rotor. The velocity (V) of the gas flow has two components, V_1 which generates shaft energy and V_2 which generates propulsive energy. In a turboprop engine the aim is to increase V_1 and minimize V_2 (reverse for a turbojet). Therefore, the basic design of the engine is different from that of a turbojet. To drive a propeller the total work of the gas turbine is taken out as equivalent shaft horsepower instead of kinetic energy as the turbojet. The propulsive efficiency is at its maximum at speeds below 450 knots. Above this speed propeller efficiency drops off because of Mach effects. The proportion of thrust between propeller and jet is about 90% propeller and 10% jet. Equivalent shaft horsepower may be written as follows:

$$ESHP = \frac{THP}{\eta_p} = BHP + \frac{\text{net jet power}}{\eta_p} \qquad \text{Equation 4.5.1}$$

where

η_p = propeller efficiency

$$\frac{\text{net jet power}}{\eta_p} = \frac{\text{Thrust}}{550} \frac{X}{X} \frac{\text{Velocity}}{\eta_p}$$

4.6 RAMJET ENGINES

4.6.1 INTRODUCTION

The ramjet is an air breathing engine which has the same basic operating cycle as the turbojet. It compresses the air by ram pressure, adds heat at a high pressure, converts the heat energy to velocity which produces the thrust. The ramjet is able to operate without a mechanical compressor by converting the kinetic energy of the incoming air into pressure. This makes the ramjet the simplest of all engines because there are no moving parts. The disadvantage of the engine is that it will not operate statically; therefore, it requires some means of boosting it to a speed in the region of 300 knots. The engine is capable of operating at very high thrust and very efficiently at supersonic speeds. The power output could be as high as 100 horsepower per pound of engine weight. The ramjet is sometimes referred to as an athodyd engine derived from the words "Aero-THermODYnamic-Duct".

The Figure 4.6.1 shows a simple drawing of a ramjet and its components with relative pressure,velocity, and temperature variations throughout the engine.

Ramjet, Pressure, Velocity and Temperature Variation

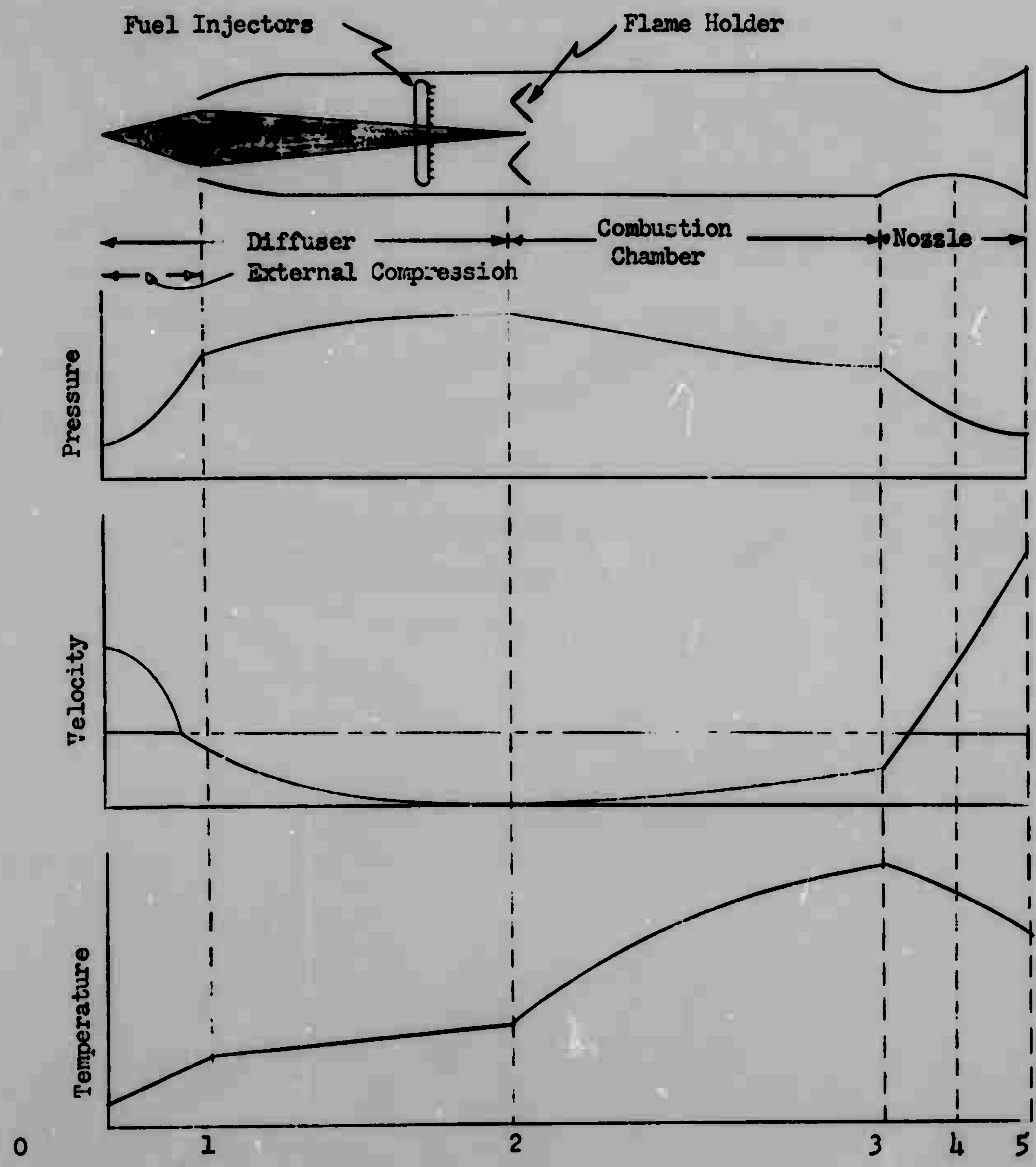


Figure 4.6.1

4.6.2 PRINCIPLES OF OPERATION

The ramjet consists of a diffuser, fuel injector, flame holder, combustion chamber and deLavel Nozzle. In referring to Figure 4.6.1 it can be seen that the pressure will rise between station 0 and the flame holder. This is normally accomplished by two stages. First, the external compression due to the air changing direction. Second, further compression is obtained in the diverging section of the diffuser due to the air slowing down. The velocity is the lowest at the flame holder, which is favorable to flame propagation, and this is where the burning takes place. The burning mixture recirculates within the sheltered area and ignites a new charge as it passes the edge of the flame holder. As the burning gases pass through the combustion chamber the temperature is raised, thus increasing the volume. As the volume is increased the gases must speed up to allow room for a new charge of air. The velocity of the air is again increased as it passes through the converging-diverging nozzle. As the gases leave the exit nozzle, the temperature is many times higher than that of the entering air and as a result so is the velocity.

4.6.3 THRUST

The gross thrust, F_g, is the actual thrust of the jet stream. The gross thrust is the product of the mass gas flow W_5/g, and its effective velocity V_j.

$$F_g = \frac{W_5 \ V_j}{g} \qquad \text{Equation 4.6.1}$$

In the case of the ramjet the gross thrust is meaningless because it is never obtained, but is useful in understanding ramjet performance. The net thrust, F_n, is the F_g minus the momentum drag imposed by accelerating the air swallowed by the engine to the forward velocity of the engine or ram drag.

$$F_n = F_g - \frac{W_o V_o}{g} = \frac{W_5 V_j}{g} - \frac{W_o V_o}{g} \qquad \textbf{Equation 4.6.2}$$

The gross thrust can be represented by the curve in Figure 4.6.2

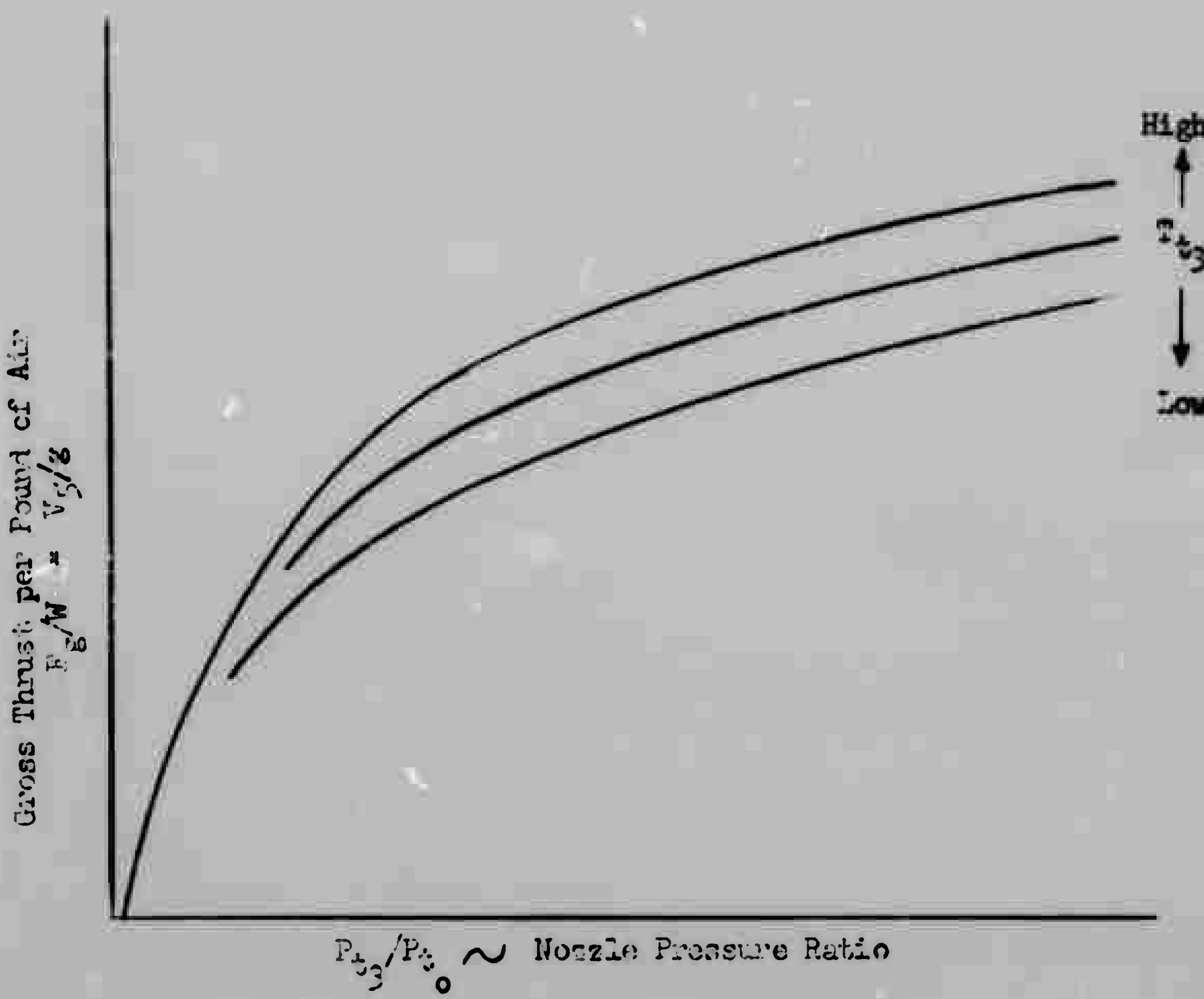


Figure 4.6.2

This curve shows that as the pressure ratio, P_{t_3}/P_o, across the nozzle is increased the thrust increases also. Also, as the combustion temperature is increased the thrust per pound of air also increases. To determine the F_n which acts on the aircraft, the ram drag must be subtracted from the gross thrust.

If we assume that the inlet weight flow, W_o, is equal to the outlet weight flow, W_j, we can construct the curve in Figure 4.6.3. Only one combustion temperature is shown.

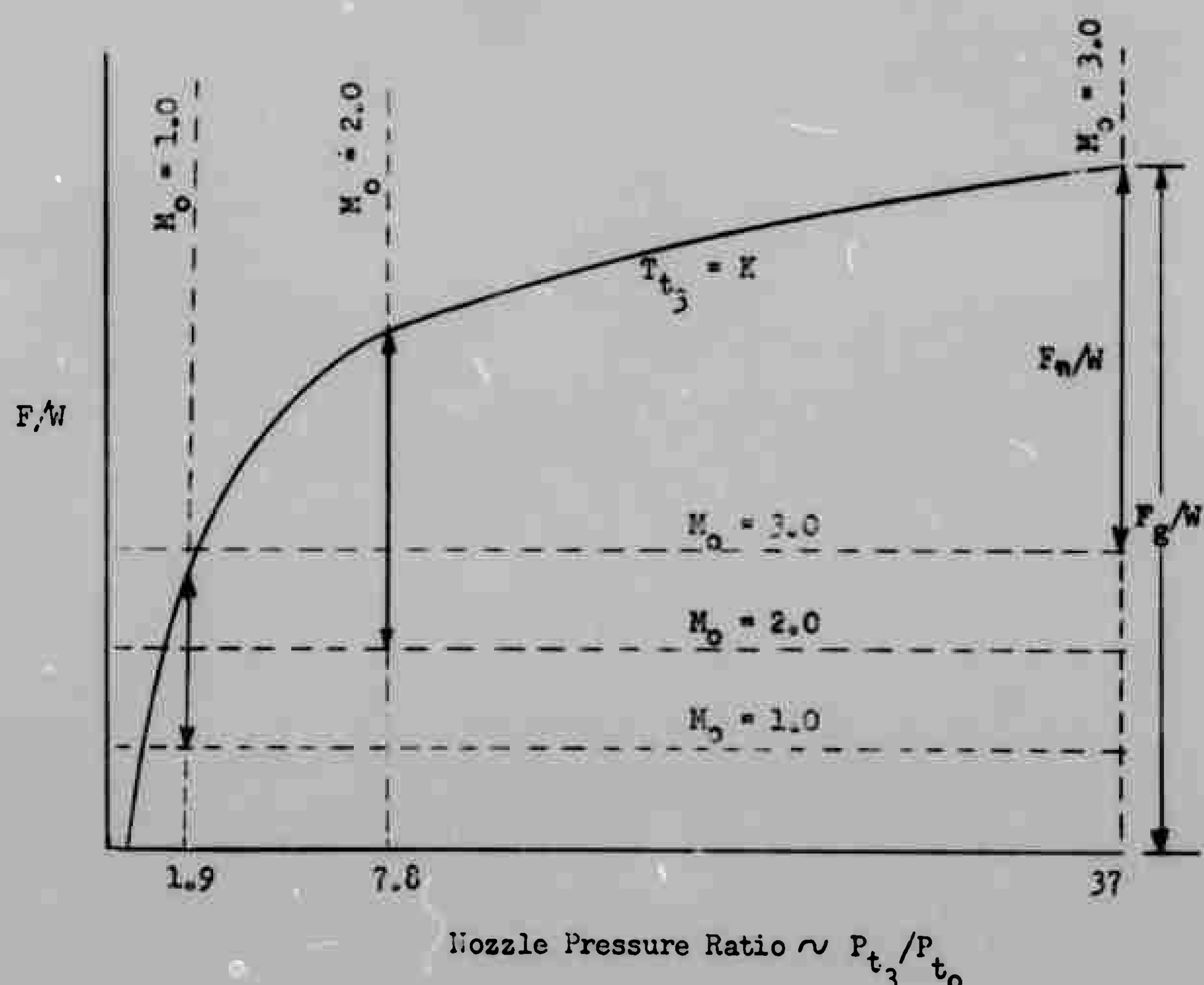


Figure 4.6.3

In the case above, the ram drag per pound of air flow is equal to V_o/g and is shown by lines of constant flight speeds. The lines can also be shown as constant Mach numbers if the engine is operating in an isothermal region. These are represented as horizontal dashed lines. If there were no pressure losses in the engine, the pressure ratio across the nozzle would be equal to the ram pressure ratio due to the forward speed. Some theoretical pressure ratios are:

MACH	RAM PRESSURE RATIO
0	1.00
1	1.89
2	7.83
3	37.00
4	157.00

These values are added to the curve, vertical dashed lines, in Figure 4.6.3 and the ideal gross thrust may be obtained as shown . This procedure may be repeated at different values of combustion temperature.

Figure 4.6.3 shows that the thrust output, F_n/W, at subsonic speeds is low. At a given combustion temperature, F_n/W increases with speed until at about 2.5 to 3.0 Mach it will start to decrease. This is because of the great increase in ram drag at high speeds as shown in Figure 4.6.4.

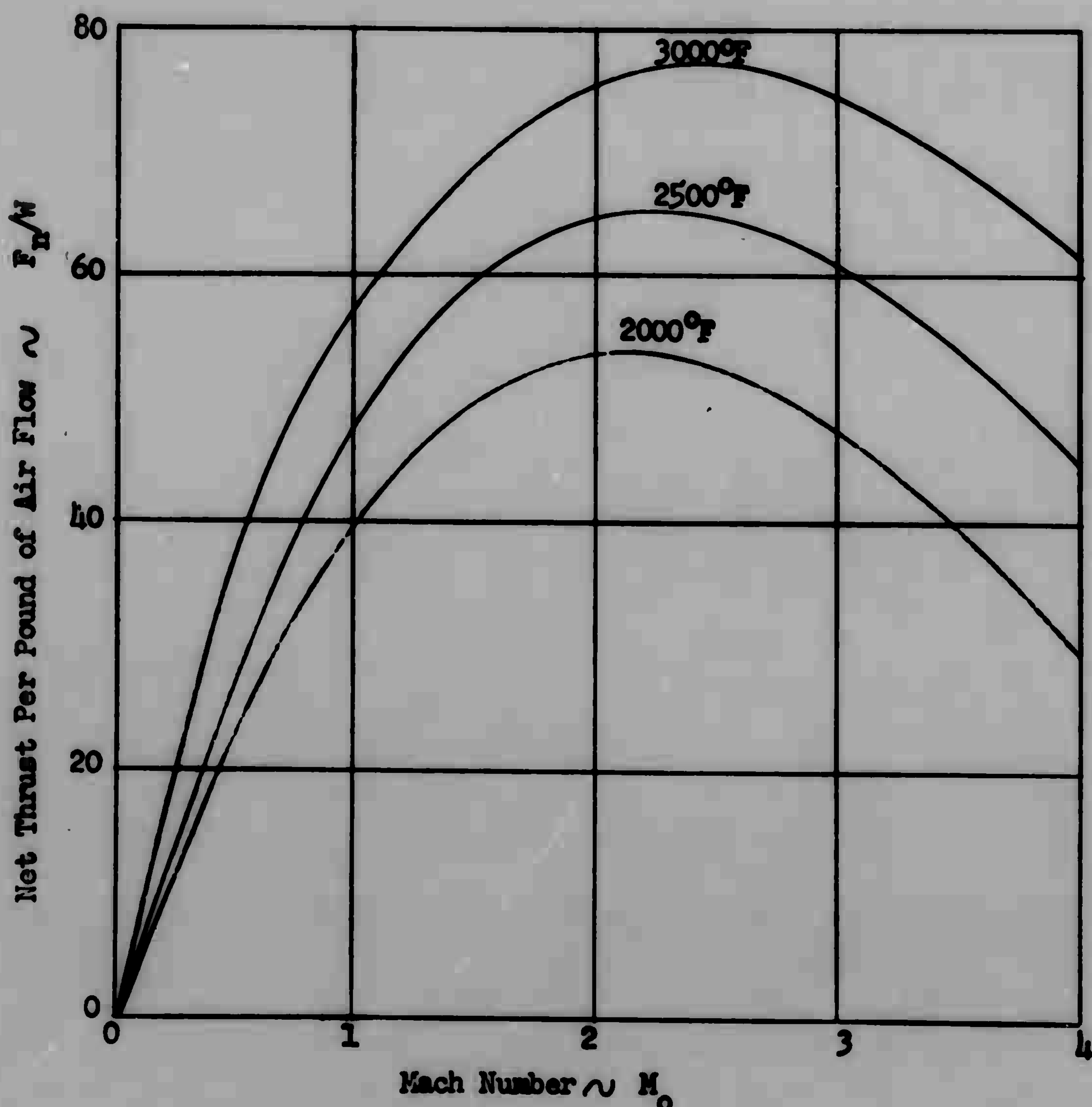


Figure 4.6.4

4.6.4 LOSSES DUE TO INEFFICIENCIES

The ideal performance cannot be obtained from a ramjet for several reasons. Some of these are:

(a) Ram pressure ratio not fully realized in the inlet. This will normally reduce the actual pressure ratio resulting in increased specific fuel consumtion and reduced thrust.

(b) Pressure loss across flame holder. This will reduce the pressure ratio and increase specific fuel consumption while reducing thrust.

(c) Pressure loss due to burning. This will result in the same as a and b above.

(d) Combustion not perfect. This will increase fuel consumption required to produce the required combustion temperature.

(e) Theoretical jet thrust not realized at a given pressure ratio. This will reduce the thrust per pound of air at the pressure ratio causing an increase in fuel consumption and a reduction in thrust.

4.6.5 INLET DIFFUSERS

In the ramjet, especially at supersonic speeds, the airflow at most operating points is established by the inlet. Since most ramjets do not incorporate variable inlets which can adjust the air flow to the requirements of the engine, optimum performance cannot be obtained at all operating conditions. The purpose of any supersonic inlet is to convert kinetic energy of the air approaching the inlet to a high pressure by efficiently slowing the air to subsonic speeds. The physical significance for subsonic flows is that for any initial Mach number it is possible to accelerate the flow to the acoustic speed by the addition of heat. If more heat is added than is necessary to reach sonic speed in the flow, the effect is to lower the initial Mach number. This is known as "choked flow". If more heat is added, the flow is literally pushed back out of the duct and sufficient spill over occurs at the duct inlet to allow the initial Mach number to satisfy the flow and heat conditions. This condition is known as "sub-critical" operation. This is shown in Figure 4.6.5.

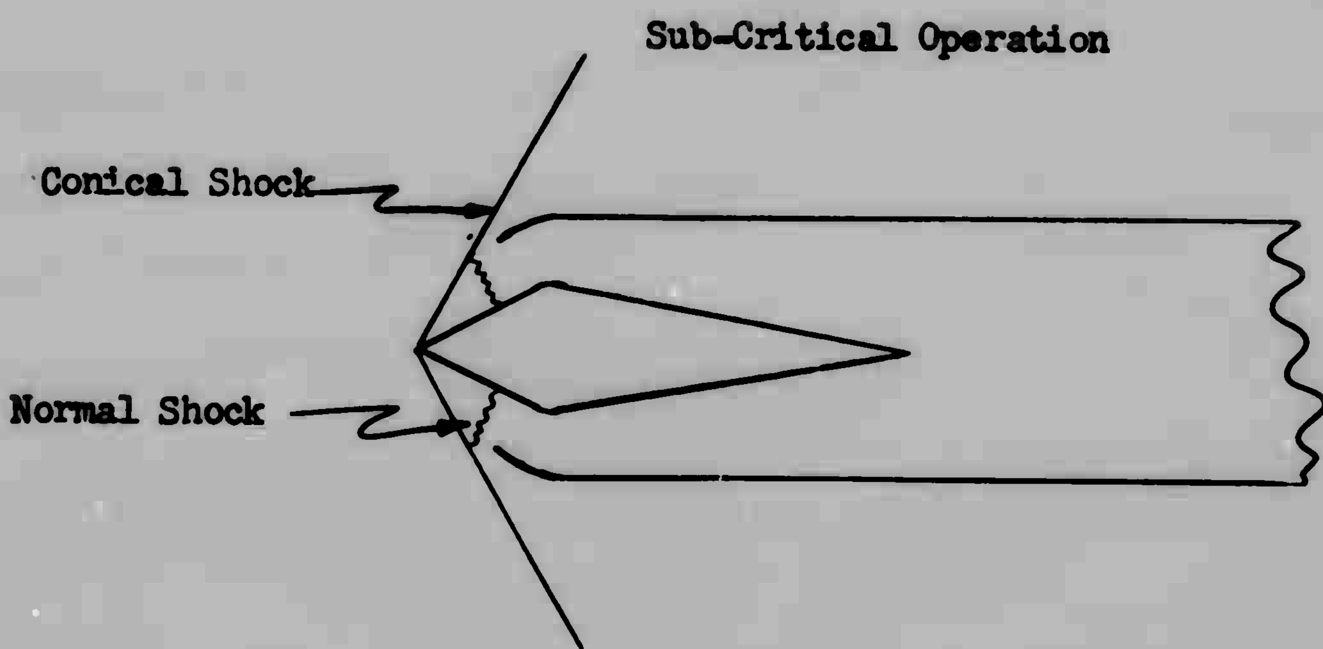


Figure 4.6.5

Figure 4.6.6 below shows the design condition known as "critical" operation. This gives a Mach number of one at the burner outlet, but if additional heat is added in order to satisfy the flow requirements by lowering the burner inlet Mach number, it tends to become sub-critical as shown in Figure 4.6.5. Thus, changing the amount of heat added has effectively changed the inlet configuration since it has caused a change in the shock-wave angle. It should be kept in mind that it is possible to choke the flow by the addition of heat only if the maximum temperature rise available from the fuel is equal to or greater than that required by the flow conditions to give a Mach number of one at the burner exit.

Critical Operation

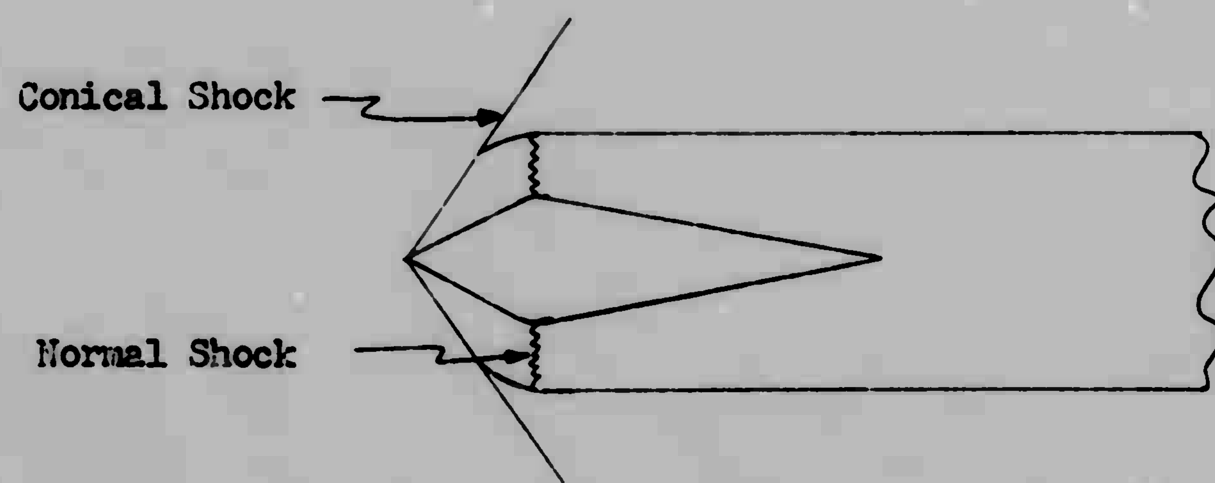


Figure 4.6.6

The next figure, 4.6.7, shows a condition known as "supercritical operation". This condition occurs when the Mach number is much greater than the design condition. The shock wave angle decreases and a "swallowed shock" takes place. This has the disadvantage of changing the flow conditions in the inlet with attendant losses. The air flow during a super critical operation remains constant even if the back pressure is reduced.

Supercritical Operation

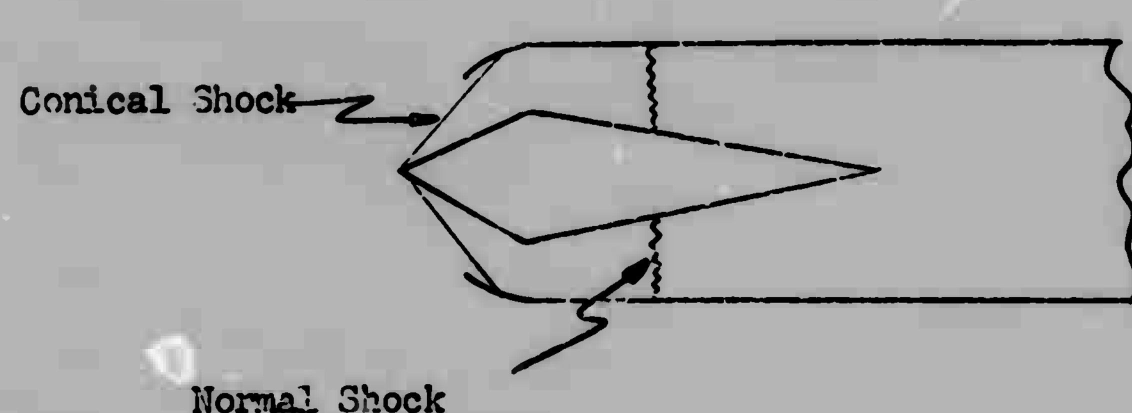


Figure 4.6.7

Unstable flow could be expected if a constant fuel flow greater than that necessary to choke the flow is maintained. This unstable flow is referred to "diffuser buzz". Let us assume that a ramjet operating at a constant Mach number requires a fuel air ratio of .05 to maintain Mach one at the burner exit but the fuel flow is constant and greater than that required for the fuel air ratio of .05. In this case, as mentioned earlier, the heat addition would lower the initial Mach number and mass flow causing sub-critical operation. With this reduction in mass flow and with a constant fuel flow, the fuel-air ratio would continue to increase until it passed the value of maximum temperature rise as shown in the Figure below.

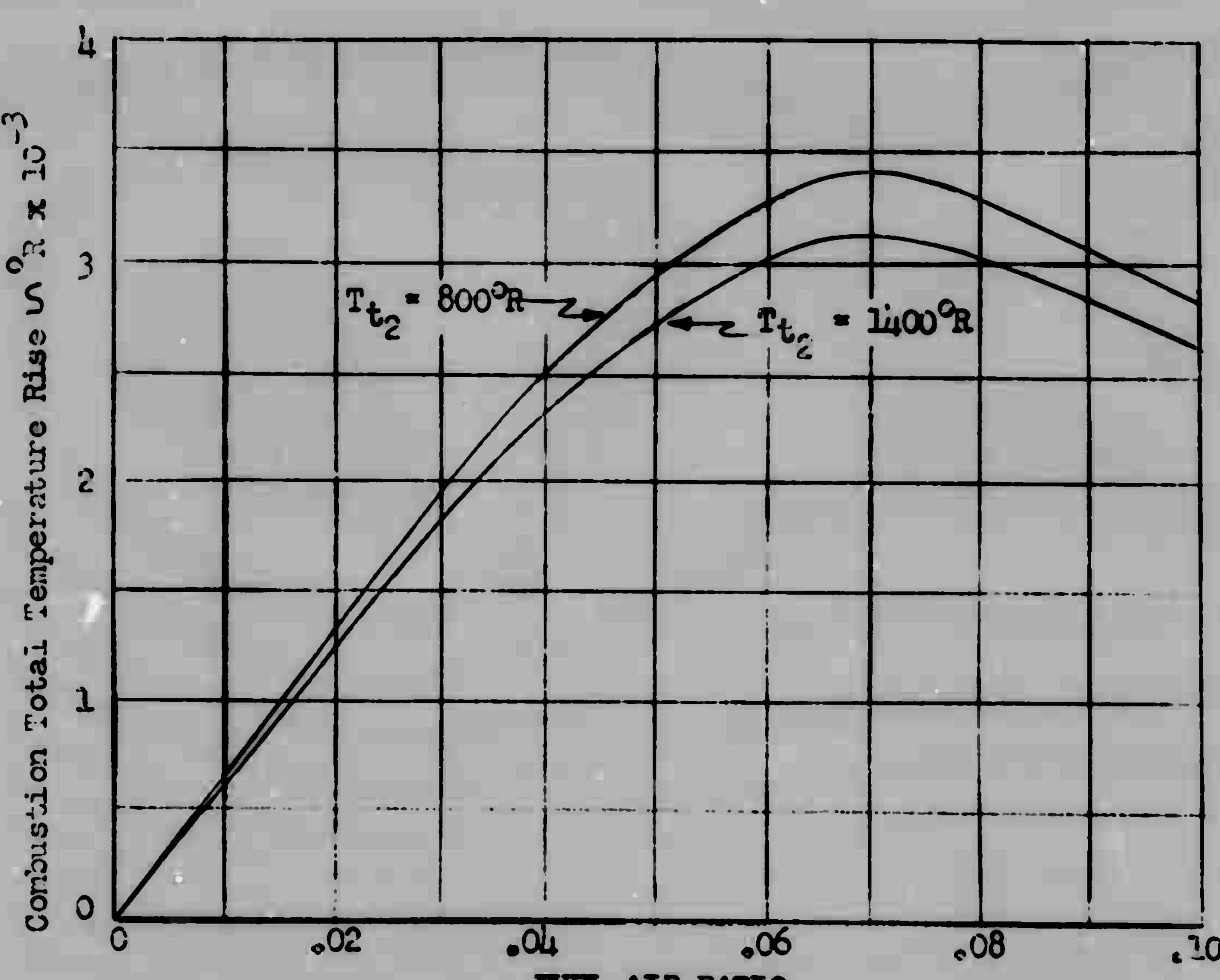


Figure 4.6.8

The temperature would tend to decrease once past the peak allowing the inlet Mach and mass flow to increase. This would decrease the fuel-air ratio,thus repeating the process. This "buzz" could cause combustion failure and be detrimental to engine thrust because of the variation in the mass flow.

There is sonic velocity in the exhaust nozzle of a supersonic ramjet whether or not the minimum exhaust nozzle area corresponds to the burner exit. In this case,the burner exit Mach number is fixed by the ratio of the burner to the jet nozzle and the "buzzing" could still take place. The fuel control of a ramjet engine, which would give the proper fuel air-ratio at all times, has always been a major problem in development.

4.6.6 COMBUSTION SECTION

The purpose of the combustion chamber is to raise the temperature of the air by burning fuel. This is accomplished as explained previously, but there is also a pressure drop across the combustion chamber. The combustion in a ramjet is continuous. When the temperature is increased as the air passes through the combustion chamber the air will also speed up. This causes a pressure drop. There is also a pressure drop due to friction as in the turbojet. The pressure drop across a combustion chamber with a fixed nozzle exit may look as shown in Figure 4.6.9.

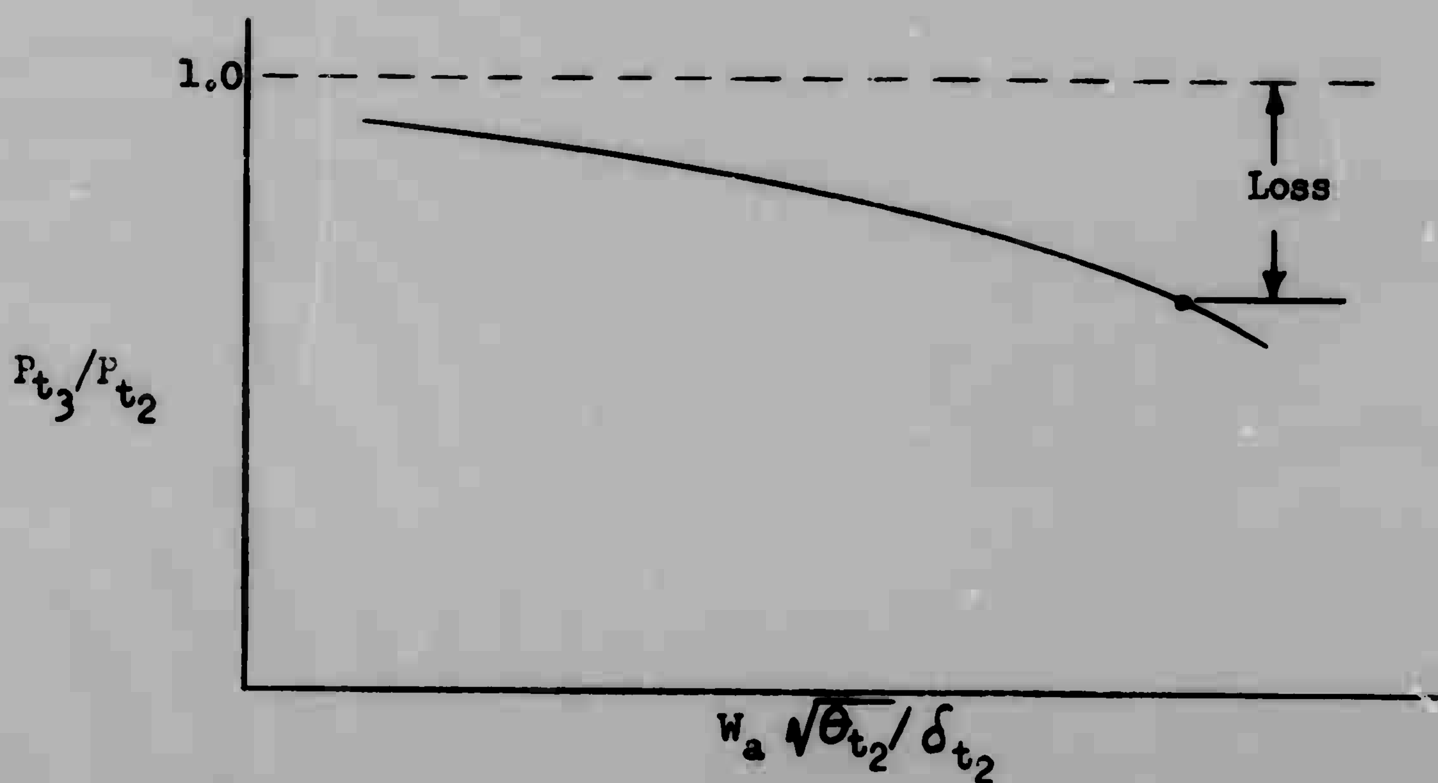


Figure 4.6.9

If the value of P_{T_3}/P_{T_2} is kept as near one as possible the pressure drop is kept at a minimum. There are many ways by which pressure losses may be reduced but only at the expense of a loss of thrust per pound engine weight. The jet nozzle area and the combustion temperature at the design conditions are two of the most important parameters that must be balanced to obtain the optimum engine output.

Loss of efficiency is also realized if combustion is not complete before the mixture leaves the engine. This normally results in an increase in the fuel air ratio required to obtain a given increase in combustion chamber temperature. The combustion efficiency is the ratio of the actual temperature rise to the ideal temperature rise.

$$\eta_c = \frac{\text{actual temp. rise}}{\text{ideal temp. rise}} \qquad \text{Equation 4.6.3}$$

The combustion efficiency is affected by the following (causes lower η_c)

1. Decreased T_{t_2}
2. Decreased P_{t_2}
3. Increased combustion inlet velocity
4. Change of fuel air ratio from design

Figure 4.6.10 shows a plot of η_c versus fuel air ratio.

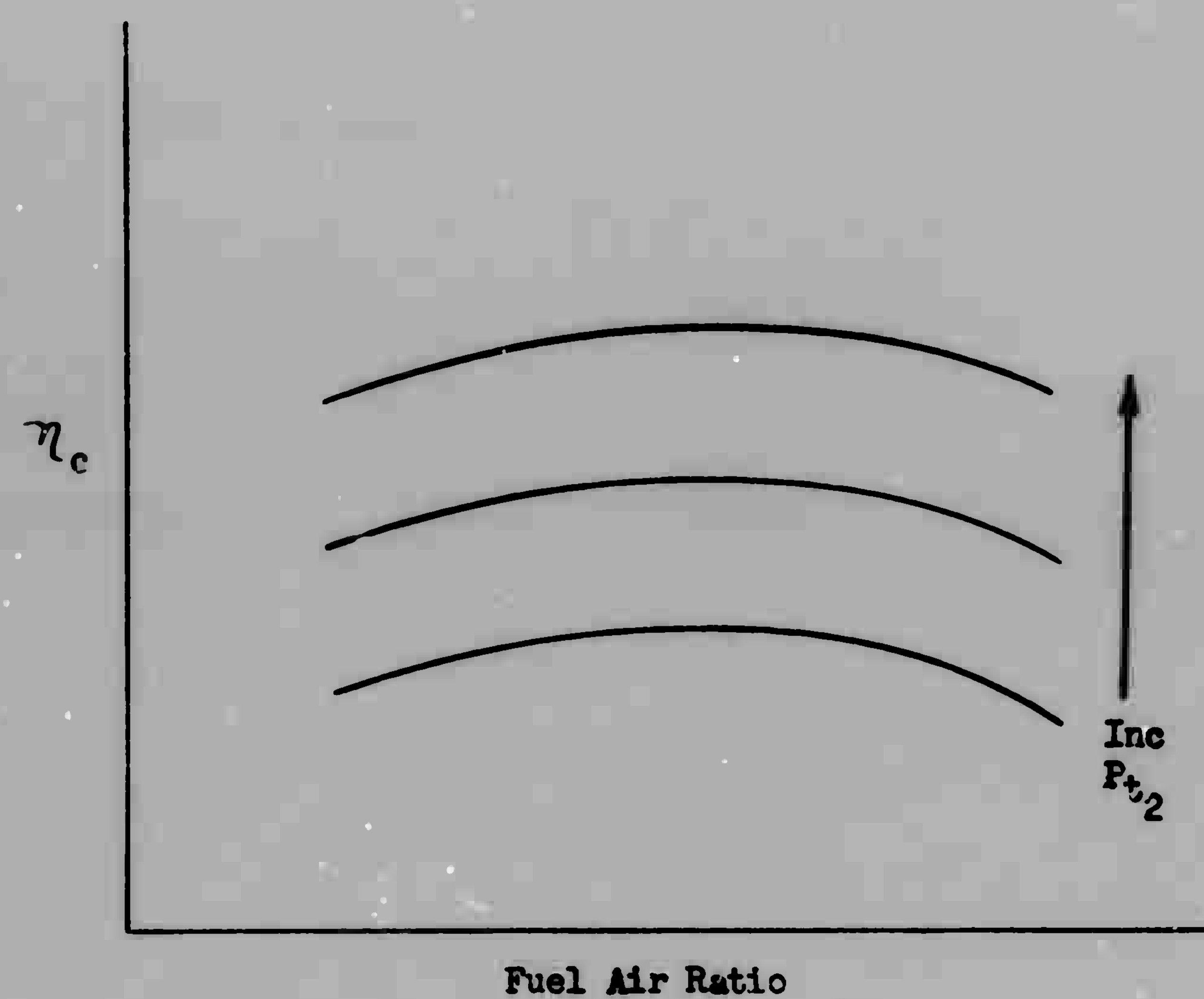


Figure 4.6.10

4.6.7 NOZZLE

The jet nozzle accelerates the hot gases from the combustion chamber to a high velocity at the nozzle exit. Thrust performance and air flow capacity are two characteristics which are important for jet performance. The thrust of a nozzle does not equal the thrust of an actual nozzle as shown in Figure 4.6.3. The reason for the difference is the following losses.

(1) Friction

(2) Angularity of flow at the exit

(3) Under expansion

(4) Over expansion

(5) Lack of one dimensional flow

Considering all these losses as a group, thrust efficiency of the nozzle may be expressed by a so called velocity coefficient.

$$C_V = \text{velocity coefficient} = \frac{(F_g/W_j)\ \text{actual}}{(F_g/W_j)\ \text{ideal}} \qquad \text{Equation 4.6.4}$$

where:

$(F_g/W_j)_{actual}$ = Actual gross thrust per pound of gas flow

$(F_g/W_j)_{ideal}$ = Ideal gross thrust per pound of gas flow

A plot in Figure 4.6.11 shows how C_V varies with nozzle pressure ratio

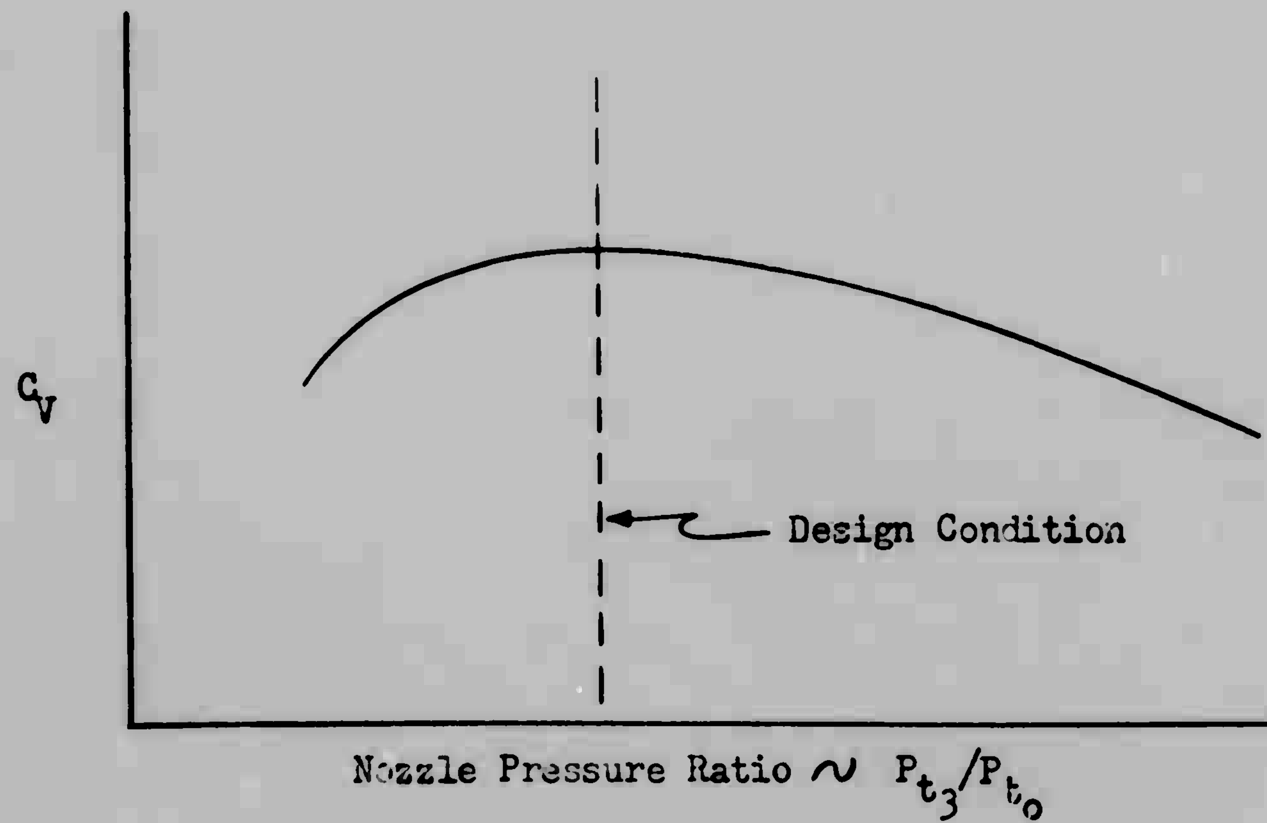


Figure 4.6.11

For a more complete discussion of nozzles, reference should be made to Section 5 on Supersonic Aerodynamics.

4.6.8 OVERALL ENGINE OPERATION

By establishing the value of (1 + fuel air ratio)x $\sqrt{T_{t_3}/T_{t_2}}$ for a given operation then the values of $W_{a_2}\sqrt{\theta_{t_2}}/\delta_{t_2}$, P_{t_3}/P_{t_2} and η_d are found. These are the parameters that are necessary for computing the engine thrust and economy. Figure 4.6.12 may be constructed and will give an understanding of the operation of a supersonic ramjet. At a fixed speed an increase in fuel air ratio will decrease diffuser loss and loss across the combustion chamber.

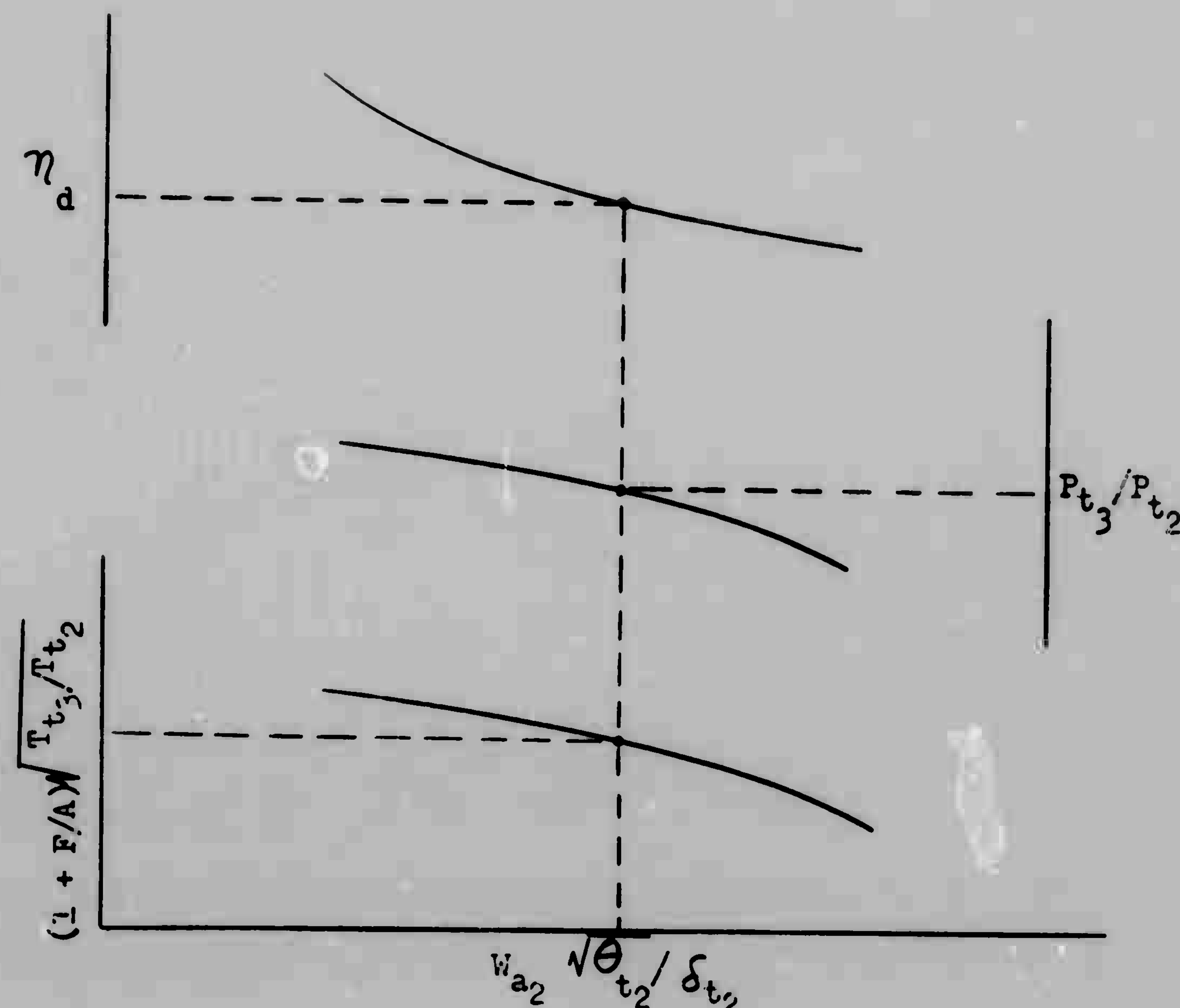


Figure 4.6.12

The above mentioned condition will also move the operating point toward the "buzz" limit. There is a continual compromise between sufficient buzz margin and optimum performance. The tendency of the designer is to obtain minimum loss and maximum efficiency. This occurs at the buzz limit and produces a very interesting engineering problem.

If the process in Figure 4.6.12 is repeated for a series of flight speeds and fuel air ratios, the curve in Figure 4.6.13 may be constructed.

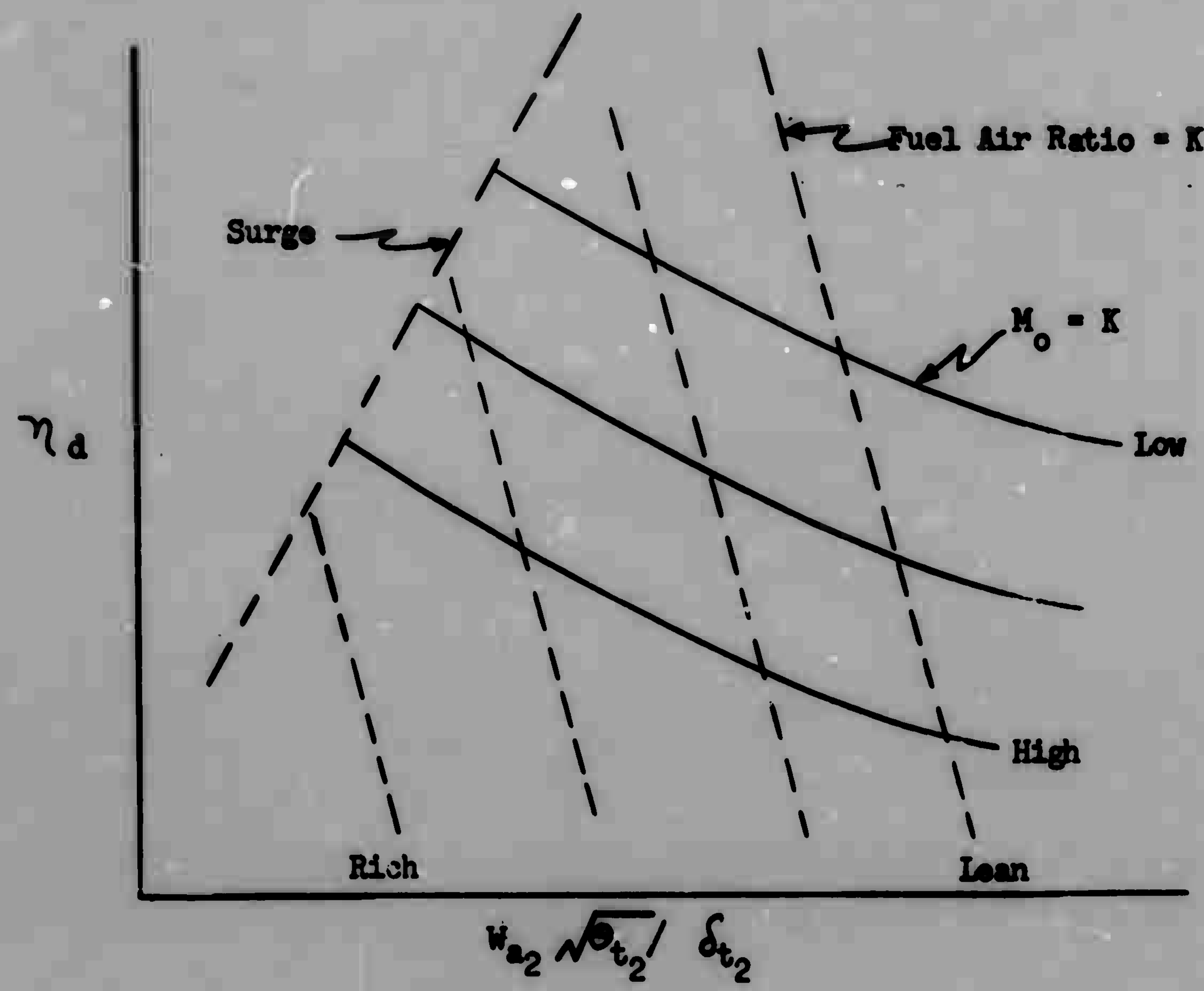


Figure 4.6.13

It can be seen that for a decreasing fuel-air ratio at a flight speed $W_2\sqrt{\theta_{t_2}}/\delta_{t_2}$ increases along with η_d. Also decreasing the Mach number at a constant fuel air ratio moves the operating point toward the buzz zone. From this it can be seen that in order to stay out of the buzz zone the fuel air ratio must be decreased with a decrease in Mach number. A curve may be constructed showing the net thrust versus Mach number for each of the above mentioned points. This curve usually looks as in Figure 4.6.14.

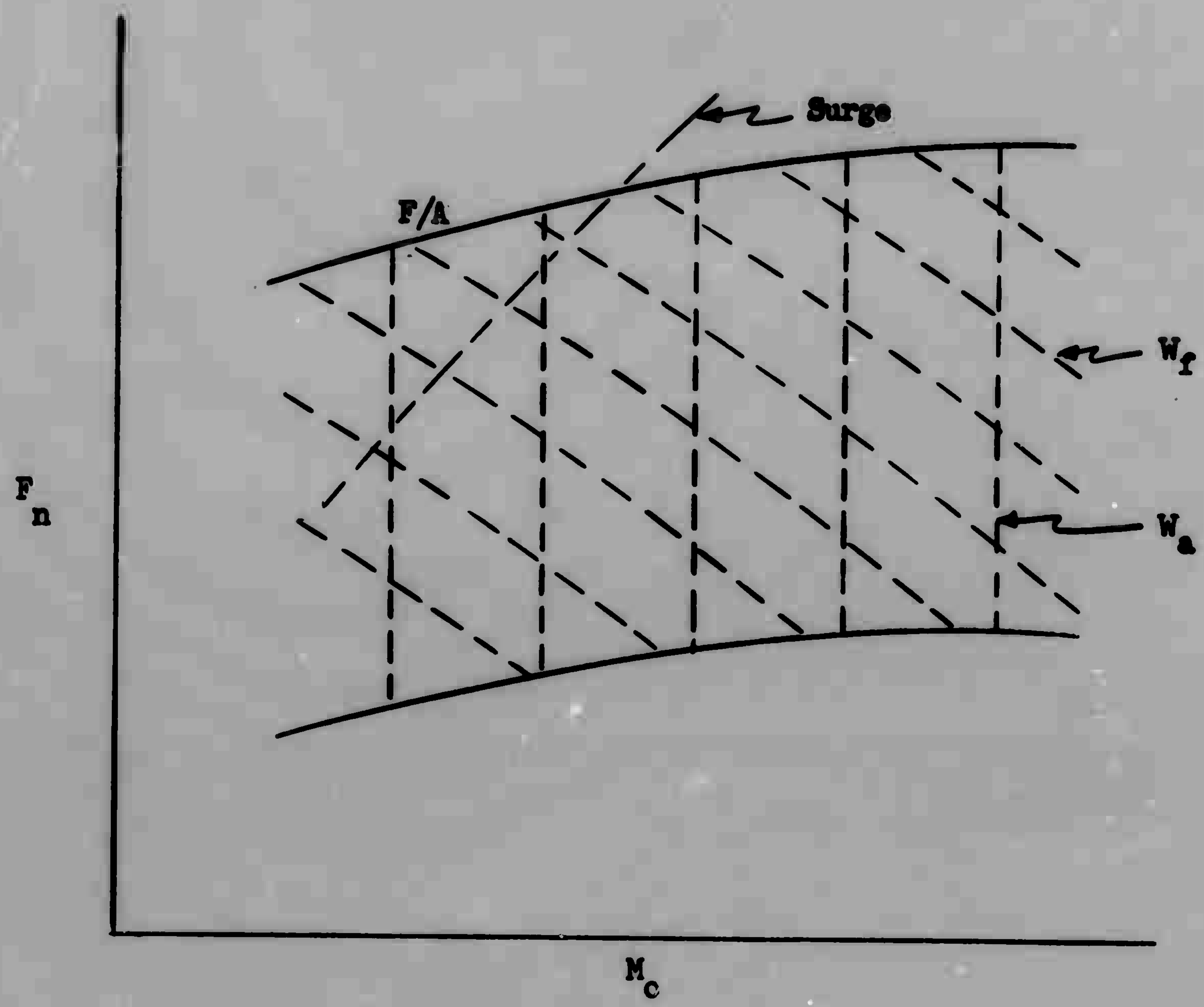


Figure 4.6.14

The pressure-volume diagram for a ramjet is nothing but a Brayton cycle as shown below.

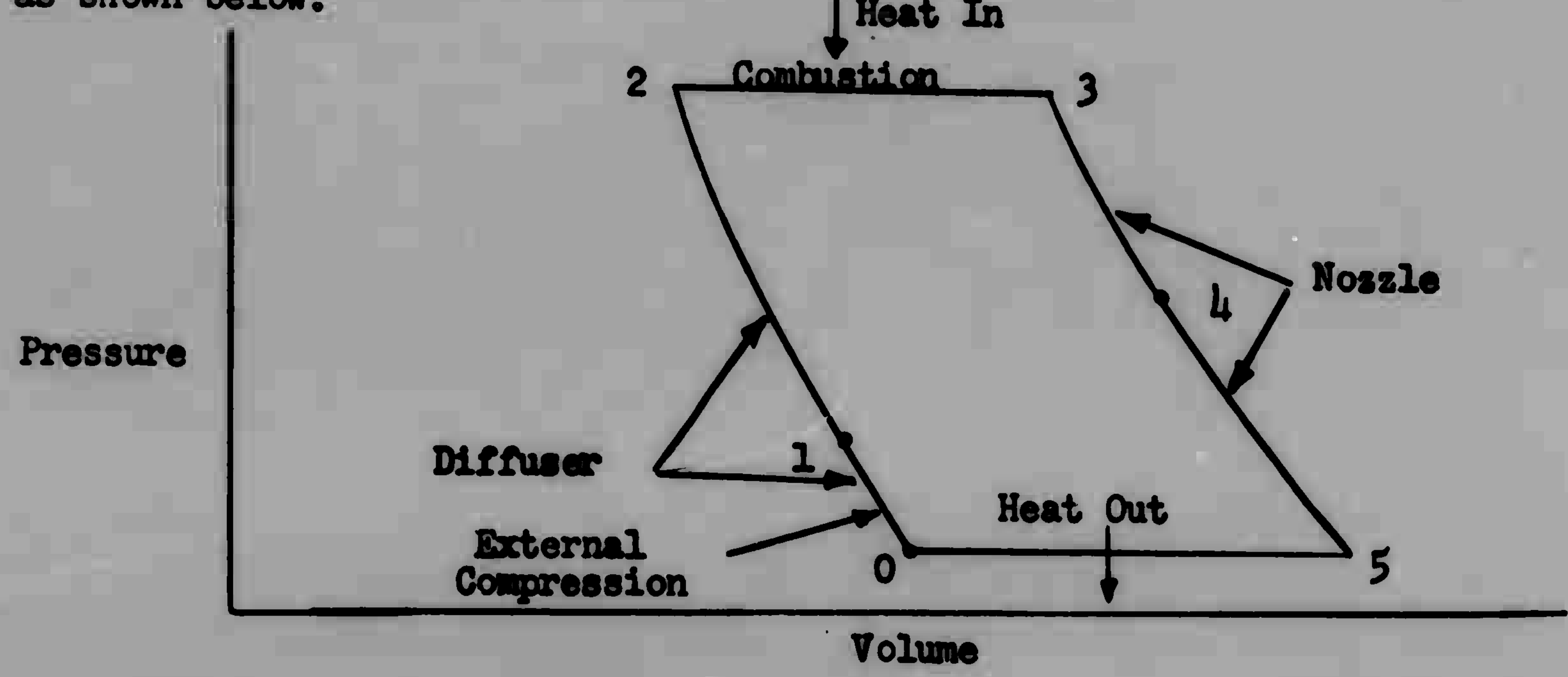


Figure 4.6.15

The thrust of a ramjet is the product of the mass flow and the increase in air velocity between the diffuser inlet and nozzle exit. You can also say that anything which will increase mass flow and temperature will create thrust. The most efficient operation of a ramjet is near the surge or buzz limit. The fuel control must be able to keep the fuel air ratio close to the buzz limit without the risk of operating in the surge.

4.7 ROCKET ENGINES

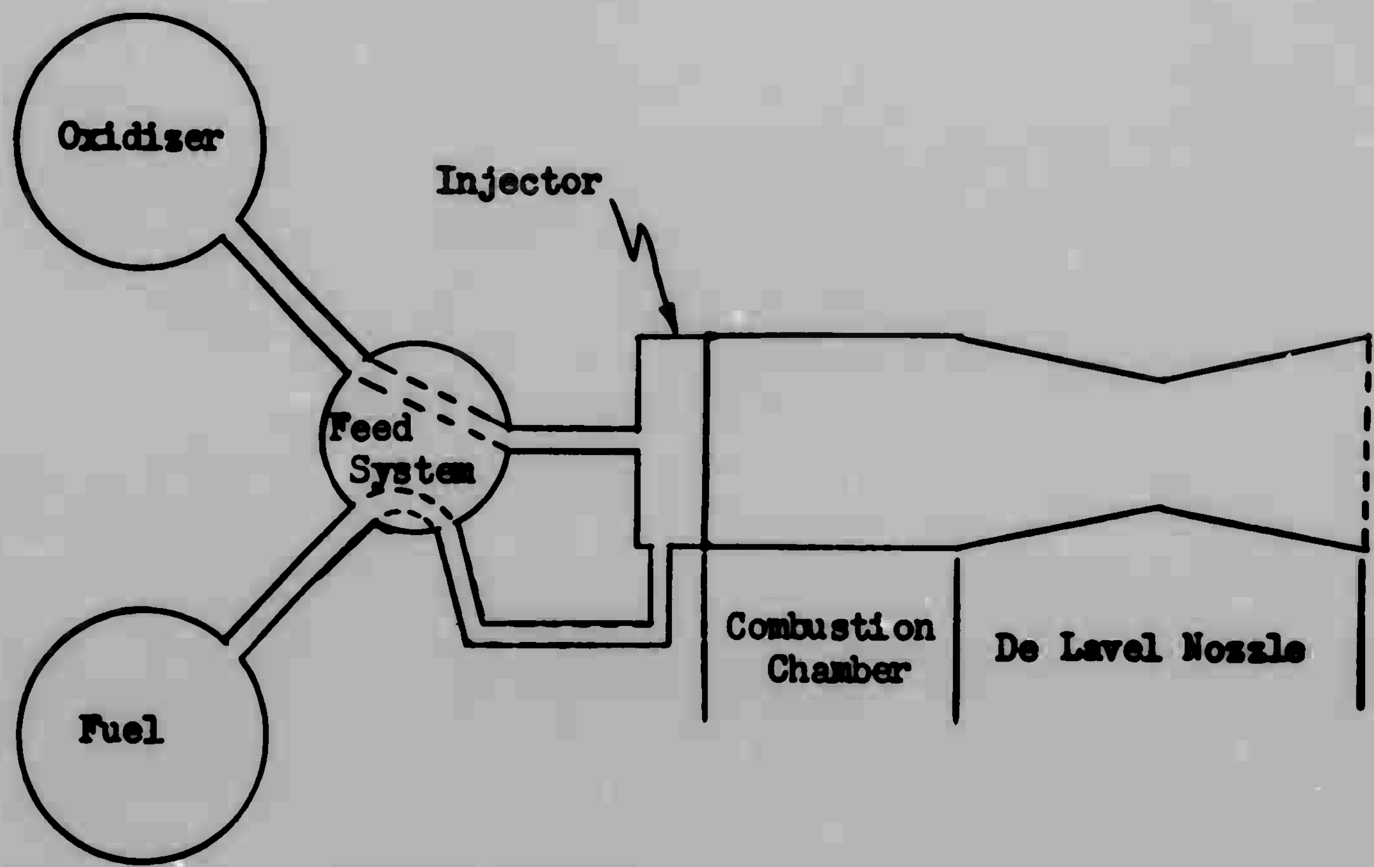
4.7.1 GENERAL

As stated previously, Newtons' third law of motion states for each action there is an equal and opposite reaction. The rocket engine is based on this principle. The propellants are injected into a combustion chamber at relatively low velocity where they undergo the combustion process. This combustion process is required to accelerate the gases to a high velocity required at the nozzle (Newton's second law). The thrust obtained from the engine is the reaction to the combustion process. In this regard, the rocket engine is very similar to the other jet type engines such as the ramjet or turbojet. The rocket, however, carries its own oxidizer enabling it to operate outside the atmosphere. The turbojet and ramjet will only operate inside the atmosphere where sufficient air is available for combustion.

There are two principal types of rocket engines in use today; the solid propellant and the liquid propellant. Each of these types have their advantages and disadvantages about which there will be some discussion.

4.7.2 LIQUID ROCKET ENGINE

Liquid propellant engines are classified according to the number of propellants which are employed. We will concern ourselves in bipropellant liquid rocket engines. Figure 4.7.1 is a simplified drawing of a typical liquid propellant engine.



Simplified Liquid Rocket Engine

Figure 4.7.1

The principal engine components are the propellant tanks, propellant feed system, ejectors, combustion chamber and nozzle (de Laval type).

A rocket engine is a converter for thermochemical potential energy into exhaust jet kinetic energy. This is accomplished by going through several distinct steps; these are; Propellant feed, injection, ignition, combustion, and expansion.

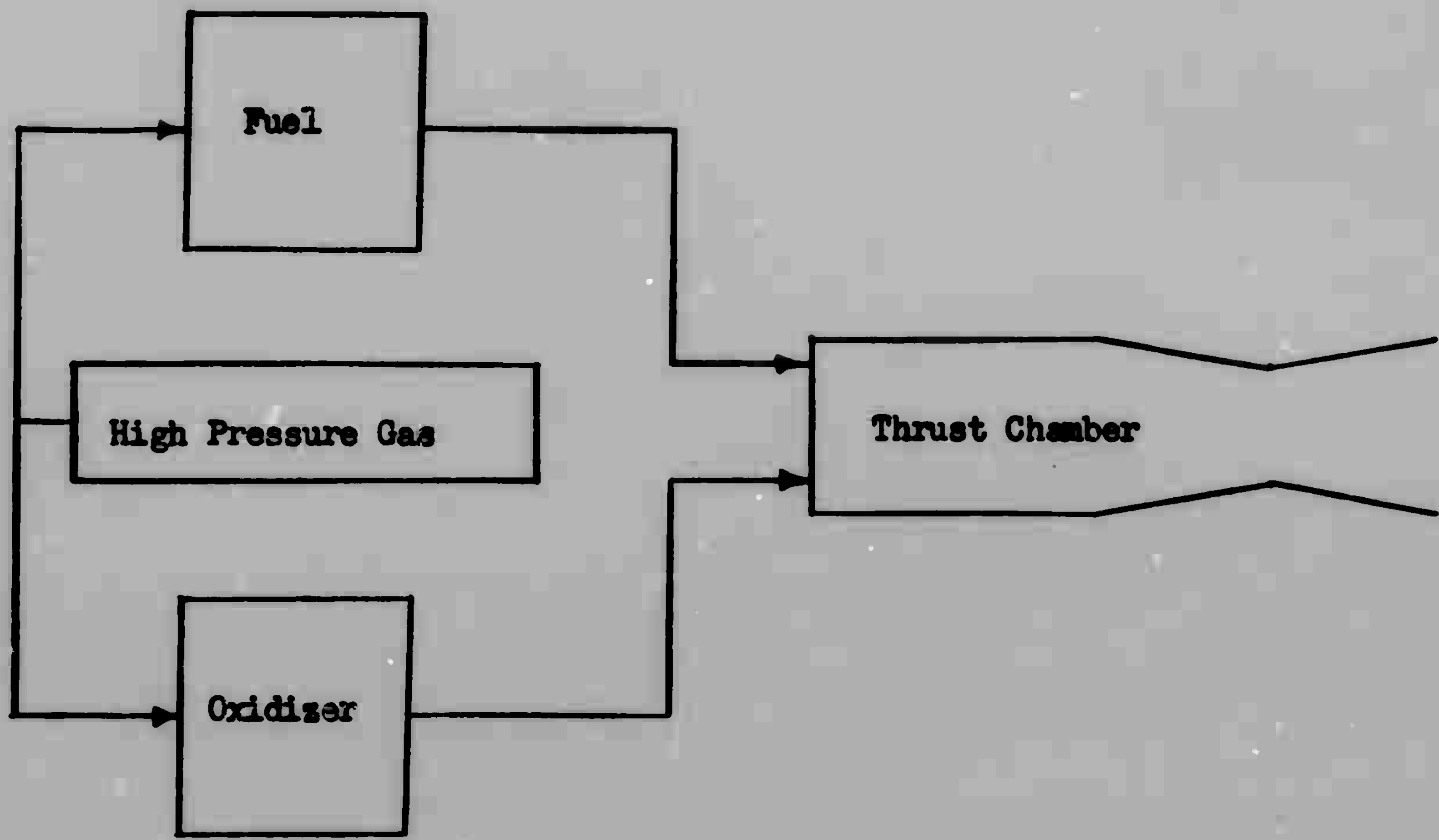
The oxidizer and fuel are at rest initially with respect to the rocket engine, and therefore, represent the potential energy of the chemical. The propellant feed system usually includes a turbopump or some high pressure gas which is used to force the propellant into the injectors. The pressure in the combustion chamber is usually quite high; therefore, output of the feed system must be great enough to force the propellant into the combustion chamber. The turbopumps used for this operation normally are small but very powerful.

The injector is so arranged in the combustion chamber that the oxidizer and fuel are well mixed. Ignition then occurs at the face of the injector as the propellant enters the combustion chamber. Some type of ignition system may be provided, but continuous ignition is maintained. The combustion process is rapid with burning taking place in the combustion chamber and nozzle, with some residual burning taking place in the exhaust gases. The bulk of the burning taking place in the combustion chamber before the gases enter the nozzle.

4.7.2a LIQUID PROPELLANT FEED SYSTEM

The method used to feed propellant into the combustion chamber may range from the simple pressure feed system to the complicated pump feed system. There may be many things which determine the system to be used; such as burning time, type of propellant, design requirements, reliability and weight.

Figure 4.7.2 shows a sample drawing of a pressure feed system.



Pressure Feed System

Figure 4.7.2

As can be seen from the drawing,the system contains a tank of high pressure gas which is used to force the propellants into the combustion chamber. This system has the advantage of simplicity, high reliability and few moving parts. The one overriding disadvantage is the weight. The pressure required to force the propellant into the combustion chamber is very high; therefore, the pressure in these tanks must be higher,requiring very strong and heavy tanks.

The other disadvantage is the short burning duration available. This system is normally used for operations requiring low thrust application, short duration thrust requirements and reliability.

The next system, pump feed, is used very extensively in the rocket engine. The system consist of centrifugal pump powered by a turbine. The turbine is driven by hot gases produced by the burning of propellants within a gas generator. Design refinements have been made to this system whereby the propellant is supplied from the main tanks after the pumps are started. This results in a simplified design and an improved mass ratio. Figure 4.7.3 shows a simplified drawing of the turbopump feed system.

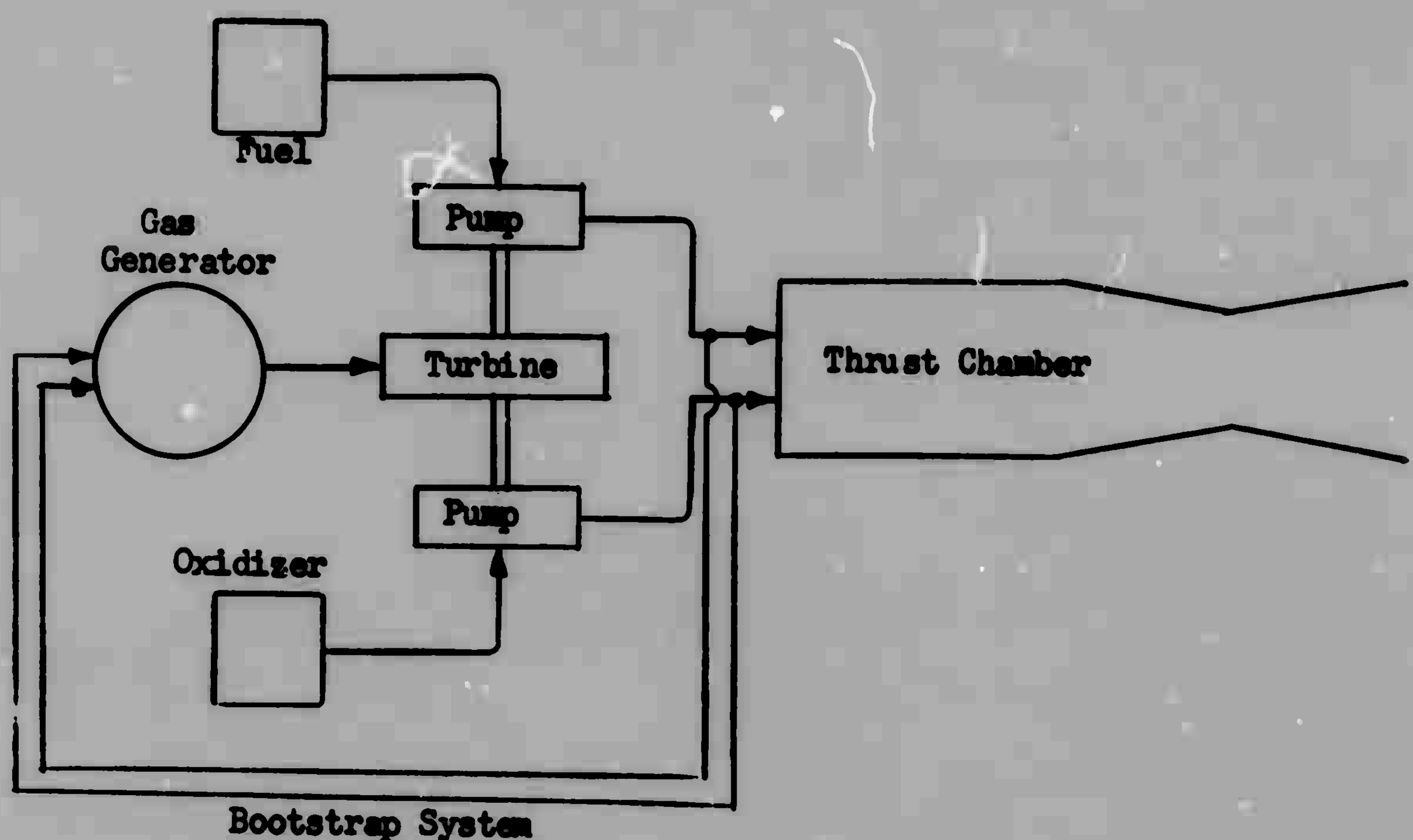


Figure 4.7.3 Pump Feed System

It can be seen from the drawing that the gas generator is replenished after the turbopumps build up pressure to the combustion chamber. This allows the pump to work until all the fuel is expended, thus giving a very long burning duration. The system can be made much lighter than the pressure feed system; however, the system requires that all parts work perfectly which affects reliability.

4.7.2b. PROPELLANT TANKS

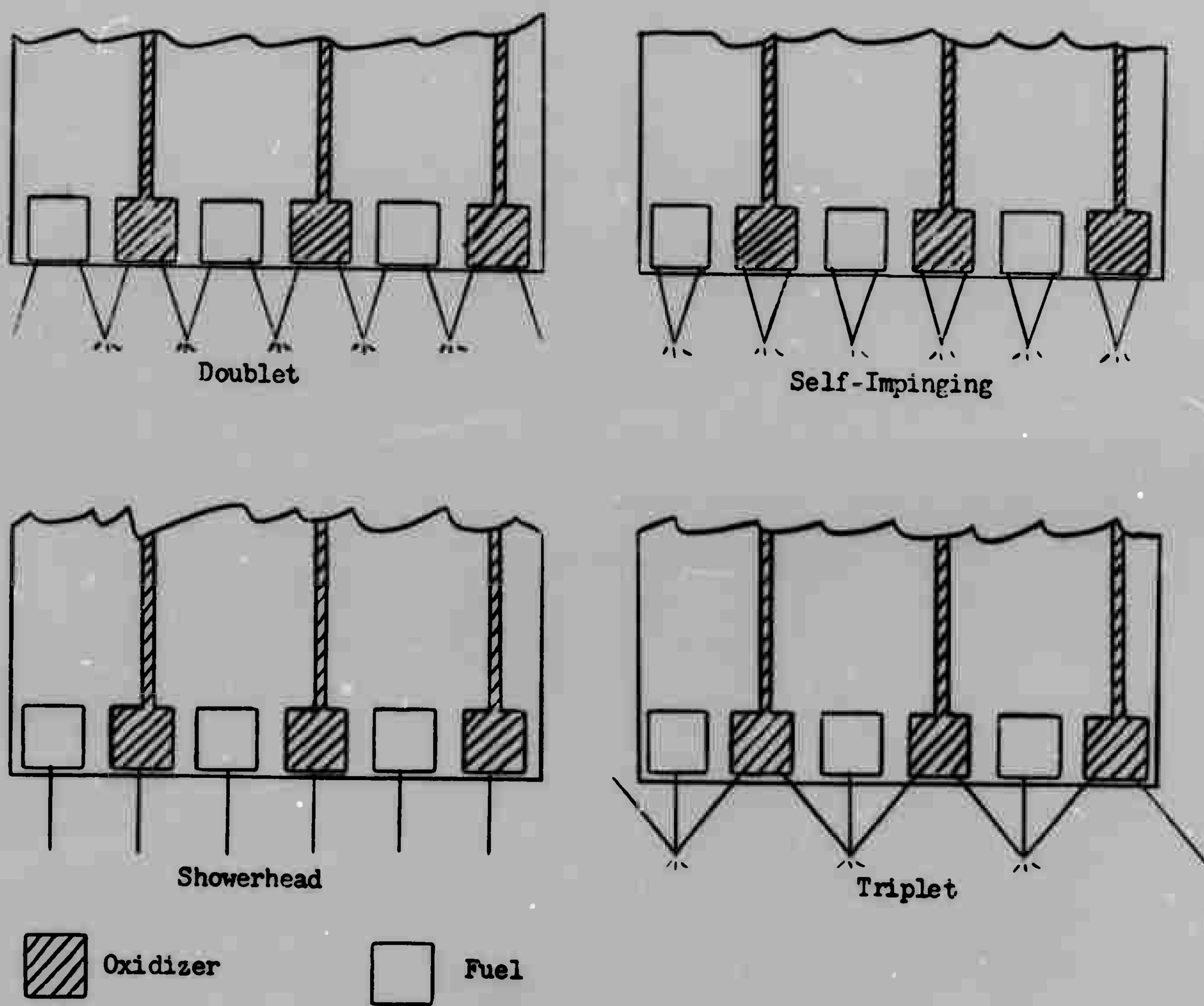
Spherical tanks would be ideal as far as stress concentration and weight, but would not be economical in rocket construction. The propellant tank must be strong and light weight. Two types are used: The first is the pressurized type used in the pressure feed system, and the second is the low pressure type used with pump feed system. These are very thin-walled and have a tendency to collapse under severe external load. These tanks are normally pressurized for storage and transportation.

4.7.2c. THRUST CHAMBER

This is the section of the engine where the propellant is burned at very high pressure to form gaseous products which in turn are accelerated and ejected at high velocities. The thrust chamber assembly includes propellant inlet manifolds, injectors, combustion chamber and the de Lavel nozzle.

The function of the injector is similar to a carburetor of an internal combustion engine. To be effective, an injector must distribute an even propellant pattern or hot spots will develop as a result of disturbing the boundary layer. The size, number and pattern of the orifices must be carefully

designed to avoid adverse effects on the injectors strength. Figure 4.7.4 shows a few of the designs possible.



Injector Designs

Figure 4.7.4

The design of the combustion chamber may be influenced by many factors. Some of the most important are:

(1) Manufactoring ease

(2) Size

(3) Weight

(4) Nozzle inlet design

(5) Coolant pressure drop

It is obvious that the most desirable chamber is the one that will produce the maximum thrust in relation to the weight and propellant flow. Any increase in thrust output would be desirable if the weight could be held down. The factors which contribute to the overall performance must be weighed against the undesirable features to see if it is profitable. These selections are usually accomplished after an extensive research and development period.

The term "characteristic length" is usually used in conjunction with combustion chambers. Characteristic length is equal to the ratio of combustion chamber volume to the area of the nozzle throat. This is represented in the formula:

$$L^* = \frac{V_c}{A_t}$$

The combustion chamber volume is the volume of the chamber plus the volume of the converging portion of the de Lavel nozzle. There is usually one characteristic length for each particular propellant combination, mixture ratio, chamber pressure, and injector.

Throughout the burning process one would expect the thrust chamber to become very hot and some type of cooling would be required. The temperatures inside the chamber could run as high as 5000 ^{o}F which is a large amount of heat on the chamber walls. The heat transfer ranges in various engines run about 2 to 10 $BTU/Sec/in^2$. The throat of the nozzle is usually the hottest as can be seen in Figure 4.7.5.

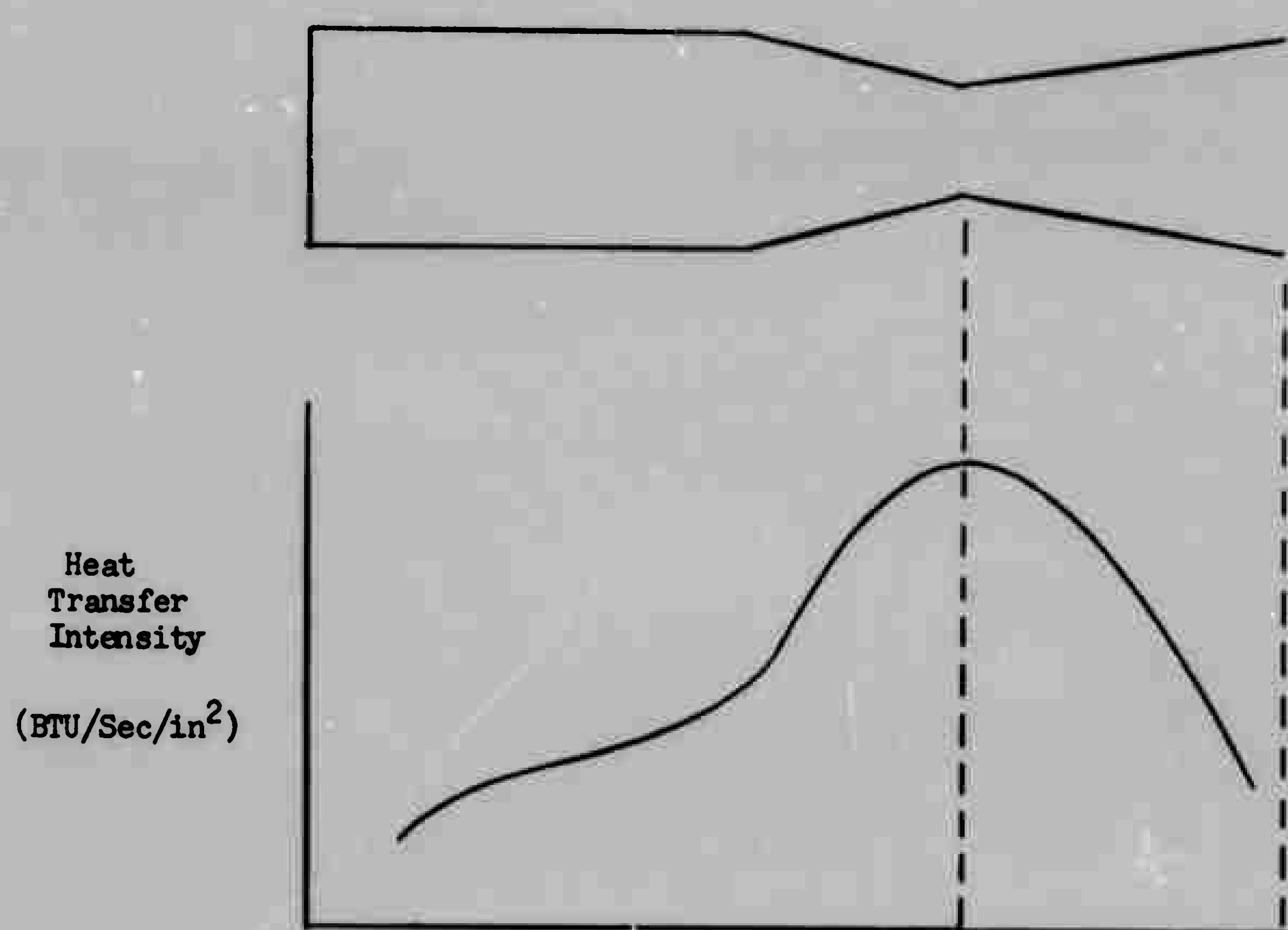
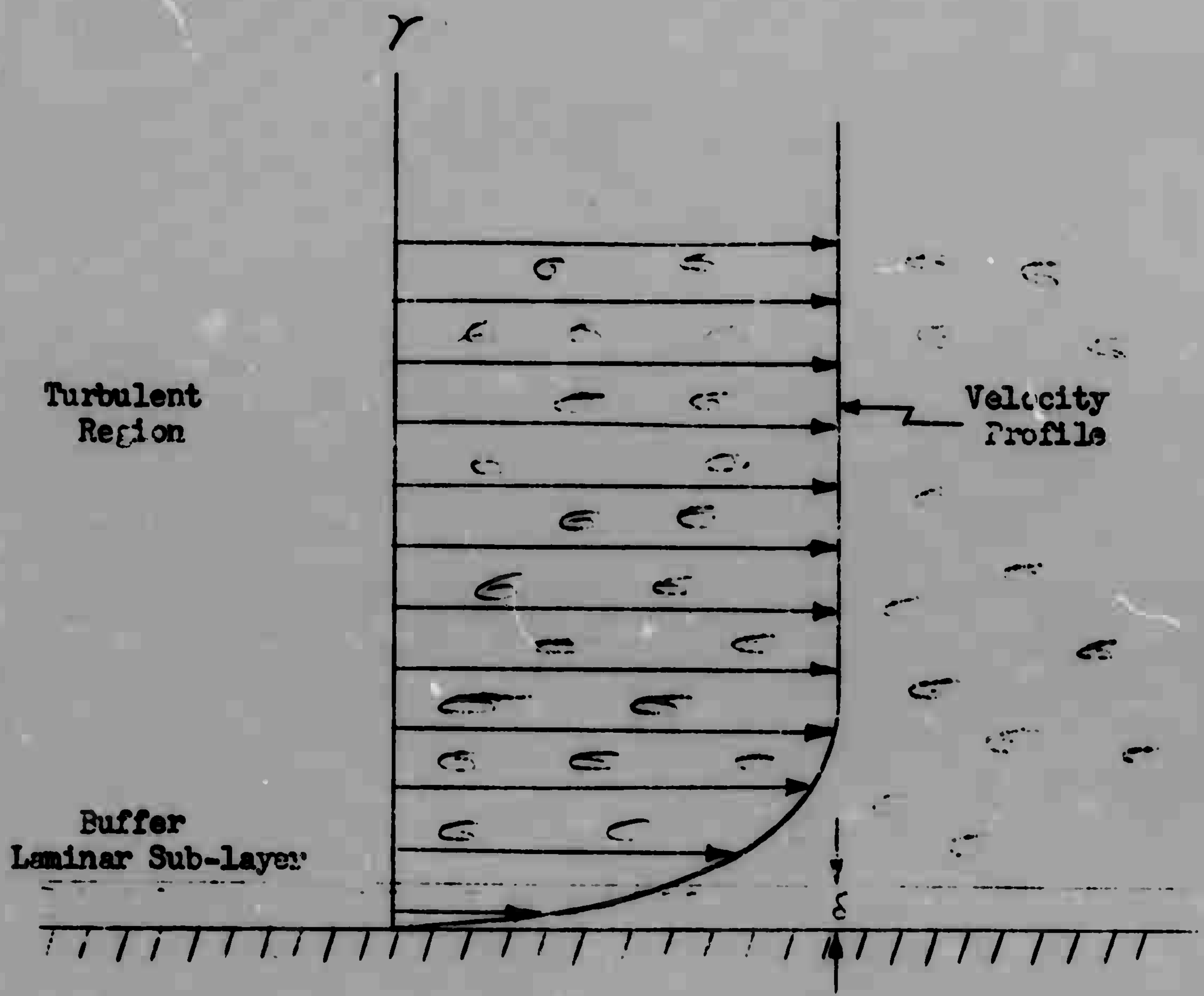


Figure 4.7.5

The thrust chamber walls are normally insulated by a "laminar-sub-layer" or a gas boundary layer. As the hot turbulent gases move through the nozzle the gas near the walls is retarded by frictional drag. Fluid viscosity slows down the layers of gas with less decellerating effects with increased distance from the walls. A typical velocity profile is shown in Figure 4.7.6.



Velocity Profile - Turbulent Flow

Figure 4.7.6

Throughout the burning process one would expect the thrust chamber to become very hot and some type of cooling would be required. The temperatures inside the chamber could run as high as 5000 °F which is a large amount of heat on the chamber walls. The heat transfer ranges in various engines run about 2 to 10 BTU/Sec/in^2. The throat of the nozzle is usually the hottest as can be seen in Figure 4.7.5.

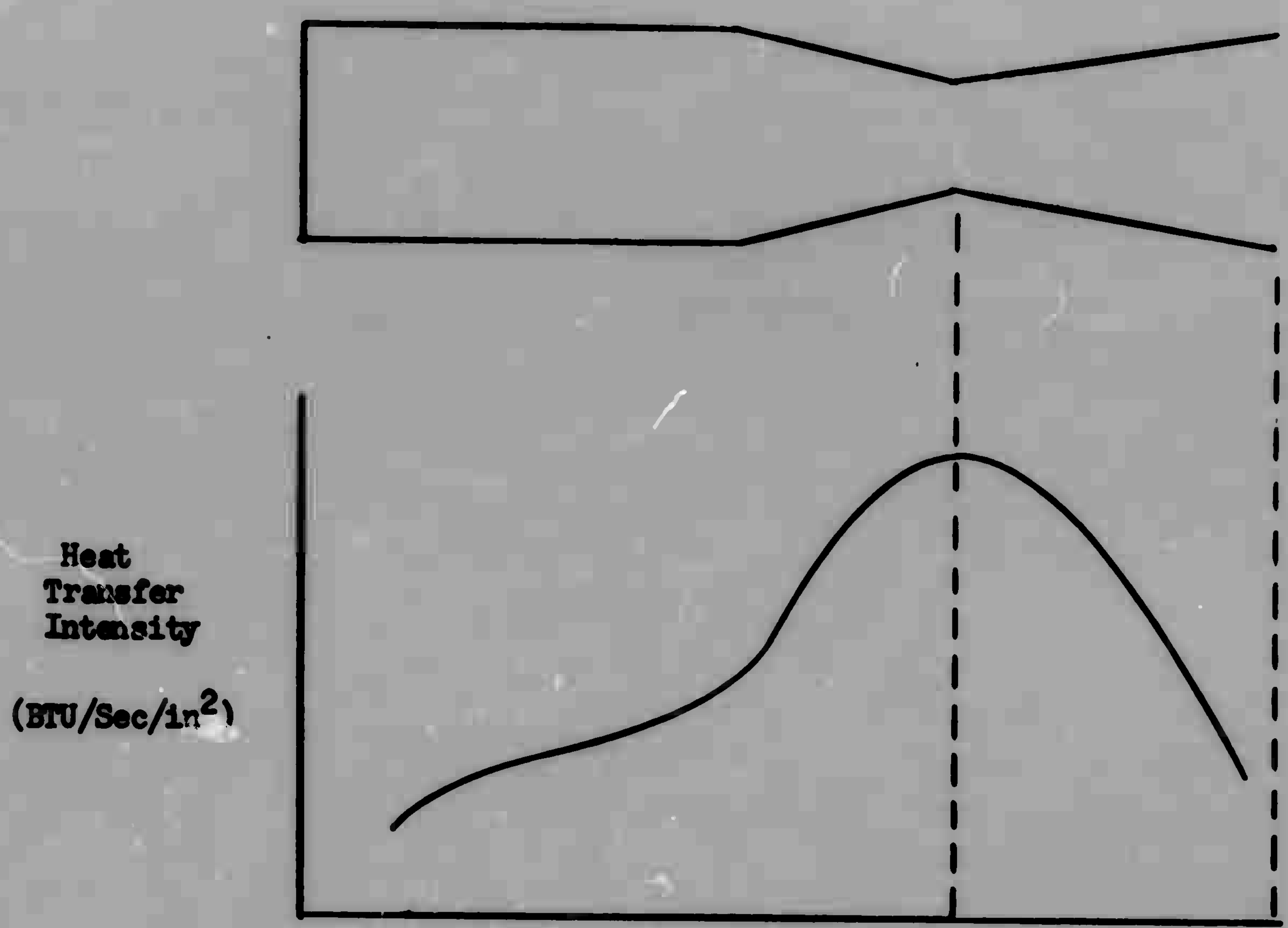
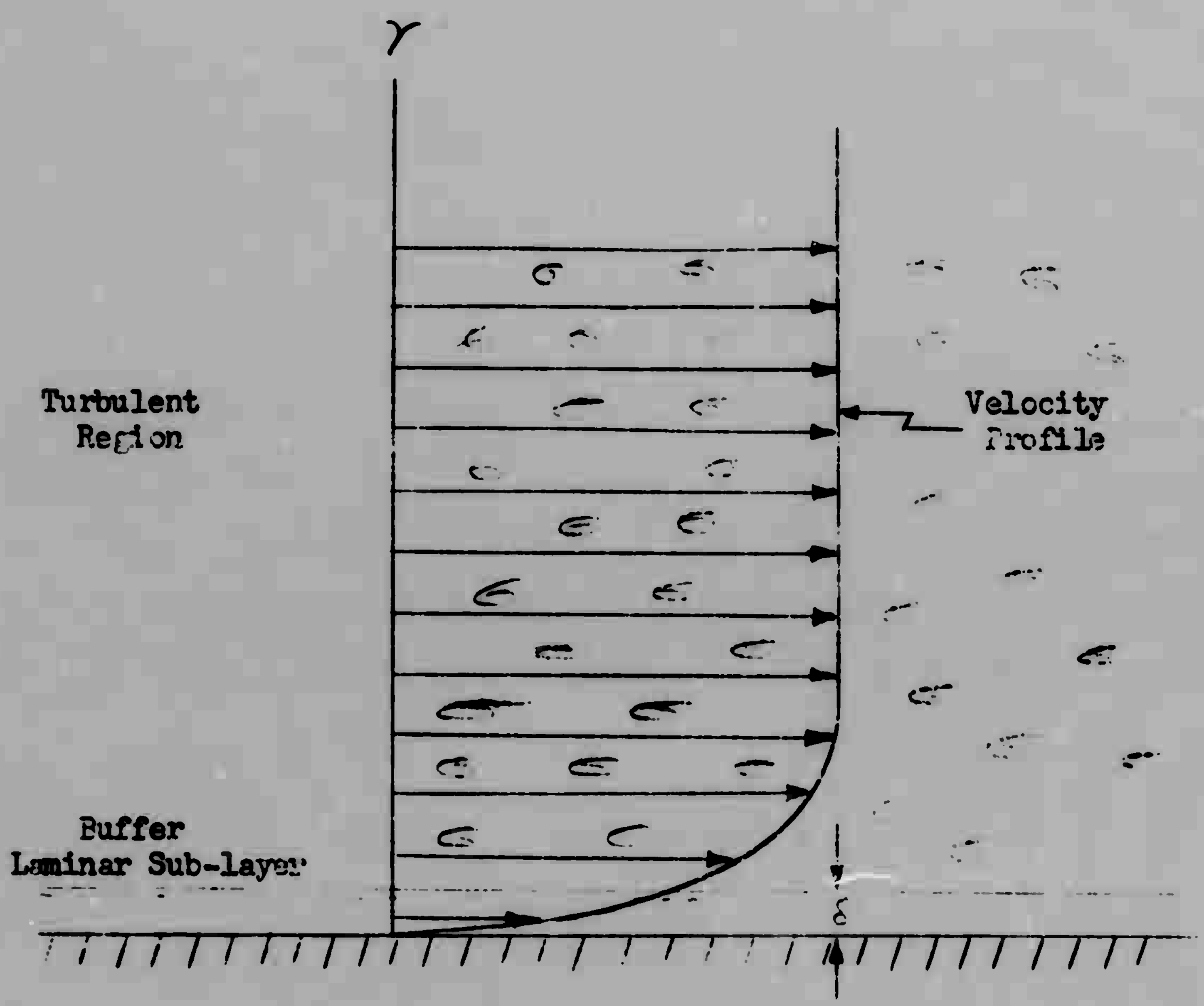


Figure 4.7.5

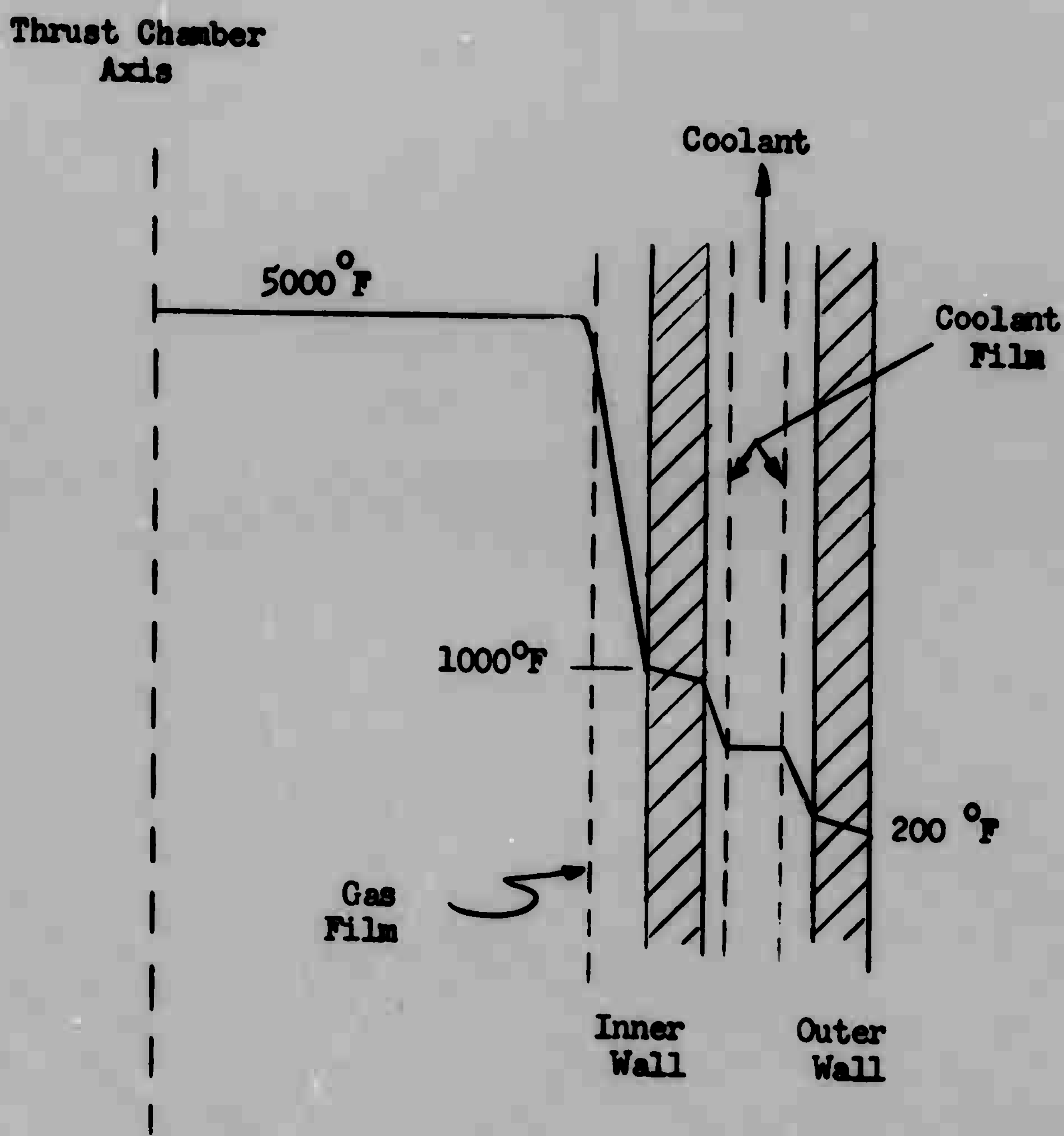
The thrust chamber walls are normally insulated by a "laminar-sub-layer" or a gas boundary layer. As the hot turbulent gases move through the nozzle the gas near the walls is retarded by frictional drag. Fluid viscosity slows down the layers of gas with less decellerating effects with increased distance from the walls. A typical velocity profile is shown in Figure 4.7.6.



Velocity Profile - Turbulent Flow

Figure 4.7.6

Since convection depends on turbulence, or molecular mixing, there can be no convection across the boundary layer because of the laminar flow. Therefore, only radiation and conduction remain and the low thermal conductivity of the gas results in an insulator. The temperature gradient through the walls is shown in Figure 4.7.7. This shows a regeneratively cooled chamber.



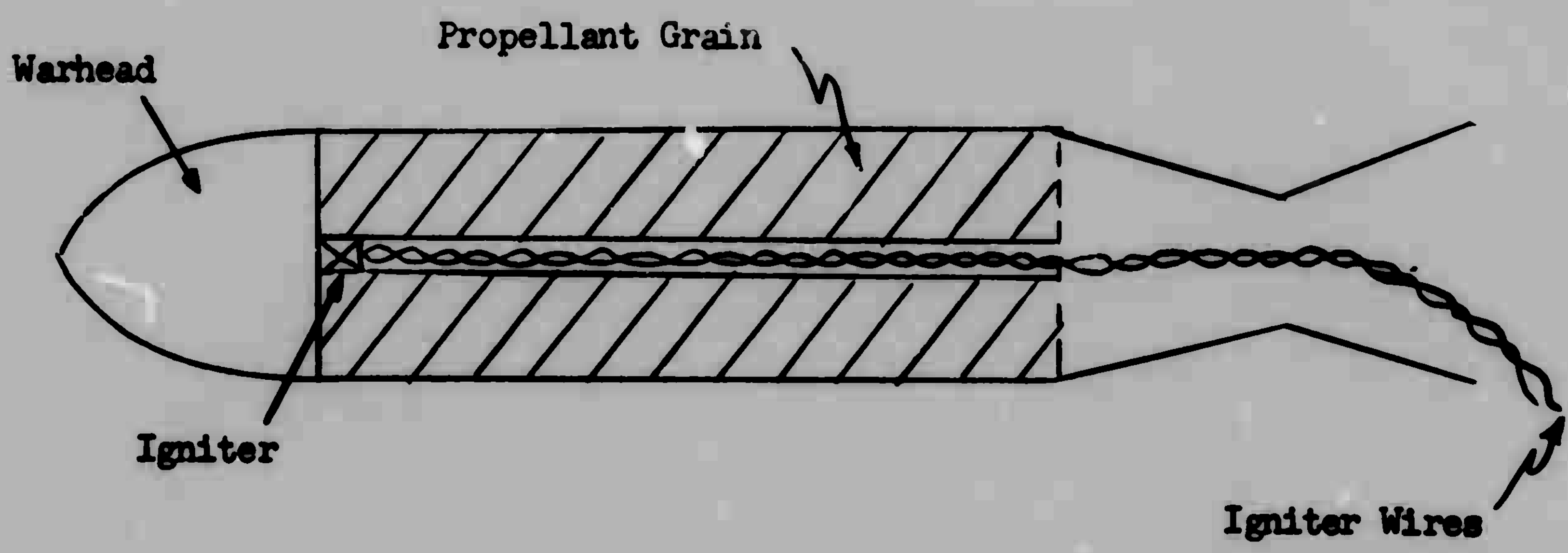
Temperature Gradient

Figure 4.7.7

The thrust chamber is usually constructed of tubes formed to the required shape then welded together. Fuel will be pumped through these tubes and will serve as a coolant. The fuel is pumped through the tubes the length of the nozzle then back through other tubes up the nozzle to a manifold which will distribute the fuel to injectors. In this process there is actually in effect a liquid cooled chamber. This allows the chamber to be used many times such as in static test.

4.7.3a SOLID PROPELLANT ROCKETS

Solid propellant rockets are used extensively to power rocket projectiles, guided missles and to boost power of an aircraft or missle. The inherent simplicity of the solid propellant is its greatest appeal, while the liquid propellant rocket has the advantage of versatility and high performance. Figure 4.7.8 shows a simplified drawing of a typical solid propellant rocket.



Solid Propellant Rocket

Figure 4.7.8

4.7.3b SOLID PROPELLANTS

The entire body of the solid propellant is called the "grain". The grain contains both oxidizer and fuel. There are two principal types of solid propellants: The composite propellant, consisting of an inorganic oxidizer dispersed in a matrix of organic plastic, and the double base propellant, consisting of the colloid of nitroglycerin - nitrocellulose. Additives are used with the chemicals to regulate burning rate, increase stability and alter the physical properties. The burning rate will normally regulate the operating pressure which is in the order of 500 to 2000 pounds per square inch. The walls of the container must be able to withstand these pressures plus a safety factor above possible surges.

The de Laval nozzle has the same function in the solid propellant rocket as it has in the liquid propellant rocket. The nozzle cannot be cooled as in the liquid rocket, so for long duration firings the nozzle must be insulated with a material such as graphite or ceramics. The walls of the rocket chamber may also be insulated to prevent damage from over heating.

The burning rate of a grain or the rate at which a solid propellant is burned is measured in a direction normal to the burning. Most propellants burn at a rate of 0.03 to 2.5 inches per second at a chamber pressure of 2000 psi. As the initial temperature is increased the burning rate increases. The temperature of a solid rocket may cause an increase in chamber pressure with a decrease in burning time if the engine has been stored for any length of time at elevated temperatures. The thermal conductivity of solid propellant

fuel is in on the order of asbestos; therefore, to have any increase in burning rate due to temperature increase, the grain will have to be stored at higher temperatures for some time.

There are temperature limitation for some solid propellants. At a very low temperature the propellant may become brittle and the shock of ignition will cause cracks which will cause detonation. At a high temperature the fuel may not be able to withstand the initial acceleration. The latter condition is very unlikely.

The grain configuration or the amount of exposed burning area is used to control the thrust-time program for a fixed quantity of propellant. The importance of proper pressure cannot be over stressed. At too low a pressure some propellants will not support combustion. At too high a pressure the rocket case may rupture. For any fixed configuration of propellant there is a "critical throat area" in the nozzle which will maintain the proper pressure for combustion.

The burning is normally controlled by the geometry of the propellant or by inhibitors. Inhibitors may be an inert or a very slow burning chemical.

Inhibitors are used to protect the walls and are referred to as "liners". The liner prevents surface burning, helps hold the propellant to the walls and acts as an insulator.

Some of the possible grain configurations are shown in Figure 4.7.9.

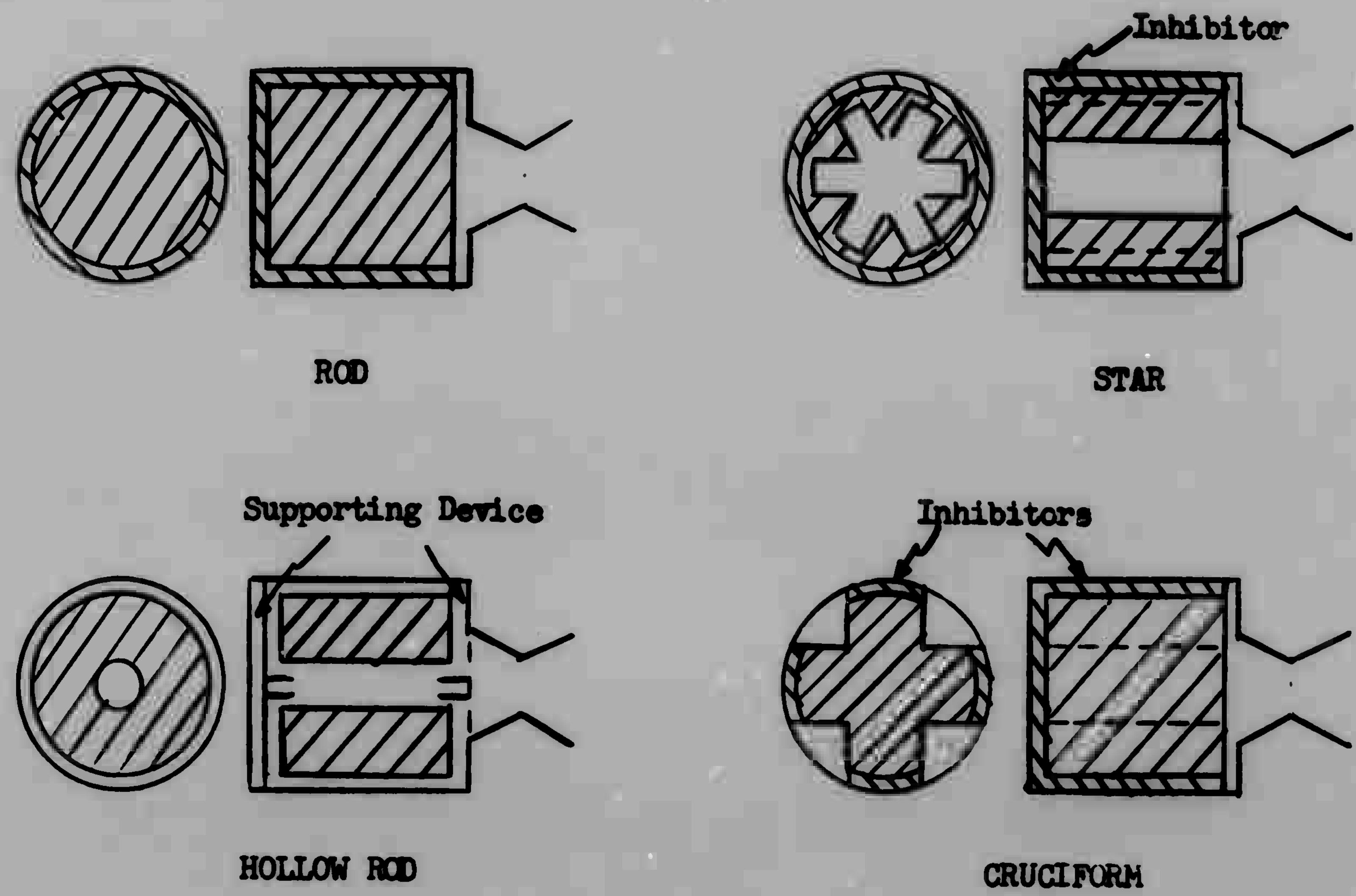


Figure 4.7.9

The typical burning characteristics of the configurations shown in Figure 4.7.9 are shown in Figure 4.7.10

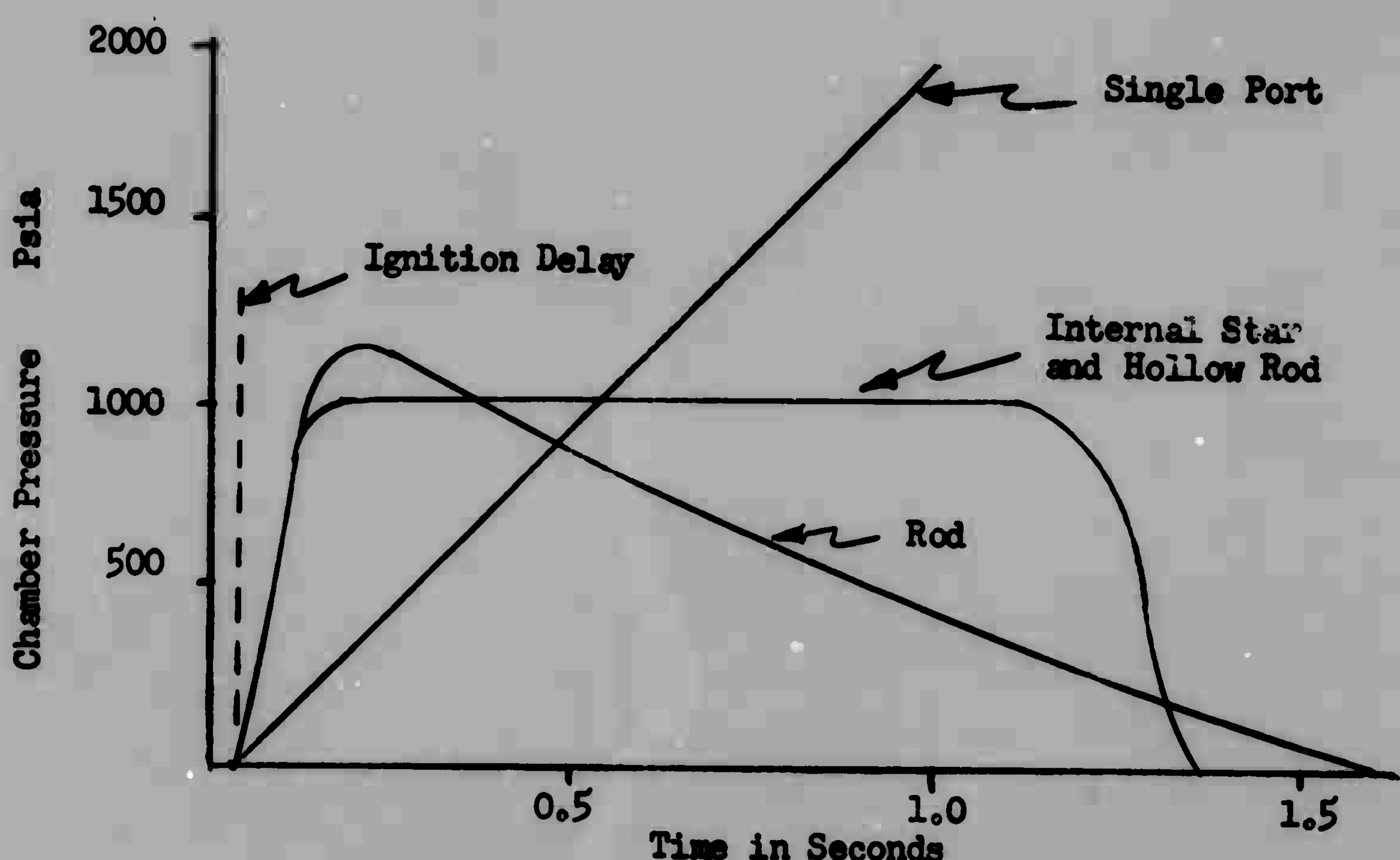
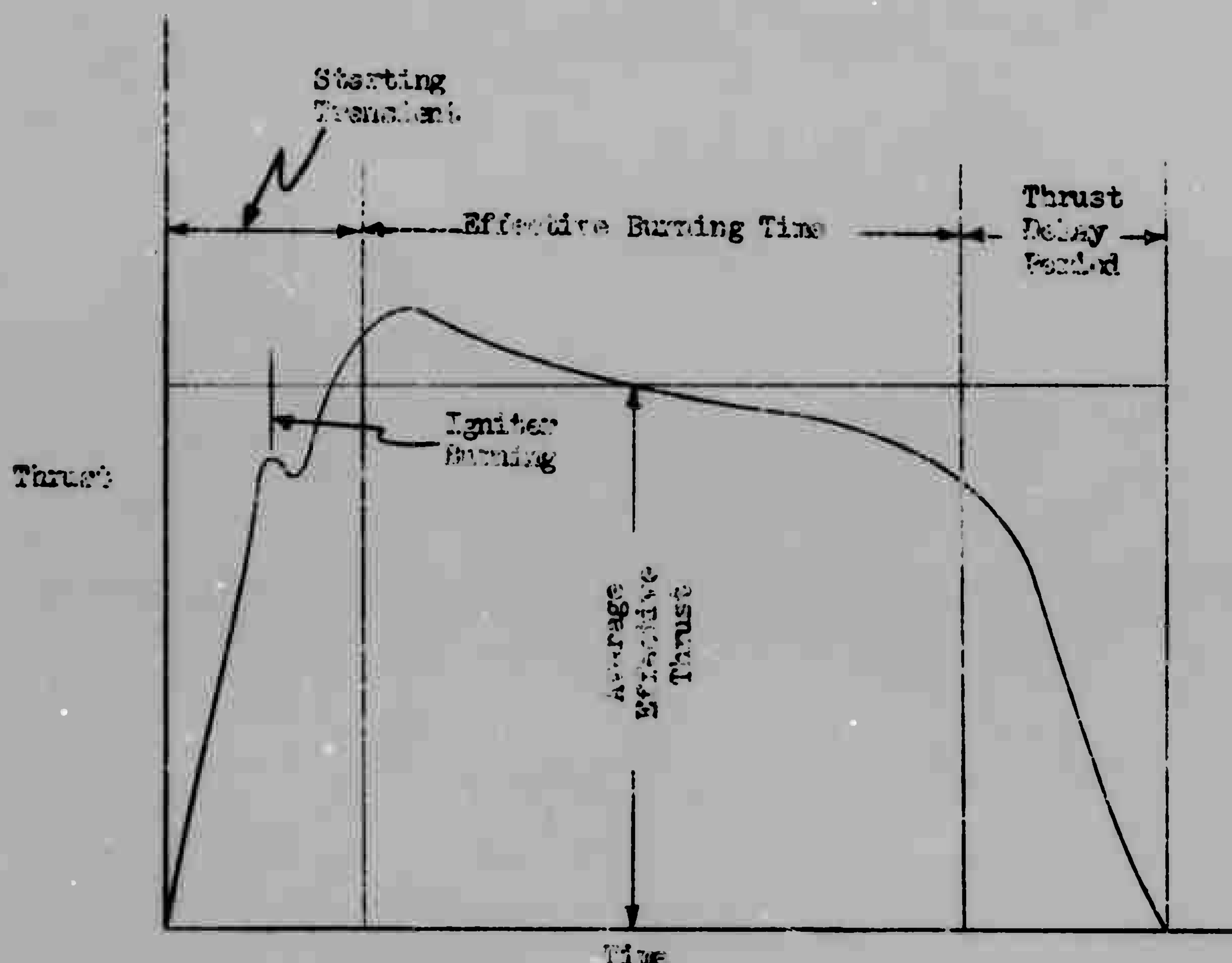


Figure 4.7.10

For many grain designs the exposed burning area changes as the propellant is burned; therefore, causing a change in burning rate and chamber pressure. This will change the thrust time program but they are controllable and prove to be advantageous at times. A typical solid propellant thrust time program is shown in Figure 4.7.11.



Thrust-Time-Program

Figure 4.7.11

Static and dynamic loads will effect the grain configuration. The propellant must be able to withstand normal handling loads as well as those imposed by thermal expansion. Most dynamic loads, such as axial acceleration and combustion gas friction push the grain toward the nozzle end of the chamber. The grain must be able to avoid crumbling which will seriously limit grain geometry, especially a configuration which will allow burning normal to the chamber axis.

4.7.4 ROCKET FUNDAMENTALS

4.7.4a FUNDAMENTAL THRUST EQUATION

Force is defined as a rate of change of momentum, that is, the thrust is equal to the product of mass flow rate ($\dot{m}$) and theoretical exhaust gas velocity (v_w) which is the average velocity across the nozzle exit. The exit velocity and the mass flow rate remains constant.

therefore $F_1 = \dot{m}\ v_w$ Equation 4.7.1

where

$\dot{m}$ = slugs per second

v_w = feet per second, velocity of jet wake

F_1 = momentum thrust in pounds

The momentum thrust does not represent all of the thrust. It often happens that the exit pressure is either greater or smaller than the ambient pressure. Although the exit pressure remains constant with respect to time, the ambient pressure will decrease with height. Any unbalance between the two

pressures will cause a force on the engine. If the difference in pressure were multiplied by the exit area the force could be expressed as follows:

$$F_2 = (p_e - p_a) A_e$$

where

p_e = exit pressure in psia

p_a = ambient pressure in psia

A_e = exit area in square inches

F_2 = pressure thrust in pounds

The total thrust of a rocket engine is the sum of the momentum and pressure thrust, and is expressed by the fundamental thrust equation as

$$F = \mathring{m}\ v_w + (p_e - p_a) A_e \qquad \text{Equation 4.7.2}$$

To obtain the optimum thrust from a rocket engine p_e must equal p_a. This is called optimum expansion. This would only be possible at all altitudes if the nozzle configuration could be varied. When p_e is less than p_a over expansion is said to occur. Normally a rocket will be designed to assure maximum total impulse for the powered phase of the flight. A rocket which changes altitude, such as an ICBM is usually designed to produce over expansion at low altitude and under expansion at higher altitude. There is some intermediate altitude where $p_e = p_a$

and this is the altitude where the optimum thrust is obtained. It must be kept in mind that the optimum thrust is the maximum thrust obtainable at a particular altitude. The rocket engine output or thrust will always be greater at altitude than at sea level. A typical curve is presented in Figure 4.7.12. Line one shows a curve for a rocket engine if p_e could be kept equal to p_a at all times. Line 2 shows a typical rocket of fixed A_e. It can be noted that where $p_e = p_a$ the two are equal, indicating that this is the maximum thrust obtainable at that altitude.

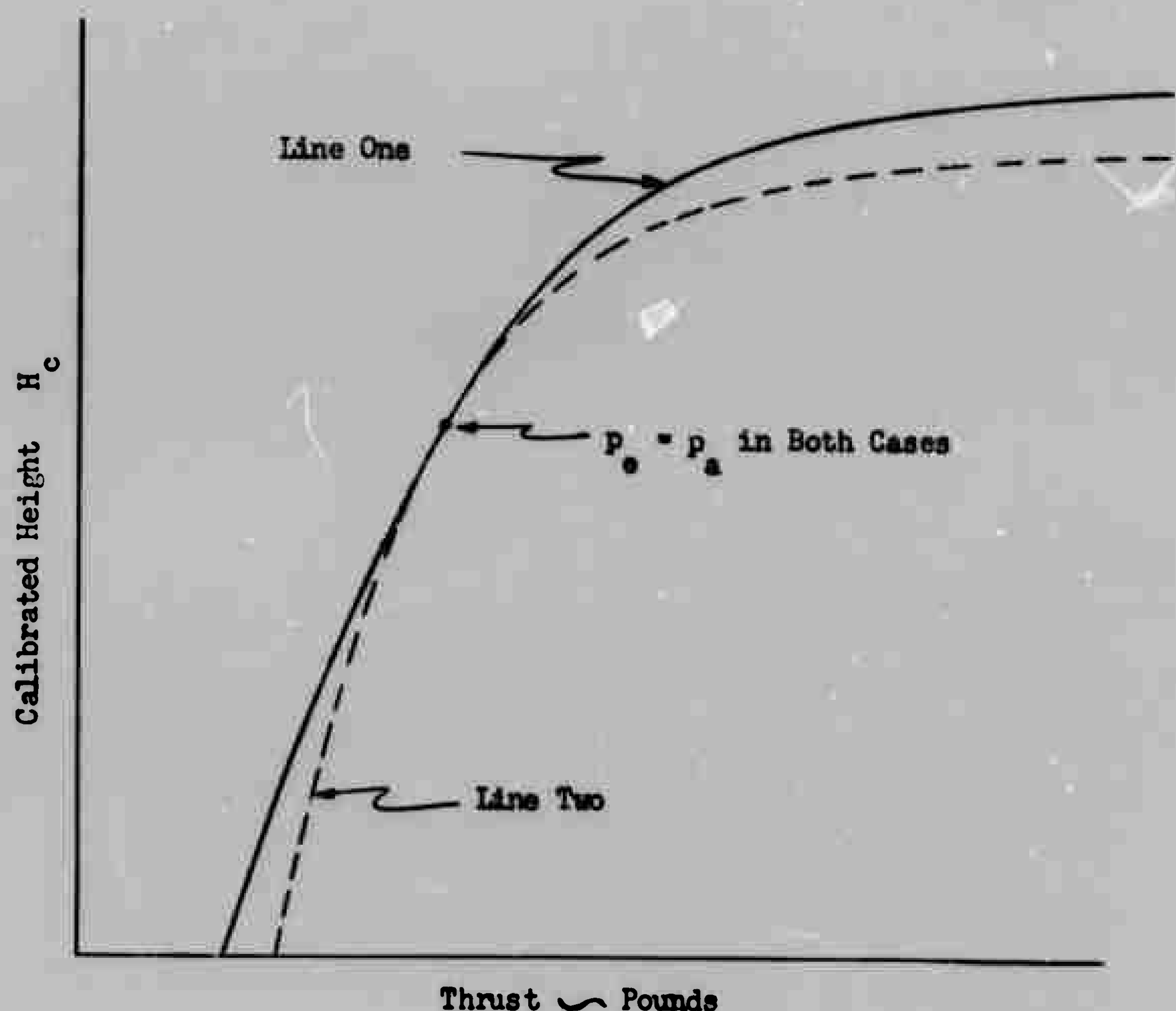


Figure 4.7.12

4.7.4b SPECIFIC IMPULSE

The performance of a rocket is conveniently expressed as specific impulse. The units are lb/lb/sec and is shortened to "sec". The term means the amount of thrust available for each pound of propellant flow per-second.

Shown as

$$I_s = \frac{F}{\dot{W}} \qquad \text{Equation 4.7.3}$$

The effective exhaust velocity "c" is defined as $F/\dot{m}$; therefore

$$I_s = \frac{\dot{m}\, c}{\dot{m}\, g} = \frac{c}{g}. \qquad \text{Equation 4.7.4}$$

To avoid g becoming very small in space and I_s becoming infinitely large, the value of g is set as g_o or 32.2 ft/sec^2;

therefore

$$I_s = \frac{c}{g_o}. \qquad \text{Equation 4.7.5}$$

Specific impulse will vary with altitude even though g is fixed because thrust will vary. A typical curve shows how altitude will change F, c and I_s. The values of $\dot{m}$, v_e, P_e and P_c will remain constant but P_a will decrease.

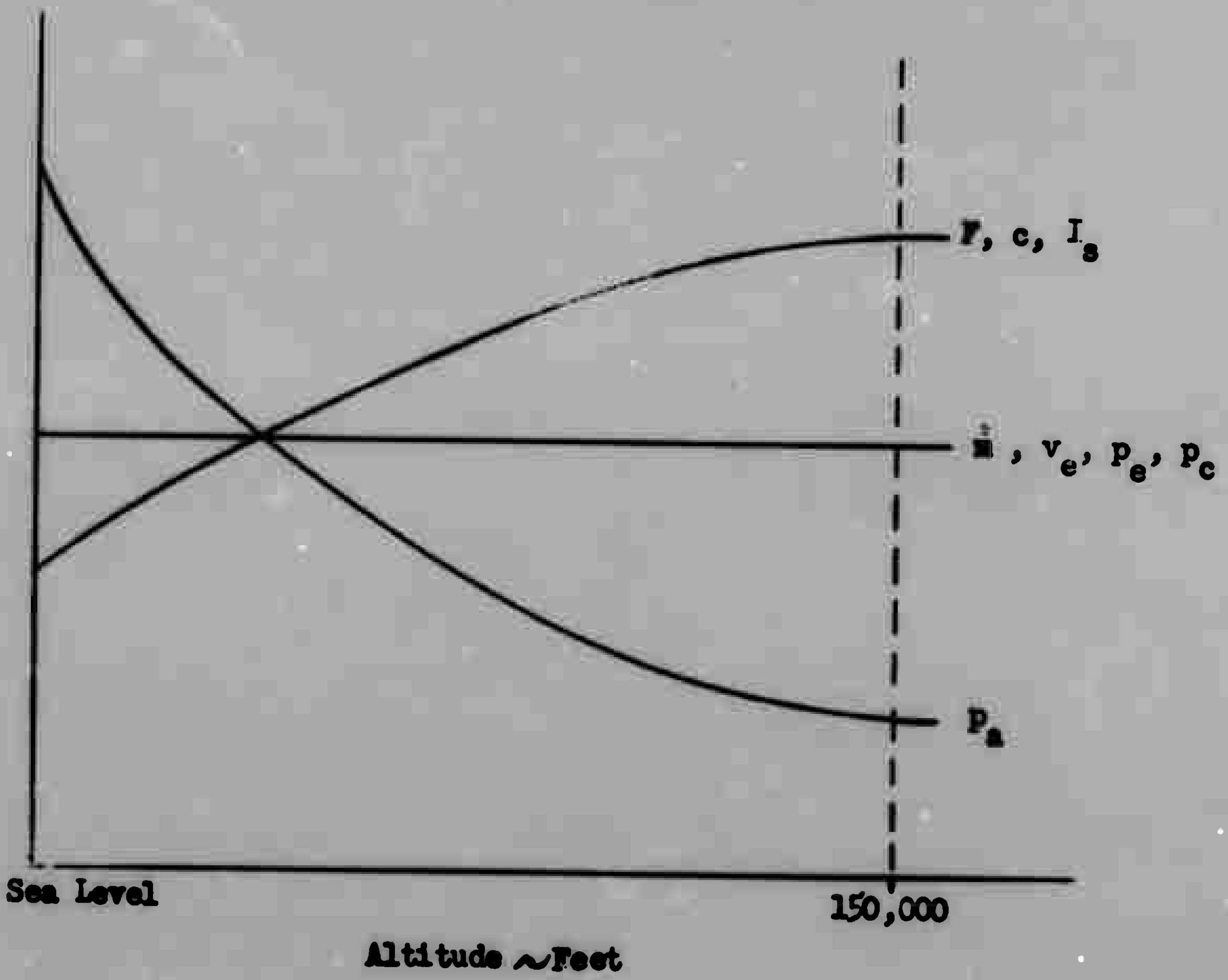


Figure 4.7.13

4.7.5 EFFICIENCY

For a rocket engine the propulsive efficiency may be written as follows:

where $F = \dot{m}\ V_w$

and

mechanical work $= \frac{1}{2}\dot{m}\ (V_o^2 + V_w^2)$

where

V_o = velocity of aircraft

V_w = velocity of jet wake

therefore

$$\eta_p = \frac{\dot{m}\,(V_w \; V_o)}{\frac{1}{2}\dot{m}(V_o^2 + V_w^2)} = \frac{2\left(\frac{V_o}{V_w}\right)}{1 + \left(\frac{V_o}{V_w}\right)^2} \qquad \text{Equation 4.7.6}$$

$$\eta_p = \frac{2\left(\frac{V_w}{V_o}\right)}{1 + \left(\frac{V_w}{V_o}\right)^2} \qquad \text{Equation 4.7.7}$$

if

$V_o > V_w$ the η_p begins to decrease

when

$V_w = 0$ then $\eta_p = 0$

also

$V_w = V_o$ then $\eta_p = 100$ percent

When $V_w = V_o$ then all the energy is utilized in propelling the airplane and no energy is consumed for propelling particles in the wake. The velocity of these wake particles with reference to the earth is then zero.

At zero speed $V_o = 0$ and $V_w/V_o = \infty$ and $\eta_p = 0$ and all of the energy of the rocket is in the jet being used to propel the particles in the wake. Generally η_p of a rocket is low because of the high jet velocity required.

It is desirable to have a η_p and high thrust, but the problem is to get high thrust with as low a V_w/V_o as possible. Figure 4.7.14 shows η_p plotted against the dashed line for a turbojet, ram jet or turboprop as obtained from section 4.3.4.

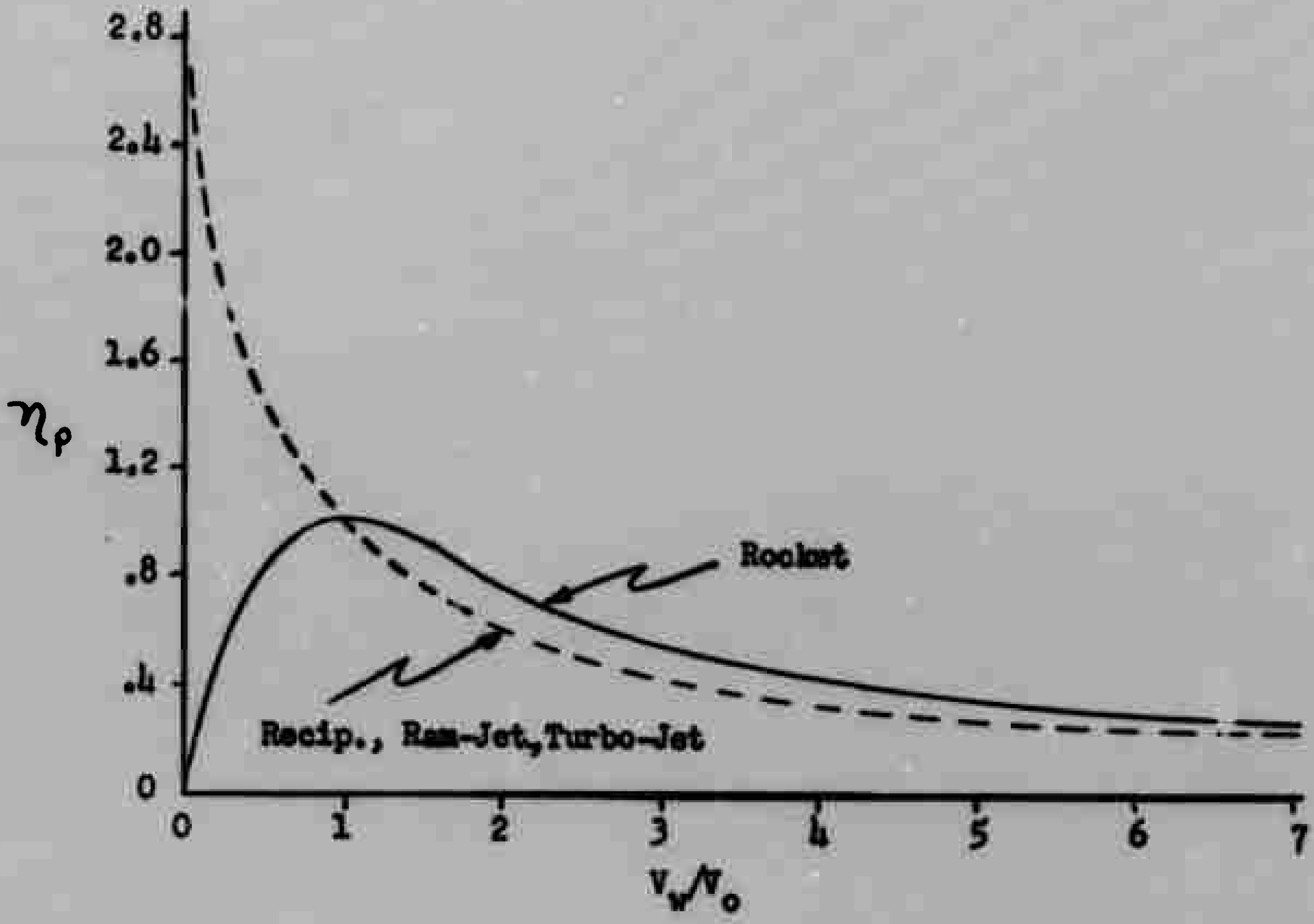


Figure 4.7.14

BLANK PAGE

APPENDIX A

INSTRUMENTATION

A.1 INTRODUCTION

The flight of aircraft requires certain knowledge of engine performance and of the relative motion of the air with respect to the aircraft. In early flights the engine performance was evaluated by sound, the airspeed was evaluated by listening to the wind rushing over the airframe and the fuel quantity was obtained by a simple dip-stick. As obviously limited as these methods are, considerable flying was done under these conditions. It was apparent that better methods were required and were possible. This was the beginning of aircraft instrumentation. Since then a steady evolution of cockpit instrumentation has come about to give us the instruments we have today. Strangely enough the basic theory has not changed significantly since the early days; only the mechanics have been refined. It is the refined versions of these early instruments that we rely on today for flight test.

Many flight tests, particularly performance of low speed aircraft, can be done with only cockpit instrumentation and an agile observer. However, a great number of stability tests which require many readings within a few seconds can not possibly be recorded and even the performance of a modern day fighter exceed the recording ability of a practiced observer. Truly, qualitative evaluation has its place in flight testing particularly in the initial phases but it is necessary to have reliable quantative data to support qualatative results. It is necessary, therefore to provide a means

of taking continuous or frequent intermittant instrument readings. The obvious first step would be to photograph cockpit type of instruments which could be read at liesure at a later time. This in fact has been done for many years and is still being done to some extent, however, the poor response and great lag of some photographable instruments has seriously limited this method of recording data. Other more adequate methods have been devised, such as the oscillograph and magnetic tape. These will be discussed later.

A.2 INSTRUMENT CHARACTERISTICS

Instruments in general have certain common characteristics. All are subject to inaccuracies commonly referred to as instrument error. They are also subject to a more subtle inaccuracy which is known as hysterisis.

All instruments are subject to some inaccuracies. That is, when they are compared to a standard true value, the instrument reads some value slightly different from the standard. The difference between the two is the instrument error. While instrument error in an aircraft instrument is undesirable from an operational point of view, large errors are not objectionable for flight test purposes. If the errors are known and are predictable the instrument readings can be corrected to the true value with no compromise of the data.

Most all instruments are mechanical in some part of their operation. All mechanical linkages have some slop or looseness built into them. For instance, a gear at one end of a gear train (Fig. A.1) may be moved several degrees without moving the gear at the other end. This is generally called hysterisis

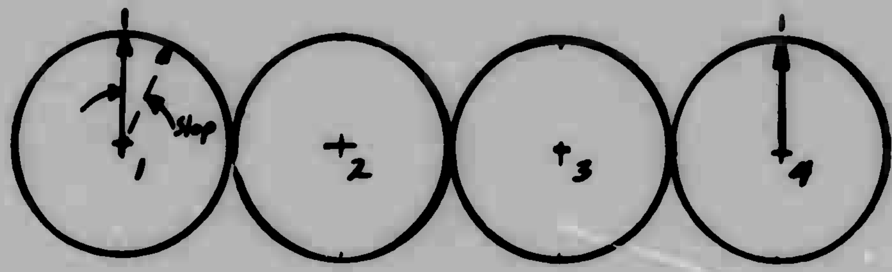


Fig. A.1

because of its similarity to its electrical counterpart. Hysterisis manifests itself in instruments in the following manner. Suppose that the arrow on gear number 4 approaches the mark from the left in which case gear number 1 falls on the mark as shown. Now let gear number 4 approach the mark from the right. Due to the slop in the system the pointer on gear number 1 falls to the right of the mark as shown by the dotted arrow. The difference between the positions of the two arrows is the hysterisis.

Hysterisis is inherent in all mechanical systems. It can be minimized by careful design but it can not be completely eliminated. Hysterisis represents an uncertainty of a given reading and is unpredictable within the limits given by the above procedure. It is therefore desirable to use instruments with near zero hysterisis and to calibrate these instruments in such a way as to determine the hysterisis band of the instrument.

A.3 INSTRUMENT CALIBRATION

A calibration consists of comparing an accepted standard true value to the actual reading of an instrument. In order to evaluate both the error and hysterisis of the instrument the calibration is run both increasing and decreasing in magnitude. The error is generally expressed as correction to be added and is calculated by subtracting the instrument reading from the standard value. The error is plotted as the error versus the indicated reading.

The hysterisis band will appear as the difference between the up and down line. An example of a calibration for an airspeed indicator is shown below.

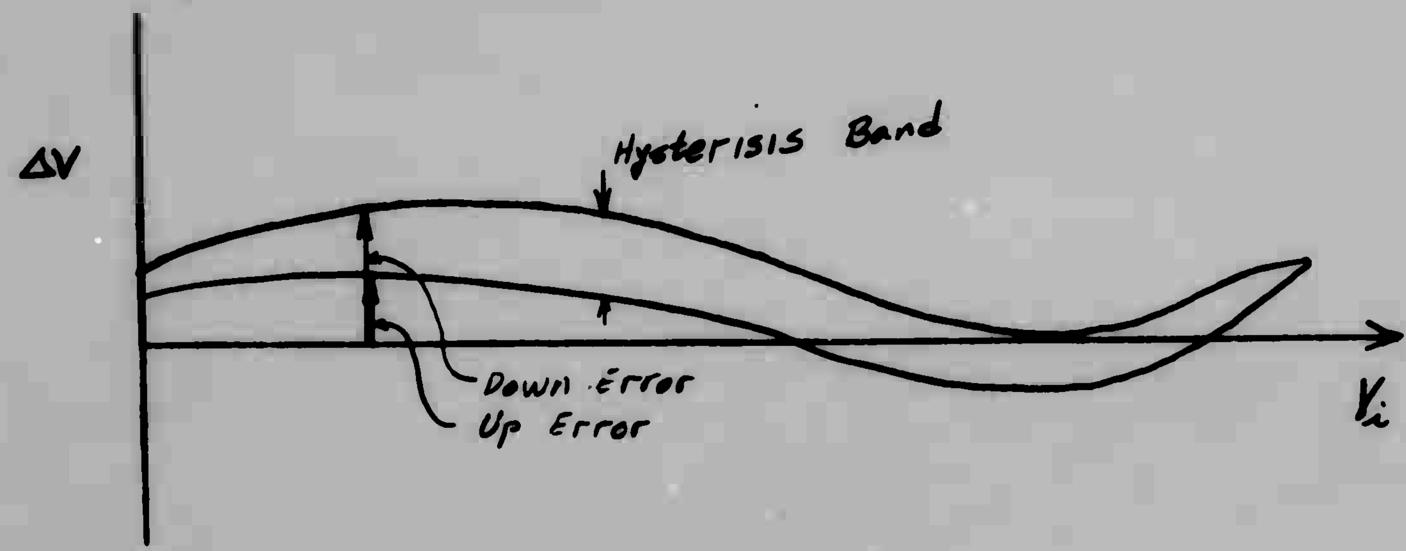


Fig. A.2

An alternate method of presenting calibrations generally used for oscillograph calibrations is to plot the instrument reading versus the standard reading. (Fig. A.3)

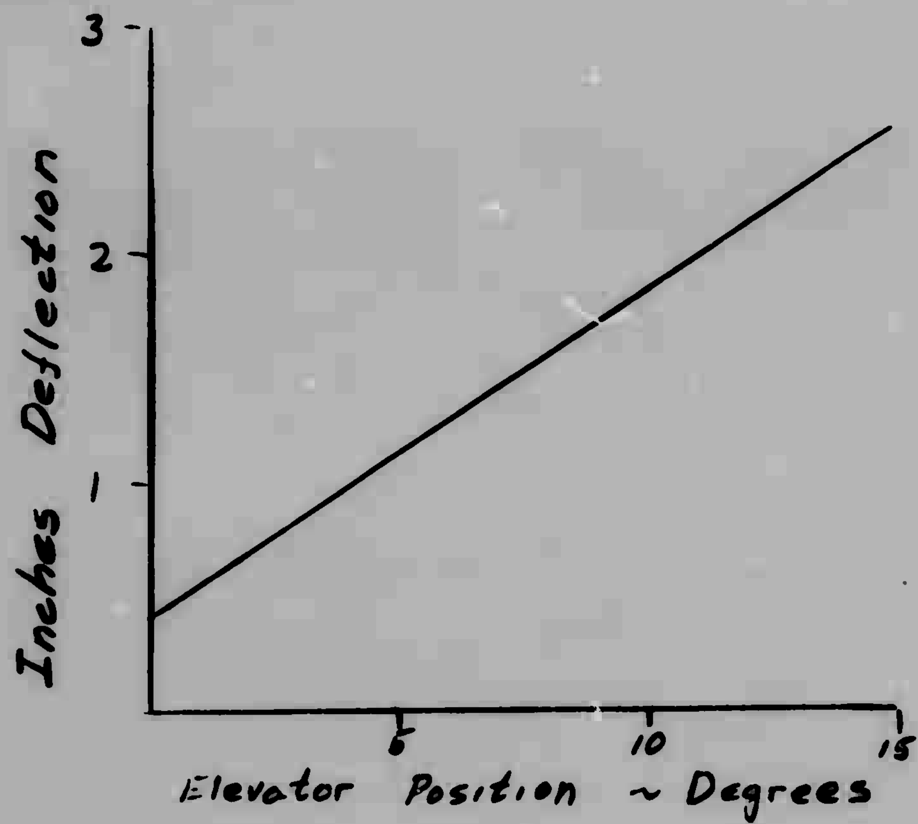


Fig. A.3

This procedure is generally used for instruments which are not specifically designed to read out in dimensional units such as an autosyn which has a general dial reading from 0 to 10 and an oscillograph trace which simply deflects through some displacement in response to a voltage input from a remote indicator.

A.4 INSTRUMENT MECHANICS

There are many schemes used in instruments to obtain reading of desired quantities. For flight test purposes they are generally divided into two convenient groups, that is, those required for performance testing and those required for stability testing. Certainly, this is not a clear cut delineation between types of instruments for there is considerable overlap between them, however, it does serve as a starting point for the discussion of instrumentation systems used.

The primary measurements of interest for performance flight testing are airspeed, altitude, time, free air temperature, engine speed and/or manifold pressure, fuel flow and fuel used. Other measurements like turbine discharge pressure and temperature may be required for some types of aircraft. The above list may be divided into three broad categories for the sake of discussing their internal workings. These are pressure instruments, frequency counting instruments and temperature measuring instruments.

A.4.1 PRESSURE MEASURING INSTRUMENTS

The majority of the measurements needed for performance flight test require accurate measurement of pressure. Parameters of this type are airspeed, altitude, manifold pressure, etc. At the heart of all conventional

cockpit pressure measuring instruments is the bellows. This may either be of the accordian type or the diaphram type. The basic principle of both is to convert a pressure into a measurable displacement. When pressure is applied to the bellows it extends from its initial position X_1 at pressure p_1. (Fig A.4) to X_2 for pressure p_2.

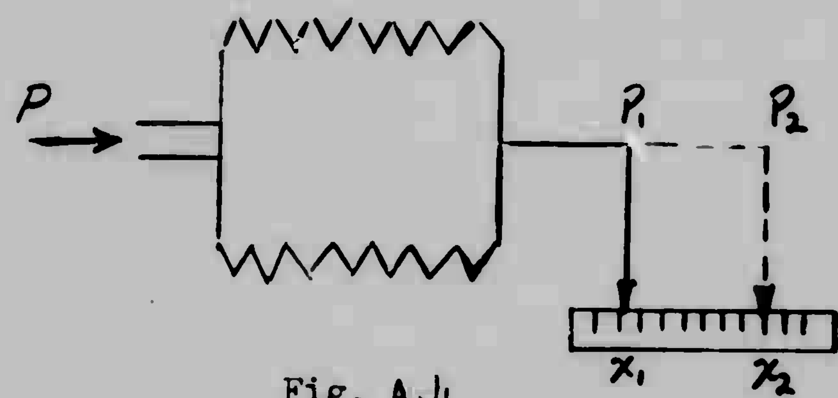


Fig. A.4

This is the system used in the manifold pressure gage where the total displacement of the bellows may be less than two tenths of an inch for a 400 inches of mercury gage. From this we see the great requirement for mechanically amplifying these small displacements into readable quantities on a dial. Since this is done mechanically through a large number of gears and levers some amount of hysterisis may be expected. That is, a given pressure arrived at by increasing from a lower pressure has a lower dial reading than the same pressure arrived at from a higher pressure. It is well to restate here that for flight test work a large instrument error is not undesirable if it is repeatable, that is, it occurs every time. It is the unpredictable or nonrepeatable errors which are of concern in test work.

A.4.2 FREQUENCY COUNTING AND RATE INSTRUMENTS

Instruments of this type are fuel counters and tachometers.

The fuel counter is of the frequency counting type. That is, the revolutions of an anemometer type rotating element in the fuel lines are counted and fed directly to a cockpit and photo panel counter.

The tachometer is a rate sensing instrument and hence operates on a different principle. A small generator whose voltage output is proportional to its speed is mounted on the engine or other unit whose speed is to be measured. Va[illegible] types of sensors may be used in the cockpit the simplest of which would be a volt or amp meter. More refined techniques are used to sense the generator output but they are beyond the scope of this writing.

Fuel flow rates are not measured directly on cockpit indicators for flight test because the present state of the art does not give sufficient accuracy. Rather, the fuel counters are timed for a given number of counts. The fuel flow is then calculated by dividing the fuel quantity given by the counters by the time increment.

A.4.3 TEMPERATURE MEASURING INSTRUMENTS

Temperature measuring instruments are of two types, the thermocouple or the resistance wire. The resistance wire system is usually used in free air temperate systems. It utilizes a wire whose resistance changes with temperature. This resistance change is measured by a sensitive voltage measuring device for cockpit visualization.

The thermocouple takes advantage of the electrical potential that results when two dissimilar metals are brought in contact with each other. If the ends of two different metals are brought together each junction will generate an electrical potential, however, they will oppose each other so that there is no current flow in the circuit. However, if the temperature of one of the junctions is different from the other a current flow will result which is proportional to the temperature difference. This then is the thermocouple principle of temperature measurement. With the proper selection of the materials used, the voltage level of the system can be easily raised to a measurable level to allow accurate temperature measurement. While in principle the thermocouple system is neat and simple requiring no external power it has a few disadvantages. First, it requires special wire from the temperature source junction to the opposite reference junction. Second, because the indicated temperature only gives the temperature difference between the two junctions it is necessary to know the temperature of the reference junction. For accurate measurements the reference junction is generally immersed in an ice bath. This requires additional maintenance of the ice bath.

Blank Page